MAT 2379 3X (Spring 2012)
Assignment 2 - solutions
Deadline: **Thursday, May 30, 2013 (in class)**
There are a total of 5 questions.

[7] 1.

(a) The false positive rate and the false negative rate for this test are respectively

$$P(+|D') = \frac{25}{175} = 0.14286 \quad \text{and} \quad P(-|D) = \frac{5}{175} = 0.02857.$$

(b)

$$\text{specificity} = P(-|D') = \frac{150}{175} = 0.85714$$
$$\text{sensitivity} = P(+|D) = \frac{170}{175} = 0.97143.$$

(c) The prevalence of the disease among these subjects is 0.5, since half of the subjects have the disease.

Remark: In practice, the prevalence in the study of the effectiveness of a diagnostic test is artificial. This is an example of a balanced experimental design where we chose half of the subjects to have the disease and half to not have the disease. The goal of the study is to understand how the diagnostic test reacts to a disease subject and also how it reacts to a non-diseased subject.

(d) We are given that $P(D) = 0.01$. We want

$$\begin{aligned} P(D|+) &= \frac{P(D \cap +)}{P(+)} \\ &= \frac{P(+|D)P(D)}{P(+|D)P(D) + P(+|D')P(D')} \\ &= \frac{(0.97143)(0.01)}{(0.97143)(0.01) + (0.14286)(0.99)} = 0.06427. \end{aligned}$$

Remark: We see in this exercise that when the false positive rate is not very small and the disease is not very prevalent, then the diagnostic test is not a good predictive tool.

[6] 2.

(a) If both parents are slightly frizzled, then the Punnett square is

parent 1	parent 2: $\frac{1}{2}F$	parent 2: $\frac{1}{2}f$
$\frac{1}{2}F$	$\frac{1}{4}FF$	$\frac{1}{4}Ff$
$\frac{1}{2}f$	$\frac{1}{4}fF$	$\frac{1}{4}ff$

The probability that the offspring is frizzled is $P(\{FF\}) = 1/4$.

(b) If only one parent is slightly frizzled and the other is normal, then the Punnett square is

parent 1	parent 2: $\frac{1}{2}f$
$\frac{1}{2}F$	$\frac{1}{2}Ff$
$\frac{1}{2}f$	$\frac{1}{2}ff$

The probability that the offspring is frizzled is $P(\{FF\}) = 0$.

(c) if only one parent is slightly frizzled and the other is frizzled, then the Punnett square is

parent 1	parent 2: F
$\frac{1}{2}F$	$\frac{1}{2}FF$
$\frac{1}{2}f$	$\frac{1}{2}Ff$

The probability that the offspring is frizzled is $P(\{FF\}) = 1/2$.

[4] 3. By the definition of independent events, we have

$$P(A \cap B) = P(A)P(B) = (0.35)(0.75) = 0.2625$$

and by the addition rule, we have

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.35 + 0.75 - 0.2625 = 0.8375.$$

[4] 4.

(a) Since the total probability must be 1, then

$$1 = p + 2\,p + 3\,p + p = 7\,p.$$

Thus, $p = 1/7$.

(b) $P(X > 1) = P(X = 2) + P(X = 3) = 3/7 + 1/7 = 4/7 = 0.5714$.

(c) The mean of X is

$$E[X] = 0(1/7) + 1(2/7) + 2(3/7) + 3(1/7) = 11/7 = 1.5714.$$

(d) We will start by computing the variance

$$\begin{aligned} V[X] &= \left(\sum x^2\, f(x)\right) - \left(\sum x\, f(x)\right)^2 \\ &= \left(0^2(1/7) + 1^2(2/7) + 2^2(3/7) + 3^2(1/7)\right) - \left(\frac{11}{7}\right)^2 \\ &= \left(\frac{23}{7}\right) - \left(\frac{11}{7}\right)^2 = 0.81633. \end{aligned}$$

Thus, the standard deviation of X is

$$\sigma_X = \sqrt{V[X]} = \sqrt{0.81633} = 0.9035.$$

[4] 5. Let X be the number of patients among $n = 20$ for which the medication reduces the pain. Under the assumption that the medication has no effect, X has a binomial distribution with $n = 20$ and $p = 0.5$. The probability that at least 17 of 20 patients would report a significant reduction in their pain is

$$\begin{aligned} & P(X \geq 17) \\ = \; & P(X = 17) + P(X = 18) + P(X = 19) + P(X = 20) \\ = \; & \binom{20}{17}(0.5)^{17}(0.5)^3 + \binom{20}{18}(0.5)^{18}(0.5)^2 + \binom{20}{19}(0.5)^{19}(0.5)^1 \\ & + \binom{20}{20}(0.5)^{20}(0.5)^0 \\ = \; & 0.00129 \end{aligned}$$

[/**25**]