(1)

$$
\begin{aligned}
F(s) &= \frac{4}{s(2s^2+8)} + \frac{18}{2s^2+10s+8} \\
&= \frac{2}{s(s^2+4)} + \frac{9}{s^2+5s+4} \\
&= \frac{a}{s} + \frac{bs+c}{s^2+4} + \frac{9}{(s+1)(s+4)} \\
&= \frac{a}{s} + \frac{bs+c}{s^2+4} + \frac{d}{s+1} + \frac{e}{s+4}
\end{aligned}
$$

$$
a(s^2+4) + bs^2 + cs = 2
$$

$$
\therefore (a+b)s^2 + cs + 4a = 2
$$

$$
\therefore \begin{cases} a+b=0 \\ c=0 \\ 4a=2 \end{cases}
$$

$$
\Rightarrow a = \frac{1}{2}; b = -\frac{1}{2}; c = 0
$$

$$
d = (s+1)\frac{9}{(s+1)(s+4)}\bigg|_{s=-1} = 3
$$

$$
e = (s+4)\frac{9}{(s+1)(s+4)}\bigg|_{s=-4} = -3
$$

$$
\begin{aligned}
\therefore F(s) &= \frac{1}{2s} - \frac{s}{2(s^2+4)} + \frac{3}{s+1} - \frac{3}{s+4} \\
&= \frac{1}{2s} - \frac{1}{2}\frac{s}{s^2+2^2} + \frac{3}{s+1} - \frac{3}{s+4}
\end{aligned}
$$

$$
\therefore f(t) = \frac{1}{2}u(t) - \frac{1}{4}\cos 2t + 3e^{-t} - 3e^{-4t}
$$

(2) Take Laplace transform on both sides:

$$s^2 \cdot 4Y(s) + 8sY(s) + 20Y(s) = \frac{12}{s}$$
$$s^2Y(s) + 2sY(s) + 5Y(s) = \frac{3}{s}$$

Noting that

$$L(\ddot{y}) = s^2Y(s) - sy(0) - \dot{y}(0)$$
$$= s^2Y(s)$$
$$L(\dot{y}) = sY(s) - y(0)$$
$$= sY(s)$$

$$\therefore Y(s) = \frac{3}{s(s^2 + 2s + 5)}$$
$$= \frac{a}{s} + \frac{bs + c}{s^2 + 2s + 5}$$

$$\therefore as^2 + 2as + 5a + bs^2 + cs = 3$$
$$(a + b)s^2 + (2a + c)s + 5a = 3$$

$$\begin{cases} a + b = 0 \\ 2a + c = 0 \Rightarrow a = \frac{3}{5}; b = -\frac{3}{5}; c = -\frac{6}{5} \\ 5a = 3 \end{cases}$$

$$\therefore Y(s) = \frac{3}{5} \cdot \frac{1}{s} + \frac{-\frac{3}{5}s - \frac{6}{5}}{s^2 + 2s + 5}$$
$$= \frac{3}{5} \cdot \frac{1}{s} + \frac{-\frac{3}{5}(s + 2)}{(s + 1)^2 + 2^2}$$
$$= \frac{3}{5} \cdot \frac{1}{s} - \frac{3}{5}\left[\frac{s + 1}{(s + 1)^2 + 2^2} + \frac{1}{(s + 1)^2 + 2^2}\right]$$
$$= \frac{3}{5} \cdot \frac{1}{s} - \frac{3}{5}\frac{s + 1}{(s + 1)^2 + 2^2} - \frac{3}{10}\frac{2}{(s + 1)^2 + 2^2}$$

$$y(t) = \frac{3}{5}u(t) - \frac{3}{5}e^{-t}\cos 2t - \frac{3}{10}e^{-t}\sin 2t$$

(3)

When b = 10

$$F(s) = \frac{1}{s^2 + 10s + 17}$$

$$p = \frac{-10 \pm \sqrt{100 - 68}}{2}$$

The poles are real numbers.

When b = 2

$$F(s) = \frac{1}{s^2 + 2s + 17}$$

$$p = \frac{-2 \pm \sqrt{4 - 68}}{2}$$

The poles are complex numbers.

So

$$\begin{aligned} F(s) &= \frac{1}{s^2 + 2s + 17} \\ &= \frac{1}{(s+1)^2 + 4^2} \\ &= \frac{1}{4} \cdot \frac{4}{(s+1)^2 + 4^2} \end{aligned}$$

$$\therefore\ f(t) = \frac{1}{4} e^{-t} \sin 4t$$

(4)

a)

$$F - kx = m\ddot{x}$$

$$\because F = 3\delta(t), \quad m = 1, \quad k = 4$$

$$\therefore \ddot{x} + 4x = 3\delta(t)$$

Take Laplace transform on both sides.

$$s^2 X(s) - sx(0) - \dot{x}(0) + 4X(s) = 3$$

$$\because x(0) = 0.1; \quad \dot{x}(0) = 0.01$$

$$\therefore s^2 X(s) - 0.1s - 0.01 + 4X(s) = 3$$

$$\therefore X(s) = \frac{3.01 + 0.1s}{s^2 + 4}$$

b)

$$\omega^2 = 4$$
$$\omega = 2$$
$$f = \frac{\omega}{2\pi} = \frac{1}{\pi} = 0.318(Hz)$$

c)

$$sF(s) = \frac{0.1s^2 + 3.01s}{s^2 + 4}$$
$$p_1 = 2j; p_2 = -2j$$

No steady value exists, because sF(s) have poles on the imaginary axis.