Lecture 2
Commerce 308: Introduction to Finance
TIME VALUE OF MONEY
Part 1

Somayeh Kaviani

Summer 2013

Agenda

- 1- What is interest
- 2- Time value of money
- 3- Discounting vs. Compounding
- 4- Annuities and Perpetuities
- 5- Some examples

The Time Value of Money

- Money NOW is worth more than money LATER!
  - Because of the risk involved with investing
    - dollar received today can be invested to earn interest (or consumed),
  - Because of the opportunity cost of money
    - The opportunity cost of money is the interest rate that would be earned by investing it.
  - Because of inflation

- We also call the interest rate the price of money.
- Time value of money quantifies the value of a dollar through time

Terminology

- PV (present value)
- FV / FV_n (future value)
- k (discount rate/ interest rate)
- n (number of periods)

Present Value

- A sum of money today is called a present value.
  - We designate it mathematically with a subscript, as occurring in time period 0
  - For example: $PV_0 = 1{,}000$ refers to $\$1{,}000$ today

Future Value

- A sum of money at a future time is termed a future value
  - We designate it mathematically with a subscript showing that it occurs in time period n.
  - For example: $FV_n = 2{,}000$ refers to $2,000 after n periods from now.

Number of Periods (n)

- As already noted, the number of time periods in a time value problem is designated by n.
  - n may be a number of years
  - n may be a number of months
  - n may be a number of quarters
  - n may be a number of <u>any</u> defined time periods

Interest rate/ Discount rate (k)

- The interest rate in a time value problem is designated by k
- Different ways to refers to k
  - Opportunity cost of capital
  - Required rate of return
  - Cost of capital
  - Appropriate discount rate
  - Hurdle rate
  - Capitalization rate
  - Etc.

Future value or present value: Investing for a Single Period

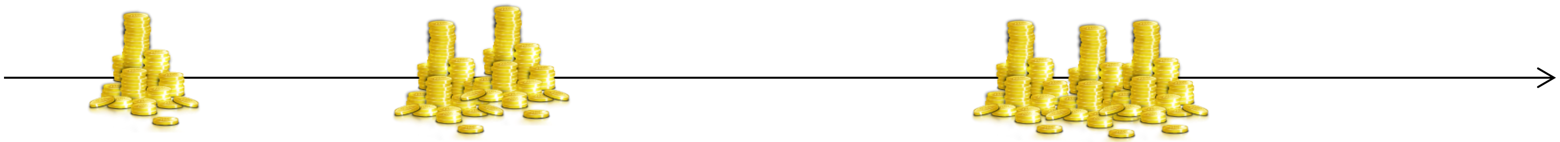
- $k \equiv$ interest rate (nominal)
- $PV_0 \equiv$ \$ to be invested today
- $FV_1 \equiv$ \$ available next period from investment

A simple illustration/ one-period investment:

$$FV_1 = PV_0 + PV_0 \cdot k = PV_0 \cdot (1+k)$$ **Compounding**

$$PV_0 = \frac{FV_1}{(1+k)}$$ **Discounting**

Multi-Period



$t = 0$

$PV = \$\boldsymbol{X}$

$k = k\%$

$t = 1$

$FV_1 = \$\boldsymbol{X} * (1 + k)^1$

$t = 2$

$FV_2 = \$\boldsymbol{X} * (1 + k)^1 * (1 + k)^1$

$FV_2 = \$\boldsymbol{X} * \ (1 + k)^2$

...

$t = n$

$$FV_n = \$\boldsymbol{PV} * \ (1 + k)^n \ \Rightarrow \ PV = FV_n / \ (1 + k)^n$$

An example

- If you invest $1,000 today at an interest rate of 10 percent, how much will it grow to be after 5 years?

Summarize the problem:

1. $FV_5 = ?$
2. $(n=5)$

$PV=1000$

Solution: $FV_n = PV_0(1+k)^n$

$FV_n = 1{,}000(1.10)^5$

$FV_n = \$1{,}610.51$

Another example…

- Assume you will receive a sum of $100,000, six years from now. How much could you borrow from a bank today and spend now, such that that money will be exactly enough to pay off the loan plus interest when it is received? Assume the bank charges an interest rate of 12 percent?
- First- Summarize:
  - $N = 6\ years$
  - $\boldsymbol{PV} = ?$
  - $FV_6 = \$100{,}000$
  - $k = 12\% = 0.12$

- Summary:

  $N = 6\ years$ | $\boldsymbol{PV} = ?$ | $FV_6 = \$100{,}000$ | $k = 12\% = 0.12$

- Solution:
- $FV_n = PV_0(1+k)^n$ so, $PV_0 = FV_n/(1+k)^n$
- $PV_0 = 100{,}000/(1.12)^6$
- $PV_0 = \mathbf{\$50,663}$

Another example…

- How long will it take for $10,000 to grow to $20,000 at an interest rate of 15% per year?

- $\boldsymbol{FV_n = PV_0(1+k)^n}$
- $20{,}000 = 10{,}000(1.15)^n$
- $2 = (1.15)^n$
- $\ln(2) = n * \ln(1.15)$
- $n = \ln(2)/\ln(1.15) = \mathbf{4.96\ years}$

Simple Interest

- Simple interest is interest paid or received on only the initial investment (or principal)
- At the end of the investment period, the principal plus interest is received

Simple vs Compound interest

- *Example: $1,000,000 invested in :*
  - *K=5%, Simple Interest,*
  - *K=5%, Compound Interest,*
  - *N=3 years*

- $Interest\ on\ interest = FV_{Compound\ Interst} - FV_{Simple\ Interest}$

- Simple Interest Formula: $FV_n = PV_0 + PV_0 \times \mathrm{k} \times n$
- Compound Interest Formula: $FV_n = PV_0 \times (1 + k)^n$

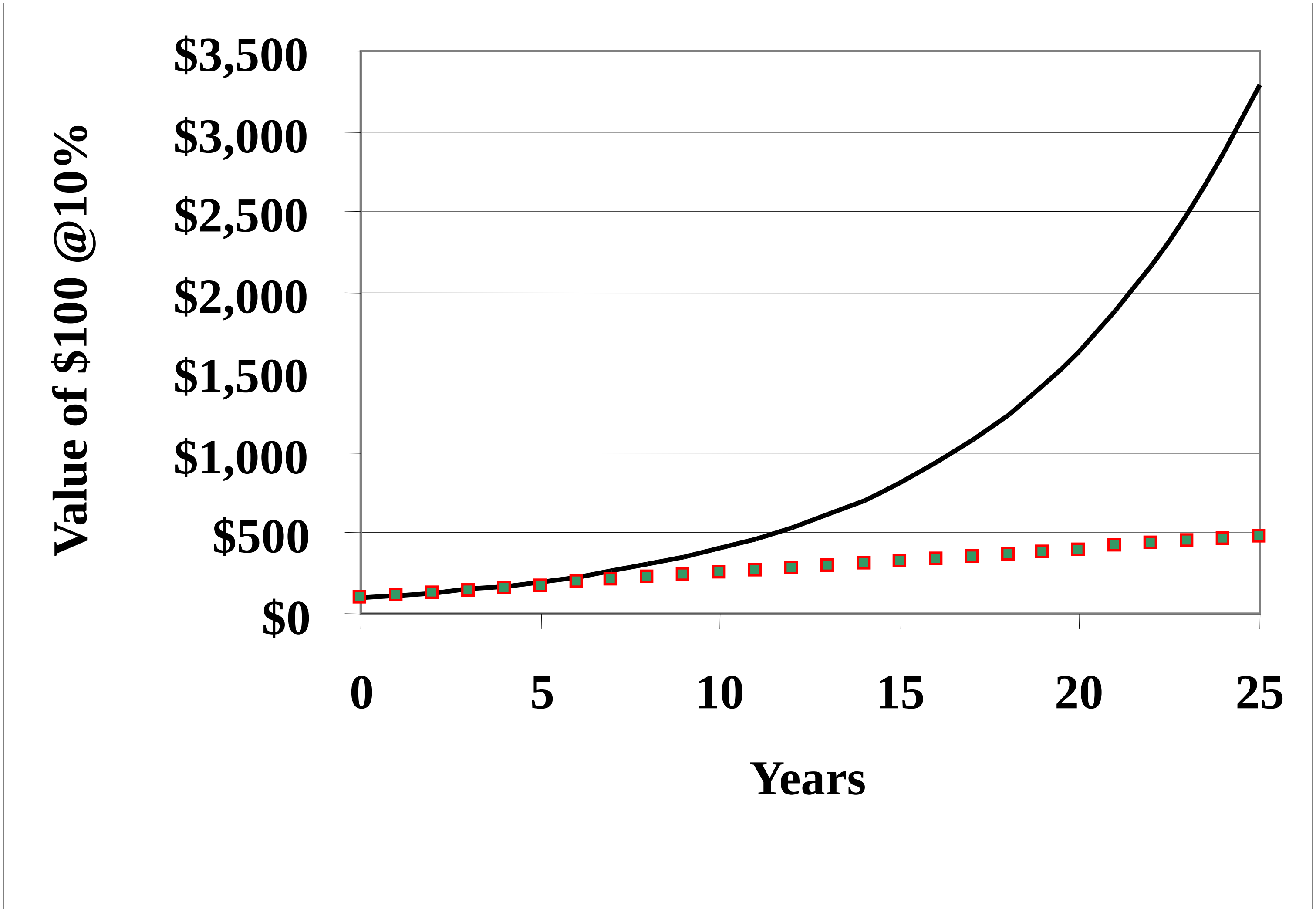
Example

$$PV_0 = 1000$$
$$k = 8\% = 0.08$$
$$n = 5$$
$$FV_{5,Compound} = ? \quad \& \quad FV_{5,Simple=?} \quad \& \quad Interest\ on\ Interest = ?$$

$$PV_5 = PV_0 \cdot (1+k)^5$$
$$= 1,000 \cdot (1.08)^5$$
$$= 1,000 \cdot 1.4693$$
$$= \$1,469.3$$

$$PV_5 = PV_0 + PV_0 \cdot k \cdot n$$
$$= \$1,000 + \$1,000 \cdot 0.08 \cdot 5$$
$$= \$1,000 + \$400$$
$$= \$1,400$$

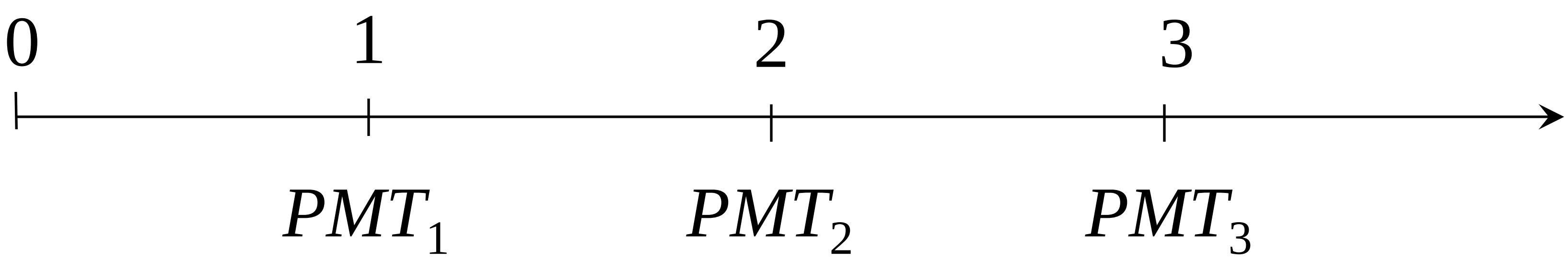
Compound vs. Simple Interest



MULTIPLE CASHFLOWS

Multiple Cash Flows

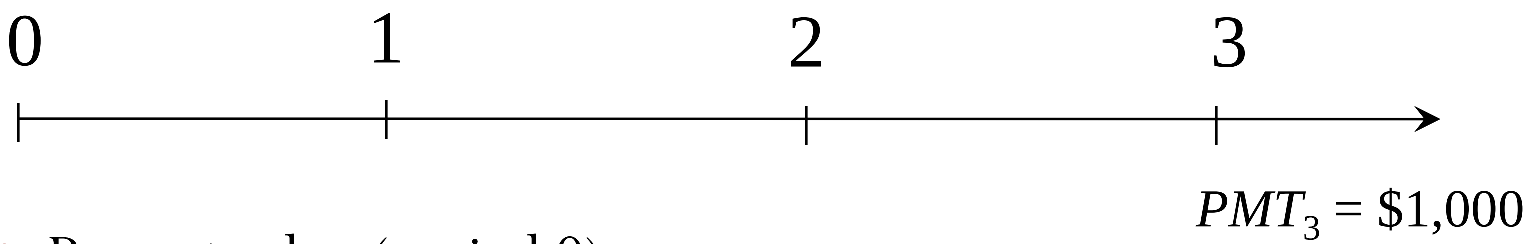
$PMT_t \equiv$ cash flow occuring at time t



PV_t will represent value of cash flow stream

$\{PMT_t\}$ at t point in time

Multiple Cash Flows



- Present value (period 0):

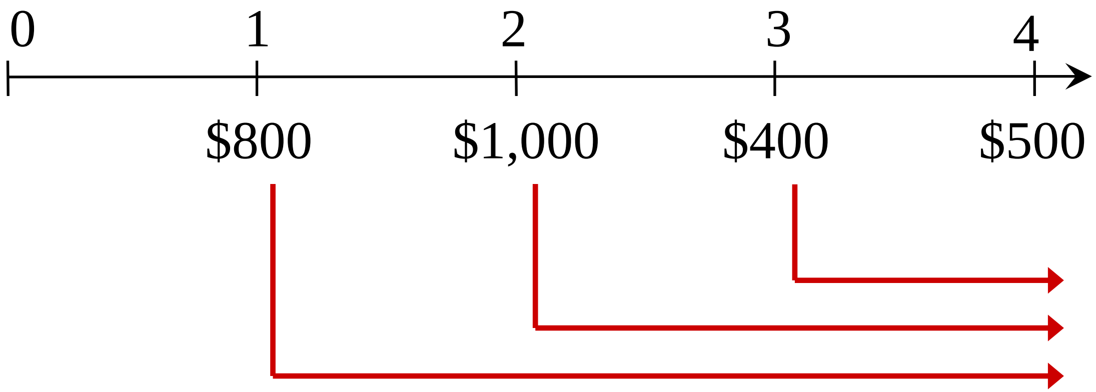
$$PV_0 = \frac{PMT_3}{(1+k)^3} = \frac{\$1{,}000}{(1.10)^3} = \$751.31$$

- Value in period 2:

$$PV_2 = \frac{PMT_3}{1+k} = \frac{\$1{,}000}{1.10} = \$909.09$$

- PV_t : price you would be willing to pay at time t for stream of cash flows

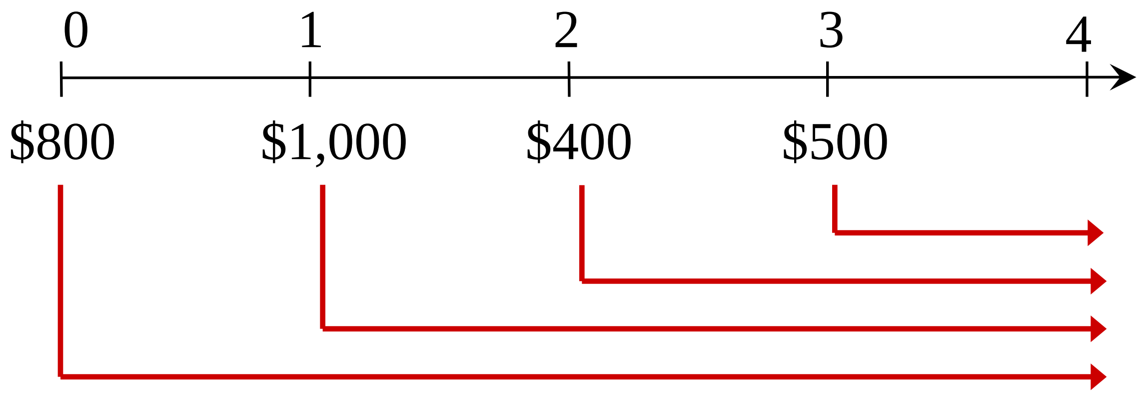
Future Value (FV_t)



$$FV_4 = ? \Rightarrow$$

$$\begin{aligned} FV_4 &= \$800 \cdot (1.08)^3 + \$1,000 \cdot (1.08)^2 + \$400 \cdot 1.08 + \$500 \\ &= \$1,007.77 + \$1,166.4 + \$432 + \$500 \\ &= \$3106.17 \end{aligned}$$

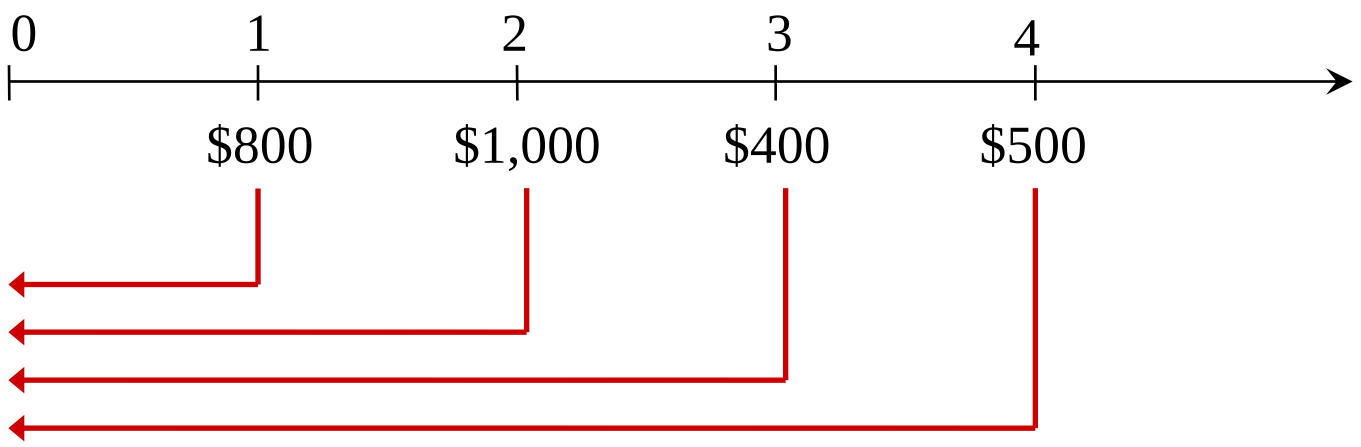
FV of Payments "DUE"



$$FV_4^{DUE} = ? \Rightarrow$$

$$\begin{aligned} FV_4^{DUE} &= \$800 \cdot (1.08)^4 + \$1,000 \cdot (1.08)^3 + \$400 \cdot (1.08)^2 + \$500 \cdot 1.08 \\ &= 1.08 \cdot \left[\$800 \cdot (1.08)^3 + \cdots \quad \cdots + \$500 \right] = 1.08 \cdot FV_4 \\ &= 1.08 \cdot \$3106.17 = 3354.66 \end{aligned}$$

Present Value (PV)



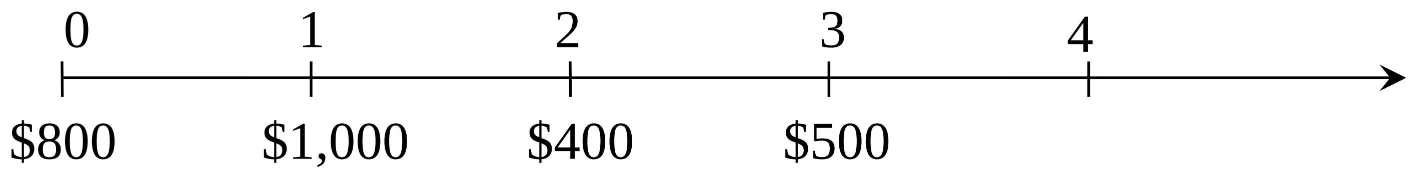
$$PV_0 = \frac{\$800}{(1.08)} + \frac{\$1,000}{(1.08)^2} + \frac{\$400}{(1.08)^3} + \frac{\$500}{(1.08)^4}$$

$$= 740.74 + 857.34 + 317.53 + 367.51$$

$$= 2,283.13 \text{ or:}$$

$$PV_0 = \frac{FV_n}{(1+k)^n} = \frac{3106.17}{(1.08)^4} = 2,283.13$$

PV of Payments “DUE”



$$PV_0^{DUE} = \$800 + \frac{\$1,000}{(1.08)} + \frac{\$400}{(1.08)^2} + \frac{\$500}{(1.08)^3}$$

$$= 1.08 \times \left[\underbrace{\frac{\$800}{(1.08)} + \frac{\$1,000}{(1.08)^2} + \frac{\$400}{(1.08)^3} + \frac{\$500}{(1.08)^4}}_{PV_0} \right]$$

$$= 1.08 \cdot PV_0$$

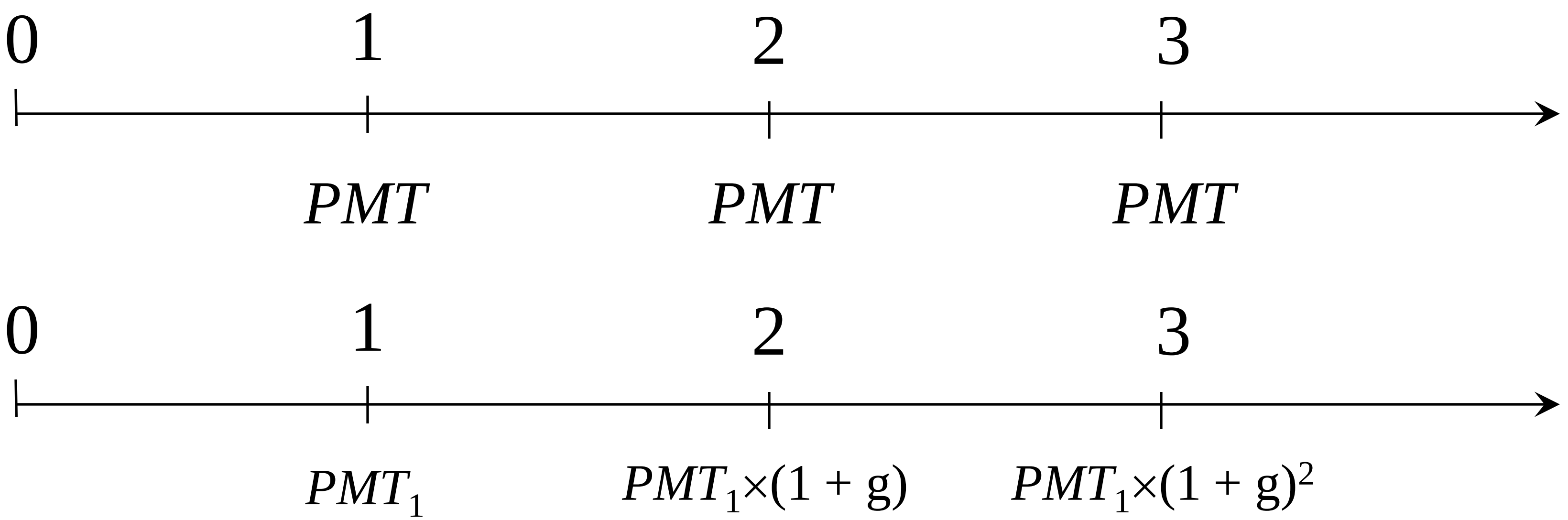
PV of end-of-period CFs × one plus the interest rate

$= 2465.78$

- Perpetuities and Annuities

Perpetuities and Annuities

- Stream of cash flows must be discounted one by one unless:
  - Cash flow are the ***same*** in all periods
  - Cash flows increase at a ***constant rate***

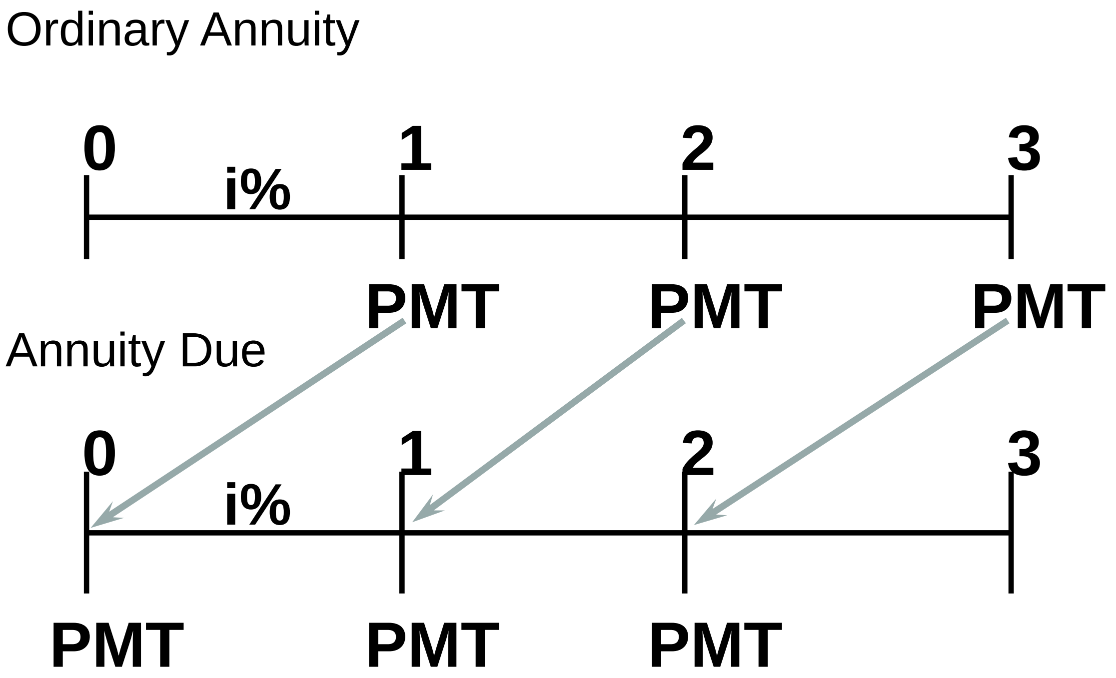


Multiple Cash Flows

- Annuity
  - an amount of money that occurs (received or paid) in equal amounts at equally spaced time intervals.
- For example:
  - If you make payments of $2,000 per year into a retirement fund, it is an annuity.
  - If you receive pension checks of $1,500 per month, it is an annuity.
  - If an investment provides you with a return of $20,000 per year, it is an annuity.
- Growing Annuity
  - pays increasing sum each period for T periods: $PMT_{t+1} = PMT_t(1 + g)$

Multiple Cash Flows

- An annuity due has payments made at the beginning of each period.



Example:

- At the age of 65 your grandfather decides to retire and use the money he saved in his RRSPs. He decided to get a fixed amount every quarter starting the day he retires. What type of payment is this?
- A. an ordinary annuity
- B. an annuity due
- C. a reverse ordinary annuity
- D. a reverse annuity due

Multiple Cash Flows

- Perpetuity
  - an annuity that continues forever (perpetually).

- Growing perpetuity
  - A growing annuity that continues forever (perpetually).

Perpetuities and Annuities

Asset Type — Present Value Formula

- **Perpetuity:** $$PV_0 = \frac{PMT_1}{k}$$
- **Annuity:** $$P_0 = \frac{PMT_1}{k}\left[1 - \frac{1}{(1+k)^n}\right]$$
- **Growing Perpetuity:** $$P_0 = \frac{PMT_1}{k-g}$$
- **Growing Annuity:** $$P_0 = \frac{PMT_1}{k-g}\left[1 - \frac{(1+g)^n}{(1+k)^n}\right]$$
- **Annuity Due:** $$PV_0 = PMT\left[\frac{1 - \frac{1}{(1+k)^n}}{k}\right](1+k)$$
- **FV Annuity Due:** $$FV_n = PMT\left[\frac{(1+k)^n - 1}{k}\right](1+k)$$

Using the calculator

N	I/Y	PV	PMT	FV
4	10	0	500	CPT→ -2320.5

- Notice that N is equal to the nb of PMTs

Some important points:

- Whenever u compute PV , it is one Period before the 1^{st} PMT.
- So if you re computing the PV of an annuity starting at year 4. the PV you get is at t=3, and to compute the PV now you will need to discount it 3 years back!

- Interest rates in the following questions are compound rates unless otherwise stated

GOLDEN rule!

- Whenever you have PMTs , N and I/Y Follow PMT.

- So if your PMT is Monthly, N should be in months and I/Y should be Monthly.

Example

The Montreal Financial Services Company offers a perpetuity of $5,000 per year with the first payment in one year. If your opportunity cost is 8% compounded annually. The present value of the perpetuity today is:

- A. 57,500
- B. 62,500
- C. 67,500
- D. 125,000

- Ans: B. 5,000 / .08 = 62,500

Example

Xiang invests $25,000 per year, starting in one year, for 20 years at an interest rate of 7%. What is the value of the investment at the end of the 20 years?

- A. $1,096,629.42
- B. $1,024,887.31
- C. $283,389.88
- D. $264,850.36

- Ans: B. N=20, I/Y = 7, PMT = 25,000, ordinary, FV = $1,024,887.308

Example

Marie is considering investing a part of her future income in a investment account that offers 0.5 percent a month. She will start work in 6 months and her contract extends for 2 years. If the investment amount is 300 dollars a month, what is the present value of this investment?

- A. $ 6768.86
- B. $ 8338.22
- C. $ 6569.30
- D. $ 8903.62

- Ans: C. N=24,I/Y=0.5,PMT=300, PV1=$ 6768.86, PV=PV1*((1+0.5%)^-6)=$ 6569.30

Next session

- 6- Interest rate handling
- 7- Effective vs. Nominal interest rate
- 8- Annuity Tricks
- 9- More examples
- 10- Mortgages and Loans