# ASSIGNMENT 2 - SOLUTIONS

## Problem 1

a) in a single shot bertrand game $P_1 = P_2 = 0$

b) strategy for firm 1

- at $t=1$ set $p_i = 10$
- at $t \neq 1$ set $p_i = 10$ as long as $(p_1 = 10, p_2 = 10)$ in every previous period; otherwise set $p_i = 0$

these strategies sustain collusion as long as

$$10N \leq \frac{10N}{2} + \frac{10N}{2}\delta_i + \frac{10N}{2}\delta_i^2 + \frac{10N}{2}\delta_i^3 + \dots$$

$$10N \leq \frac{10N}{2}\frac{1}{1-\delta_i} \rightarrow 1-\delta_i \leq \frac{1}{2}$$

this is the minimum discount rate. $\rightarrow$ $\boxed{1/2 \leq \delta_i}$ $i=1,2$

c) in a single shot bertrand game

$p_1 = 4 - \varepsilon \quad p_2 = 4$

$\hookrightarrow$ firm one gets the whole market.

d)

Strategy for firm 1 $\left\{\right.$
- at $t=1$ set $p_1 = 10$
- at $t \neq 1$ set $p_1 = 10$ as long as $(P_1 = 10,\ P_2 = 10)$ in every previous period ; otherwise set $p_1 = 4 - \varepsilon \approx 4$ ($\varepsilon$ very small)

Strategy for firm 2 $\left\{\right.$
at $t=1$ set $p_2 = 10$
at $t \neq 1$ set $p_2 = 10$ as long as $(P_1 = 10,\ P_2 = 10)$ in every previous period, otherwise set $p_2 \geq 4$ → why?

Firm 1 will not deviate if

$$10N + 4N\delta_1 + 4N\delta_1^2 + 4N\delta_1^2 + \dots \leq \frac{10N}{2} + \frac{10N}{2}\delta_1 + \frac{10N}{2}\delta_1^2 + $$

$$10N + \frac{4N\delta_1}{1-\delta_1} \leq \frac{10}{2} N \frac{1}{1-\delta_1}$$

$$10(1-\delta_1) + 4\delta_1 \leq 10$$

$$10 - 10\delta_1 + 4\delta_1 \leq 10/2$$

$$5 \leq 6\delta_1$$

$$\boxed{5/6 \leq \delta_1}$$

And firm 2 will not deviate as long as

$$6N \leq \frac{6N}{2} + 6\frac{N}{2}\delta_2 + 6\frac{N}{2}\delta_2^2 + \dots$$

$$6N \leq \frac{6N}{2}\frac{1}{1-\delta_2}$$

$$1/2 \leq \delta_2$$

e) in part d cooperation is less likely to occur compared to part b, as now $\delta_1 \geq 5/6$ while in b $\delta_1 \geq 1/2$

AZ.4

Problem 4

a) Firm 1 solves $\text{Max } \pi_1 = 4\alpha_1 + 3\alpha_1\alpha_2 - \alpha_1^2$

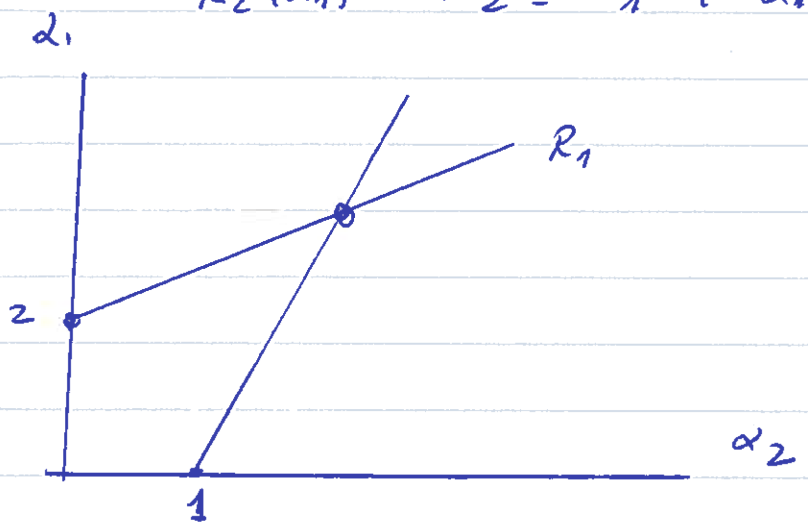
FOC $4 + 3\alpha_2 - 2\alpha_1 = 0$

$R_1(\alpha_2) = \alpha_1 = 2 + 3\alpha_2/2$

Firm 2 solves. $\text{Max } \pi_2 = 2\alpha_2 + \alpha_1\alpha_2 - \alpha_2^2$

FOC $2 + \alpha_1 - 2\alpha_2 = 0$

$R_2(\alpha_1) = \alpha_2 = 1 + \alpha_1/2$




b) we just solve for $\alpha_1$ and $\alpha_2$ in $R_1$ and $R_2$

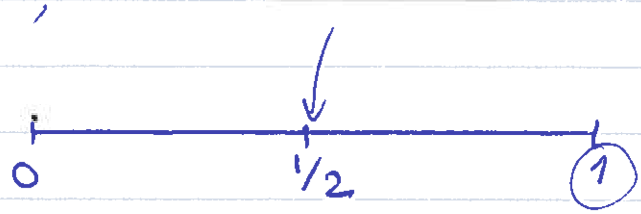
$\alpha_1 = 14 \quad \alpha_2 = 8$

$\pi_1 = 14^2 \quad \pi_2 = 8^2$.

A2.5.

Problem 5,

restaurant is here.




in this case $L$ is normalized to

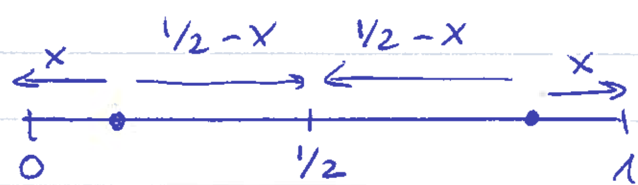
transportation cost = \$1 per Kilometer.

if you live "a" units from restaurant your utility is $U = B - a - p$.

if you dont eat at the restaurant your $U = 0$

a) suppose $0 < B < 1$

now suppose we have 2 consumers, one to the left of the restaurant, the other one to the right, both are at the same distance from the extremes, so both are at the same distance from restaurant.




the restaurant total demand is $2(1/2 - x)$

the utility from the most distant consumer to the restaurant is:

$$U = B - (1/2 - x) - p.$$

we also now that this consumer buys from the restaurant as long as.

$$B - (1/2 - x) - p \geq 0$$

so you as a monopolist set.

$$p = B - 1/2 + x.$$

then you maximize:

$$\pi = 2\underbrace{(1/2 - x)}_{q}\underbrace{(B - 1/2 + x)}_{p}$$

FOC:

$$2(1/2 - x) - 2 \cdot (B - 1/2 + x) = 0$$

$$1/2 - x - B + 1/2 - x = 0$$

$$1 - B = 2x$$

$$\frac{1-B}{2} = x$$

then $\underbrace{2 - 2x}_{2(1/2 - x)} = 1 - (1 - B) = B$

total demand

and $p = B - 1/2 + 1/2 - B/2 \quad \pi = \frac{B^2}{2}$

$$p = B/2$$

b) What if $B > 1$

then $x < 0$, that is, the indifferent consumer lies outside the city, then the monopo. can increase the price and still have the entire street purchase the product.

then the monopoly set the highest possible price subject to having the consumers living at the city extremes purchase the product.

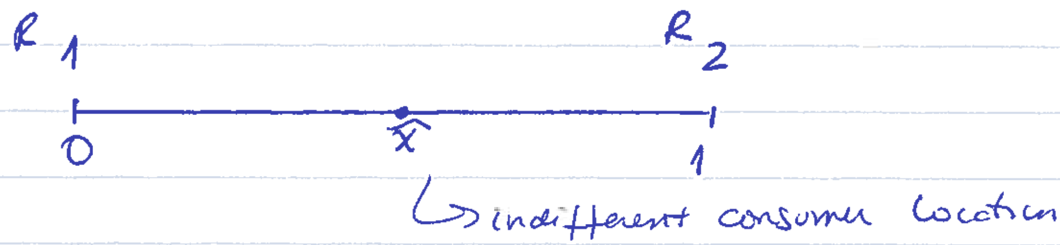
that is: $U = \beta - 1/2 - p \geq 0$

$$\boxed{\beta - 1/2 = p}$$

total demand $= 1$

profit $= (B - 1/2) \cdot 1 = \beta - 1/2$.

Problem 6




a) assume

$$0 < P_1 - R < P_2 < 1 + P_2$$

for indifferent consumer.

$$P_1 + \hat{x} = P_2 + R(1 - \hat{x})$$

$$P_1 + \hat{x} = P_2 + R - R\hat{x}$$

$$\hat{x}(1+R) = P_2 - P_1 + R$$

$$\hat{x} = \frac{P_2 - P_1}{1+R} + \frac{R}{1+R}.$$

b) suppose $P_1 = P_2$.

all consumer go to eat to 1 if

$$\hat{x} = 1$$

that is, if:

$$\underbrace{\frac{P_2 - P_1}{1+R}}_{0} + \frac{R}{1+R} = 1$$

$$\frac{R}{1+R} = 1 \Rightarrow$$ that is, if $R \to \infty$

Problem 7.

a) Firm 1 solves.

$$\text{Max } \pi_1 : (100 - q_1 - q_2 - q_3)\, q_1$$

FOC $\quad 100 - 2q_1 - q_2 - q_3 = 0$

by symmetry: $\quad q_1 = q_2 = q_3 = q$

$$100 = 4q$$

$$25 = q_1 = q_2 = q_3 \qquad p = 25$$

$$\pi_1 = 625 = \pi_2 = \pi_3$$

b) Now firm 1 solves

$$\text{Max}_{q_1} \; \pi_1 = (100 - q_1 - q_4)\, q_1$$

FOC: $\quad 100 - 2q_1 - q_4 = 0$

firm 4 solves

$$\text{Max}_{q_4} \; \pi_4 = (100 - q_1 - q_4)\, q_4$$

FOC: $\quad 100 - 2q_4 - q_1 = 0$

solving for the FOCs we get;

$$q_1 = q_4 = 100/3 \qquad p = 100/3$$

$\pi_1 = \frac{100^2}{9}$ $\qquad$ $\pi_4 = \frac{100^2}{9}$

c) note that

$$\frac{100^2}{9} < \underbrace{\overset{\pi_2}{625} + \overset{\pi_3}{625}}$$

$$1111.11 < 1250$$

so 2 and 3 do not benefit from merging

d) now we have a monopoly.

so we solve.

$$\text{Max} \quad (100 - Q) \cdot Q$$

$$\text{FOC} \quad 100 - 2Q \qquad Q = 50 \qquad p = 50$$

$$\pi = 2500.$$

clearly firm 4 benefits from the merger.
(suppose 4 and 1 equally split the merged firms benefits

e) in d we have a monopoly, which yields the maximum possible profits for the industry.

# Problem 8

a)

firm x solves $\max_{P_x} \pi_x = (\alpha - P_x - 2P_y)\,P_x$

FOC $\alpha - 2P_x - 2P_y = 0$

firm y solves $\max_{P_y} \pi_y = (\alpha - P_x - 2P_y)\,P_y$

FOC $\alpha - P_x - 4P_y = 0$

From FOCs $P_x = \frac{2}{6}\alpha \quad P_y = \frac{\alpha}{6}$

then the price for a system is $P_x + 2P_y = \frac{4}{6}\alpha = \frac{2}{3}$

then $Q_S = \alpha - \frac{2}{3}\alpha \quad Q_S = \frac{1}{3}\alpha$

also note that $Q_x = Q_S = 1/3\,\alpha$

and $Q_y = 2Q_S = 2/3\,\alpha$ → because 1 computer needs 2 USE

then $\pi_x = \underbrace{\frac{2}{6}\alpha}_{P_x} \cdot \underbrace{\frac{1}{3}\alpha}_{Q_x} = \frac{1}{9}\alpha^2$

$\pi_y = \underbrace{\frac{\alpha}{6}}_{P_y} \cdot \underbrace{\frac{2}{3}\alpha}_{Q_y} = \frac{1}{9}\alpha^2$

b) Now the systems producer solves

$\max_{P_S} (\alpha - P_S) \cdot P_S$

FOC $\alpha - 2P_S = 0 \quad P_S = \alpha/2 \quad Q_S = \alpha/2$

then $\pi_S = \frac{1}{4}\alpha^2$

c) First note that $\frac{1}{4}\alpha^2 > \frac{1}{9}\alpha^2 + \frac{1}{9}\alpha^2$

$$\frac{1}{4}\alpha^2 > \frac{2}{9}\alpha^2$$

also note that: $P_{S\,\text{before merger}} < P_S$ after merger.

$$\frac{1}{2}\alpha < \frac{2}{3}\alpha$$

and that $Q_{S\,\text{before merger}} > Q_S$ after merger

$$\frac{1}{2}\alpha > \frac{1}{3}\alpha$$

↳ so consumers are also better off. !!!

merger is welfare improving. !