Assignment 1 solution hints

Problem 1 $\quad P = 12 - Q/2 \Rightarrow$ monopolist's inverse demand function

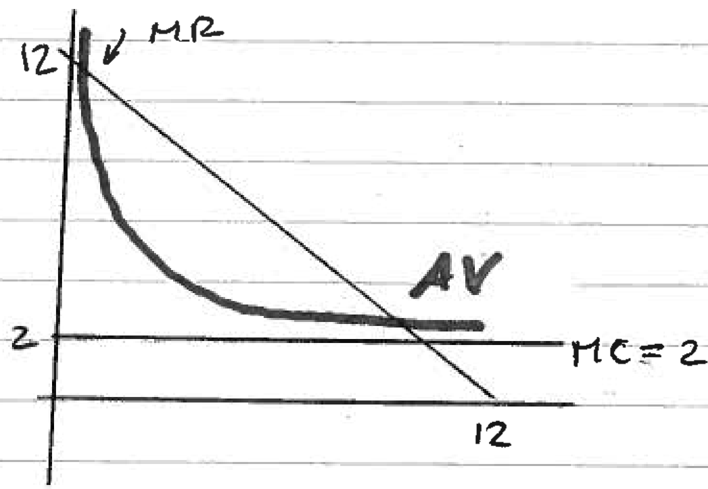
$C(Q) = 4 + 2Q \Rightarrow$ total cost.

$$\underbrace{MC}_{\text{marginal cost}} = \frac{d\,C(Q)}{dQ} = 2 \qquad \underbrace{AV}_{\text{average cost}} = \frac{C(Q)}{Q} = \frac{4}{Q} + 2$$

note that as $Q \to \infty$ $\quad AV \to 2 = MC$

$$\underbrace{MR}_{\text{marginal revenue}} = \frac{d\,(\overbrace{P(Q).Q}^{\text{total income}})}{dQ} = 12 - Q.$$

Graphycally



b) Monopolist equates

$$MR = 12 - Q = 2 = MC$$

$$Q = 10$$

then $P = 12 - Q/2 = 12 - 5 = 7$

so $\pi = P(Q).Q - C(Q)$

$$\underbrace{7.\,10}_{70} - \underbrace{(4 + 2 \times 10)}_{24} = 46$$

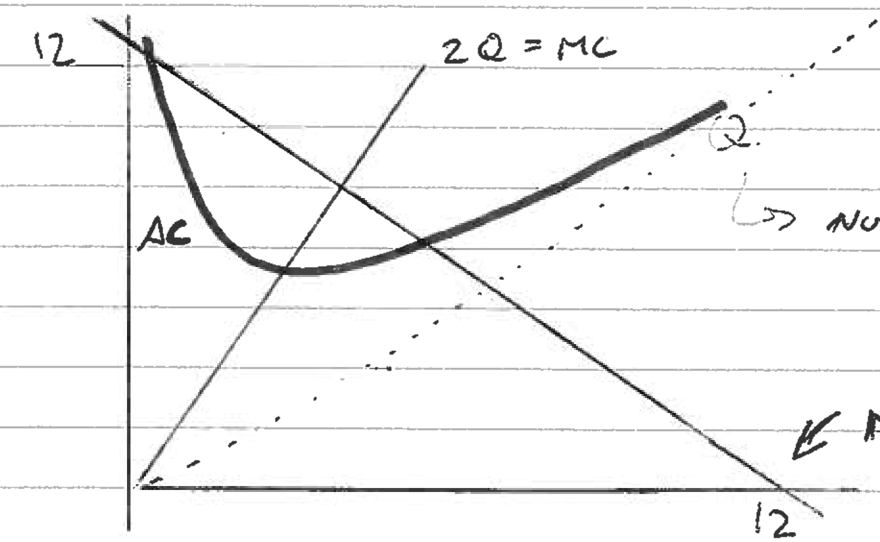
S2

c) $\varepsilon = \frac{dQ}{dP} \cdot \frac{P}{Q} = -2 \cdot \frac{7}{10} = -1.4$

$\boxed{Q = 24 - 2P}$ — $\frac{dQ}{dP} = -2$

d) Now $C(Q) = 4 + Q^2$ $\quad MC = \frac{dC(Q)}{dQ} = 2Q$

$AC = \frac{C(Q)}{Q} = \frac{4}{Q} + Q$



→ Note that when $Q \to \infty$, $A \to Q$

$\lim_{Q \to \infty} AV = Q$

$MR = 12 - Q$

e) Monopolist equates $MR = MC$

$12 - Q = 2Q$

$12 = 3Q \quad Q = 4$

then $P(Q) = \underbrace{12 - \frac{4}{2}}_{12 - \frac{Q}{2}} = 10$

$\pi = 10 \cdot 4 - (4 + 4^2)$

$\pi = 40 - 20 = 20$

f) $Q = \frac{120}{P^2}$ $\quad$ $\underbrace{\text{total income}}_{} = P(Q) \cdot Q$

$\hookrightarrow P = \frac{(120)^{1/2}}{Q^{1/2}}$ $\quad TI = \frac{(120)^{1/2}}{Q^{1/2}} \cdot Q$

$TI = (120)^{1/2} Q^{1/2}$

S3

$$MR = \frac{d\left(120^{1/2} Q^{1/2}\right)}{dQ} = 120^{1/2} \cdot \frac{1}{2} \cdot Q^{-1/2}$$

$$= \frac{120^{1/2}}{2} \cdot \frac{1}{Q^{1/2}}$$

$$MC = \frac{d\left(4 + Q/2\right)}{dQ} = \frac{1}{2}$$

Monopolist solves: $\overbrace{\frac{120^{1/2}}{\cancel{2}} \cdot \frac{1}{Q^{1/2}}}^{MR} = \overbrace{\frac{1}{\cancel{2}}}^{MC}$

$\boxed{120 = Q}$ $\quad P = \frac{120^{1/2}}{120^{1/2}} = \underline{1}$

$$\pi = P(Q) \cdot Q - TC(Q)$$

$$1 \cdot 120 - \left(4 + \frac{120}{2}\right)$$

$$120 - 64 = \underline{56}$$

S4

Problem 2

$P_N = 12 - q_N$ $\quad$ $P_S = 6 - q_S$ $\quad$ $TC = 10 + 2Q$

$Q = q_N + q_S$

these are group aggregated demands

a) we know the aggregated demands, but we don't know the individual ones, then 1st and 2nd degree price discrimination are not possible.

so the only type of price discrimination that is possible is 3rd degree price discrimination.

then $$MR_N = MR_S = MC$$

$$12 - 2q_N = 6 - 2q_S = 2$$

so: $q_N = 5 \quad q_S = 2$

$P_N = 7 \quad P_S = 4$

$$\pi = 5 \times 7 + 2 \times 4 - (10 + 2 \times 7) = 19$$

b) two possibilities:

$$Q_T = \begin{cases} \text{if } P > 6 & Q_T = 12 - P \\ \text{if } P \leq 6 & Q_T = 18 - 2P \end{cases}$$

1) only serve high demand

then $q_N = 5 \quad P_N = 7 \quad \pi = 7 \times 5 - (10 + 2 \times 5)$

$\pi = 15$

2) serve both groups

$$\max_Q \underbrace{(9 - Q/2)}\cdot Q - \overbrace{(10 + 2Q)}^{TC}$$

$P(Q) = 9 - Q/2$

→

55

$$\text{FOC} \quad \overbrace{9-Q}^{MR} - \overbrace{2}^{MC} = 0 \rightarrow P = 9 - Q/2 = 9 - 7/2$$

$$\boxed{Q = 7} \qquad P = 5.5$$

$$\pi = 5.5 \times 7 - (\underbrace{10 + 2 \times 7})$$

$$\underbrace{38.5 - 24}$$

$$14.5 < 15$$

→ So in this case set $p = 7$ and only serve high demand market.

c) by price discriminating the firm makes a profit of 17 which is 4 units higher than the profit the firm makes when it cannot price discriminate.

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Problem 3

aggregate demand in market 1

$P_1 = 100 - q_1/2$

aggregate demand in market 2

$P_2 = 100 - q_2$

$\underbrace{TC}_{\text{TOTAL COST}} = Q^2$ where $Q = q_1 + q_2$

a) $\pi(q_1, q_2) = \underbrace{(100 - q_1/2)\, q_1}_{\text{profit mkt 1}} + \underbrace{(100 - q_2)\, q_2}_{\text{profit mkt 2}} - \underbrace{(q_1 + q_2)^2}_{\text{total cost}}$

q_1 and q_2 produced in same plant

b) from FOC: $MR_1 = MR_2 = MC$

$$100 - q_1 = 100 - 2q_2 = \underbrace{2(q_1 + q_2)}_{MC}$$

$\frac{d\,TC(Q)}{dQ} = 2Q = 2(q_1 + q_2)$

$\cancel{100} - q_1 = \cancel{100} - 2q_2$

$\boxed{q_1 = 2q_2}$

replace here

$100 - 2q_2 = 2(q_1 + q_2)$

$100 - 2q_2 = 2(2q_2 + q_2)$

$100 = 8q_2$

$q_2 = 12.5 \qquad q_1 = 25$

$P_2 = 87.5 \qquad P_1 = 87.5$

$\pi = 87.5 \times 25 + 87.5 \times 12.5 - \underbrace{(37.5)^2}_{(q_1 + q_2)^2 = TC}$

$\boxed{\pi = 1875}$

c) Now production for market 1 is produced in a different plant. (so now monopolist operates two plants)

So in Mkt 1 monopolist solves

$$\max_{q_1} \pi_1 = (100 - q_1/2)\, q_1 - q_1^2$$

$$\text{FOC} \quad \underbrace{100 - q_1}_{MR} - \underbrace{2q_1}_{MC} = 0$$

$$100 = 3q_1$$

$$100/3 = q_1 \qquad p_1 = 100 - 100/6$$

$$\pi_1 = 1666.67$$

in market 2 monopolist solves.

$$\max_{q_2} \pi_2 = (100 - q_2)\, q_2 - q_2^2$$

$$\text{FOC:} \quad 100 - 2q_2 - 2q_2 = 0$$

$$100 = 4q_2$$

$$\boxed{25 = q_2}$$

$$75 = p_2$$

$$\pi_2 = 1250$$

$$\pi_T = \pi_1 + \pi_2 = 1250 + 1666.67 = 2916.67$$

S8

d) the cost function $\rightarrow$ $CT = Q^2$

has $MC = 2Q$

$\hookrightarrow$ that is MC is increasing in Q $\rightarrow$ the more Q you produce, the higher the extra cost of each additional unit.

then, when you set two plants you can escape this situation, because you distribute your production between two plants, the increasing cost technology does not affect you too much.

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Problem 6

a) → we have to analyze 2 possible cases.

1st case.

consumer born in ~~t=1 buys~~ in $t=2$

so in this situation clearly $P_2 = 50$ → why?

and $\pi = 50 + 50 + 50 = 150$

2nd case:

consumer born in $t=1$ buys in $t=1$

so again you set $P_2 = 50$ so $\pi_2 = 50 \times 2 = 100$

and set P_1 so that consumer 1 buys in period one.

which means that.

$$\underbrace{200 - P_1}_{\text{why?}} \geq \overbrace{100 - 50}^{\text{why?}}$$

$$150 \geq P_1$$

so set $P_1 = 150$

in this case

$$\pi = \pi_1 + \pi_2 = 150 + 2 \times 50 = \boxed{250}$$

so set $P_1 = 150$ $P_2 = 50$ $\pi = 250$

→ $\pi_1 + \pi_2$.

b) Again we have to analyze two possible cases.

1st case

consumer 1 buys in 2nd period

then.

if you set $P_2 = 20$ all three consumers buy at $t = 2$

and you get $\pi_{t=2} = 60 = \pi$

2nd case

consumer 1 buys in 1st period

then you set $P_2 = 50$ and get $\pi_2 = 50 \times 2 = 100$

and you set $P_1 = 40$ and you get $\pi_1 = 40$

note that at this price consumer born at $t = 1$ does not want to wait till $t = 2$

why

So the best you can do is set:

$P_1 = 40 \quad P_2 = 50$

$\pi = \pi_1 + \pi_2 = 140.$

Problem 7

each type 1 $\Rightarrow q_1 = 1-P$ each type 2 $\Rightarrow q_2 = 2-2p$

$\hookrightarrow$ so we ~~know individual~~ demands

a)

$$q_T = 15(1-P) + 15(2-2P)$$

$$q_T = 15 - 15P + 30 - 30P$$

$$\boxed{q_T = 45 - 45P.}$$

b) the monopolist cannot set different prices in different in mkts as it cannot distinguish who is who

then monopolist solves

$$\text{Max} \ \overbrace{\left(1 - \frac{1}{45} q_T\right)}^{P(Q)} q_T - \underbrace{TC(Q)}_{=0}$$

from FOC

$$\underbrace{1 - \frac{2q_T}{45}}_{MR} = \underbrace{0}_{MC=0} \quad \boxed{q_T = 22.5} \quad \boxed{P = 0.5}$$

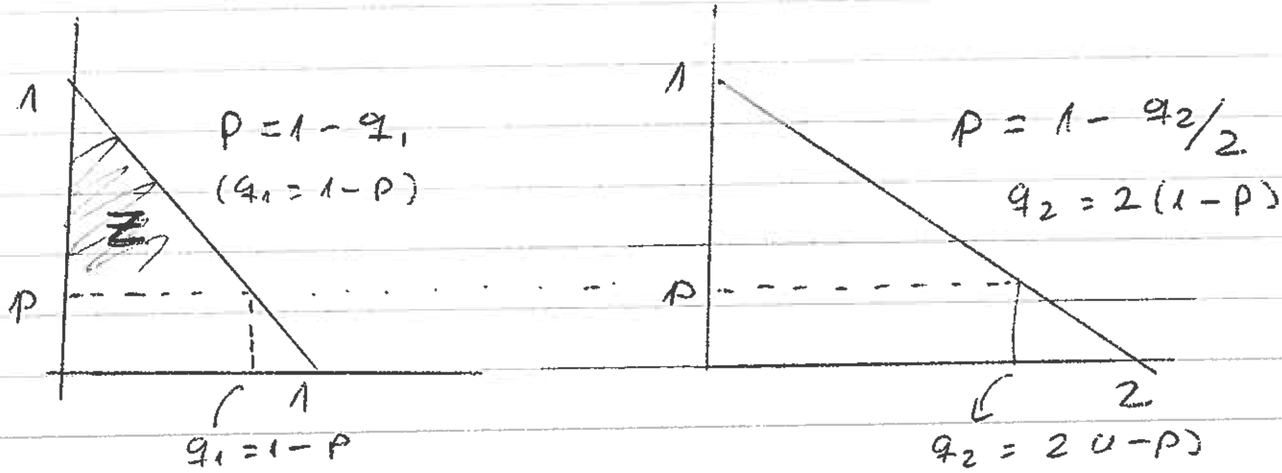
and $\pi = 22.5 \times 0.5$

$$\boxed{\pi = 11.25}$$

$\longrightarrow$

c) you don't know who is who, but you can set two part tariffs:

type 1 type 2.



• Unique two part tariff → everyone pays the same fixed fee Z and same unit price p.

where Z is the "low type" demand consumer surplus.

↳ why?

then the monopolist solves:

$$\text{Max} \quad 30 \cdot Z + 15\, p_1 q_1 + 15\, p_2 q_2 - \underbrace{0}_{TC}$$

$$\underset{p}{\text{Max}} \quad 30 \frac{(1-p)^2}{2} + 15\, p_1 (1-p) + 15\, p \cdot 2(1-p) - \underbrace{0}_{TC}$$

$$\text{FOC:} \quad \cancel{-30(1-p)} + 15(1-p) - 15 p_1 + \cancel{30(1-p)} - 30p =$$

$$15 - 15p - 15p - 30p$$

$$\boxed{p = 0.25} \qquad Z = \underbrace{0.28125}_{\frac{(1-0.25)^2}{2}}$$

$$\pi = 30 \times \frac{(1-0.25)^2}{2} + 0.25 \times 0.75 \times 15 + 0.25 * 2 \times 0.75 \times 15 \quad \rightarrow$$

S13

$\pi = 16.875$

we always have to check for this!!!

but, what if we serve high types only?

then set $\begin{cases} p_2 = 0 \\ p_2 = MC \end{cases}$ $\{ q_2 = 2$ ← quantity demanded by high type if $P = 0 = MC$

why?

and $Z_2 = \dfrac{1 \times 2}{2} = 1$

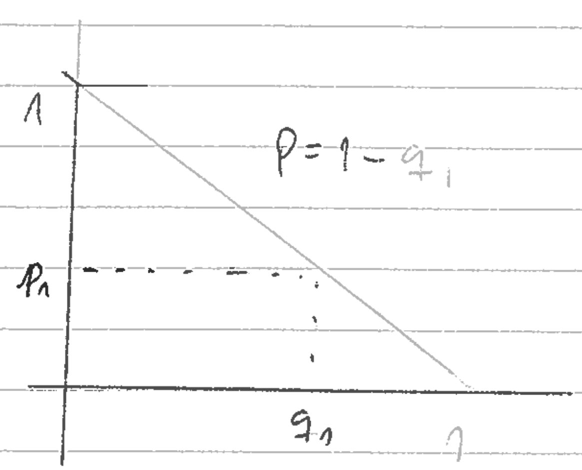
then $\pi = Z_2 \times 15 = \underbrace{15 < 16.875}$

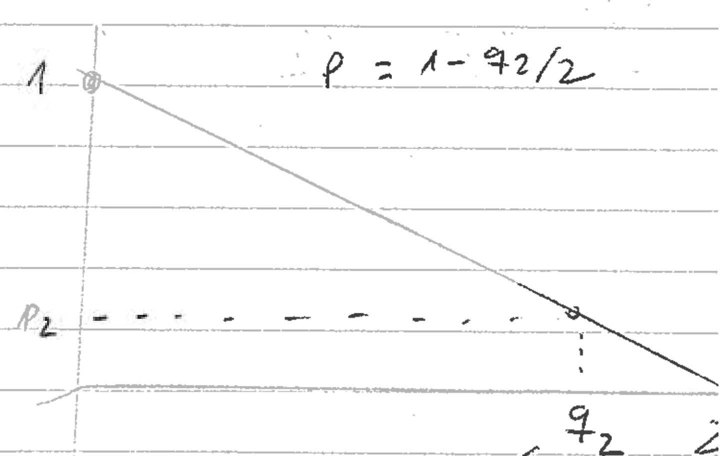
so will serve both mkts

and $Z = 0.28125$

and $p = 0.25$

d) type 1   type 2.





$q_1 = (1 - p_1)$

$q_2 = 2(1 - p_2)$

S. 14

for type 1 we will set $\nearrow Z_1$, $\searrow P_1$ for type 2 $\nearrow Z_2$, $\searrow P_2$

So the monopolist solves

$$\text{Max } \pi = 15(Z_1 + P_1 q_1) + 15(Z_2 + P_2 q_2) - \overset{0}{TC}$$

s.a. (1) $\frac{(1-P_1)^2}{2} - Z_1 \geq 0$ Individual Rationality constraint for type 1

(2) $\frac{(1-P_2)\cdot 2(1-P_2)}{2} - Z_2 \geq 0$ individual Rationality constraint for type 2

(3) $\frac{(1-P_1)^2}{2} - Z_1 \geq \frac{(1-P_2)^2}{2} - Z_2$ incentive compatibility constraint for low type

(4) $\frac{(1-P_2)\,2(1-P_2)}{2} - Z_2 \geq \frac{(1-P_1)\,2(1-P_1)}{2} - Z_1$ incentive compatibility constraint for high type

we know that (1) and (4) are the relevant constraints why and that they hold with "equality" (=). then the monopolist solves

$$\underset{P_1, P_2}{\text{Max}}\ \pi = 15\Big[\underbrace{\frac{(1-P_1)^2}{2}}_{Z_1} + P_1\underbrace{(1-P_1)}_{q_1}\Big] + 15\Big\{\underbrace{(1-P_2)^2 - \Big[(1-P_1)^2 - \frac{(1-P_1)^2}{2}\Big]}_{Z_2} + P_2\underbrace{2(1-P_2)}_{q_2}\Big\}$$

$$\frac{\partial \pi}{\partial P_2} = \cancel{-2(1-P_2)} + P_2 + \cancel{2(1-P_2)} = 0$$

$$P_2 = 0 = MC$$

now you can easily find P_1, Z_1, Z_2, π.