Section 10.1. Vector Functions and Space Curves

A **Vector Function** is a function whose domain is a set of real numbers and who range is a set of vectors.

Example Find the domain of the vector function $\vec{r}(t) = \langle t^2, \ln(t+1), \sqrt{5-t} \rangle$.

Definition If $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$, then

$$\lim_{t \to a} \vec{r}(t) = \left\langle \lim_{t \to a} f(t), \lim_{t \to a} g(t), \lim_{t \to a} h(t) \right\rangle$$

provided the limits of f, g, and h exist.

Example Find $\lim_{t \to 0} \vec{r}(t)$, where $\vec{r}(t) = e^{-t}\vec{\imath} + \sqrt{t+4}\,\vec{\jmath} + \frac{\sin t}{t}\vec{k}$.

Definition A vector function $\vec{r}$ is continuous at a if

$$\lim_{t \to a} \vec{r}(t) = \vec{r}(a)$$

Space Curves

Definition If $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$ then the curve in $\mathbb{R}^3$ with parametric equations

$$x = f(t) \qquad y = g(t) \qquad z = h(t)$$

is called the space curve defined by $\vec{r}$.

Example Describe the curve defined by the vector function $\vec{r}(t) = \langle 3 + t, 2 + 3t, 4t \rangle$.

Example Sketch the curve whose vector function is $\vec{r}(t) = \cos t \ \vec{\imath} + \sin t \ \vec{\jmath} + t\vec{k}$.

Example Find a vector equation for the line segment that joins the point $P(-2, 3, -4)$ to $Q(3, 2, -7)$.

Example Find a vector function that represents the curve of intersection of the surfaces $x^2 + y^2 = 1$ and the plane $y + z = 2$.