

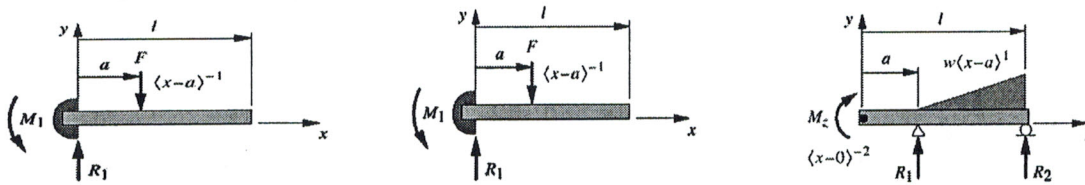
MIDTERM EXAMINATION

Date & Time: Feb. 16th (Thursday) 8:30 am ~ 10:30 am (2 hours)
 Room: E5-3101, 3102
 Materials allowed: One summary sheet (letter-size, double-sided) and a calculator
 Number of pages: 4 pages including this one
 Number of questions: 3 (100 points in total: 25 + 40 + 35)

Blank sheets are provided. Show all your work.

List of Useful Formula

① Singularity functions



② Integrals of singularity functions

$\int_{-\infty}^x \langle x-\lambda \rangle^{-2} d\lambda = \langle \lambda-a \rangle^{-1}$	$\int_{-\infty}^x \langle x-\lambda \rangle^{-1} d\lambda = \langle \lambda-a \rangle^0$	$\int_{-\infty}^x \langle x-\lambda \rangle^0 d\lambda = \langle \lambda-a \rangle^1$
$\int_{-\infty}^x \langle x-\lambda \rangle^1 d\lambda = \frac{1}{2} \langle \lambda-a \rangle^2$	$\int_{-\infty}^x \langle x-\lambda \rangle^2 d\lambda = \frac{1}{3} \langle \lambda-a \rangle^3$	$\int_{-\infty}^x \langle x-\lambda \rangle^3 d\lambda = \frac{1}{4} \langle \lambda-a \rangle^4$

③ Formulas for bending (circular cross-section)

Governing equations:	$q(x) = \frac{dV(x)}{dx} = \frac{d^2M(x)}{dx^2}$	
Area moment of inertia:	$I = \frac{\pi r^4}{4}$	
Normal stress:	$\sigma_x = -\frac{M(x)y}{I}$	
Shear stress:	$\tau_{xy} = -\frac{V(x)(r^2-y^2)}{3I}$	

④ Formula for torsion: $\tau = \frac{Tr}{J}$ where the polar area moment of inertia $J = \frac{\pi r^4}{2}$.

⑤ Static failure criteria for ductile materials

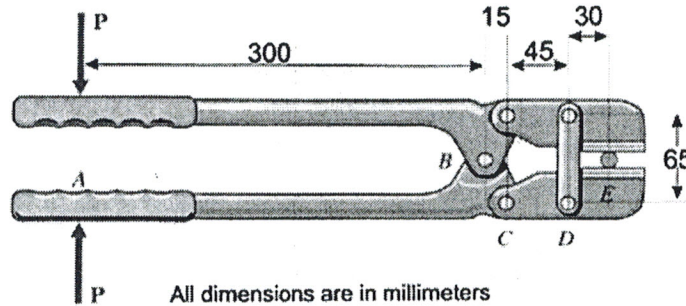
Safety factor:	$N = \frac{S_y}{\sigma'}$
Von Mises criterion:	$\sigma' = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_3\sigma_1} = \sqrt{\frac{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)}{2}}$
Maximum shear stress criterion:	$\sigma' = 2\tau_{max}$

⑥ Steady-state equivalent circuit for a DC Motor: $V_{supply} = Ri + V_{emf}$

⑦ Speed conversion: $60 \text{ rpm} = 2\pi \text{ [rad/sec]}$

1. Answer the following questions.

- (a) (15) The bolt cutter shown in Fig. 1 has a link on each side of the jaws at *D*, so that the pins are in double shear. The maximum allowable shear stress in the pins and links is 140 Mpa, while the maximum allowable normal stress is 280 MPa. Determine the minimum pin diameter and suitable link dimensions for a maximum force *P* of 250 N.



All dimensions are in millimeters
Figure 1 Bolt cutter pin design

- (b) (5) The synchronous speed of an induction motor is given by $\omega_s = \frac{120f}{p}$ in [rpm]. What are *f* and *p*? Using this formula, find the synchronous speed of the shaded pole induction motor in Fig. 2.

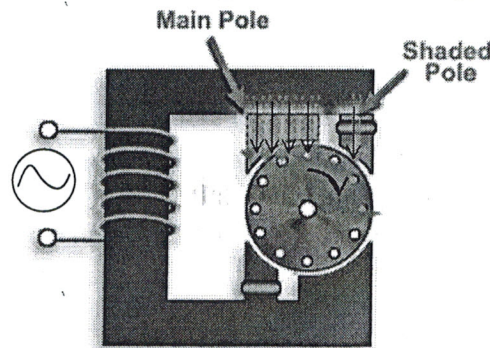


Figure 2 Shaded pole induction motor

- (c) (5) The step angle of a stepper motor is given by $\frac{360}{N_{rt}N_{sp}}$ in [degrees]. What are N_{rt} and N_{sp} ? Find the step angle of the VR stepper motor shown in Fig. 3. If the coils are switched on and off in the sequence of A-B-C-D, which direction does the rotor move and why (briefly explain)?

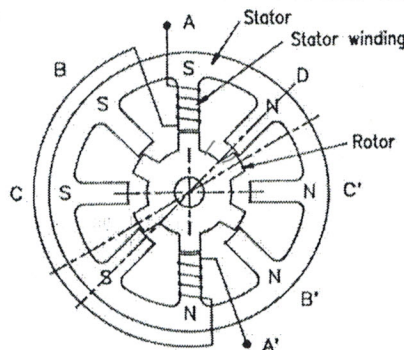


Figure 3 Variable reluctance stepper motor

2. Figure 4 shows a bracket. It consists of a rod and an arm. There is a force F applied at the end of the cross arm.

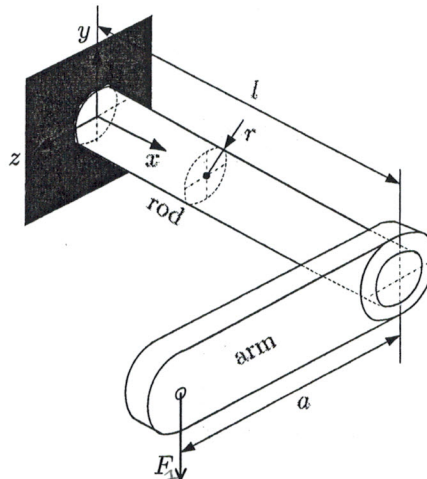


Figure 4 Uniform rod and bracket under loading

- (a) (5) Draw a free body diagram of the rod (length l component) in Fig. 4. Assume that the thickness of the arm is negligible.
- (b) (10) Write down the load function $q(x)$ for the rod using singularity functions. Find the shear and moment functions, $V(x)$ and $M(x)$. (Note: You do not need to consider torsion here.) *<only for part B>*
- (c) (15) Choose $F = 1000$ N, $l = 200$ mm, $a = 180$ mm and the radius of the rod $r = 20$ mm. Find the most highly stressed location(s) on the rod and write them in (x, y, z) coordinates. For each of these location(s), draw Mohr's circle and find the principle stresses $\sigma_1, \sigma_2, \sigma_3$ and τ_{max} .
- (d) (5) Assume that the rod is made of 1010 hot rolled steel, the yield strength of which is $S_y = 180$ MPa. Find the safety factor(s) for the static failure at the most highly stressed location(s) using both the Von Mises criterion and the maximum shear strength criterion.
- (e) (5) Now, consider that the rod is modified to have a step as shown in Fig. 5 with the length and the radius of the slender section denoted by l_1 and r_1 , respectively. If you solve (a), (b), (c) and (d) again with this new bracket, will you have the same solution? If not, which sub-problem(s) among (a), (b), (c) and (d) will you have different solution(s) and how? Briefly explain.

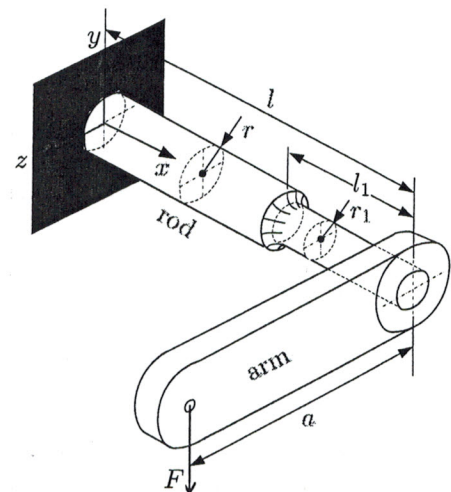


Figure 5 Modified bracket

3. The side view of an electric bicycle is shown in Fig. 6. The front wheel is driven by a wheel hub motor (Fig. 7). The type of motor is the permanent magnet (PM) brush DC motor which runs by 48 V Lithium-ion battery. The bicycle and the biker weigh 100 kg in total.
- (a) (5) If we want the bicycle to be able to drive up a 10 degrees incline with 20 Nm torque provided by the pedalling at the rear wheel axle, how much additional torque does the front wheel axle need to provide at the minimum?
- (b) (10) Assume that we want the bike to maintain 36 km/h continuous speed on a flat road without pedalling and that the no-load motor speed is 3000 rpm. As shown in Fig. 7-c), this motor is connected through the planetary gear mechanism of ratio 6:1, which produces a viscous friction torque of 0.012 Nm/(rad/s) seen from the motor. Assume that the Coulomb friction is negligible. Choose the motor power in multiples of 250 W (i.e. 250, 500, 750,...).
- (c) (15) Draw a torque-speed curve for the motor you selected in (b) and compute: i) the stall torque T_{stall} , ii) the torque constant k and iii) the stator coil resistance R . How much current does the motor consume when the bicycle is running at its rated speed on a flat road without pedalling?
- (d) (5) With the motor selected in (b), what will be the steady-state speed of the bicycle on the 10 degrees uphill with the same condition as (a) (i.e. the biker is providing the rear axle with 20 Nm torque.)?

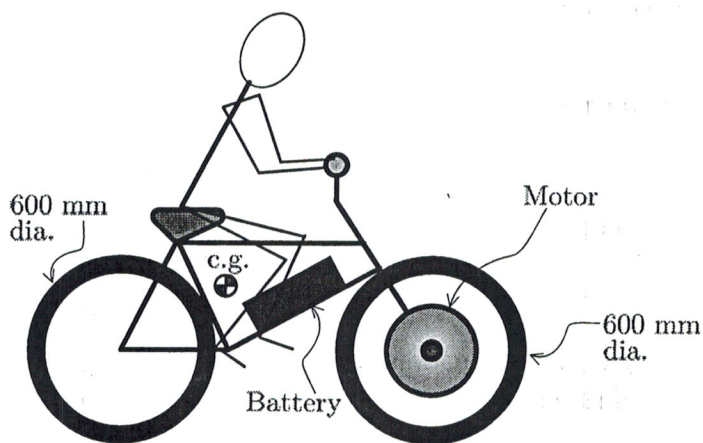


Figure 6 Electric bicycle - schematic

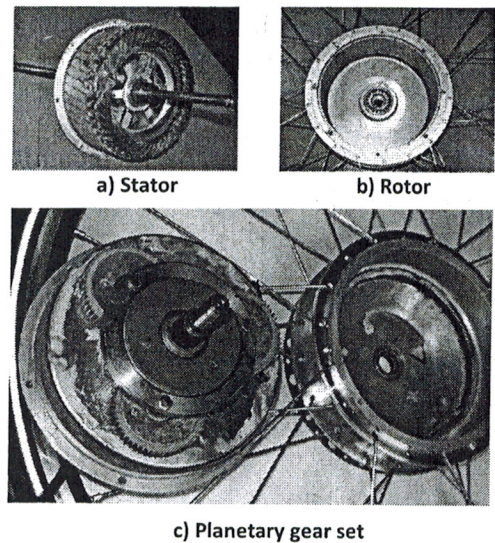


Figure 7 Wheel hub motor

1. (a) exercise #1 Problem 4.

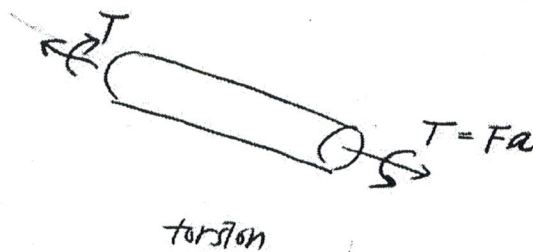
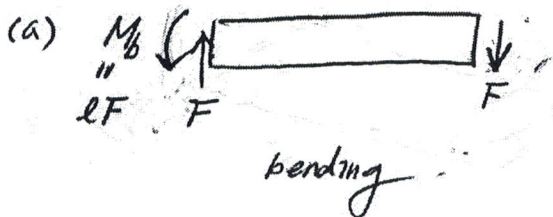
(b) $\omega_s = \frac{120f}{p}$, f : AC power frequency in [Hz], p : # of poles

$\therefore \omega_s = \frac{120 \times 60}{2} = 3600 \text{ rpm}$

(c) step angle = $\frac{360}{N_r + N_p}$, N_r : number of teeth, N_p : # of stator phases

\therefore step angle of VR stepper motor is $\frac{360}{6 \times 4} = 15^\circ$ Reluctance torque will affect the closest bar to the activated coil. Thus, CW.

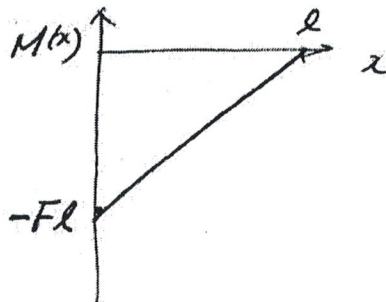
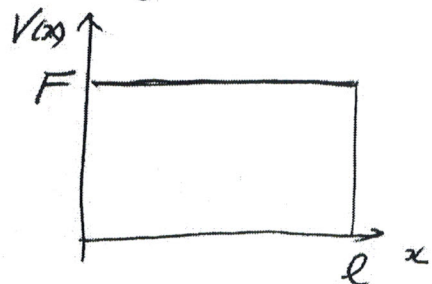
2.



(b) $q(x) = -Fl \langle x-0 \rangle^{-2} + F \langle x-0 \rangle^{-1} - F \langle x-l \rangle^{-1}$

$V(x) = \int q(x) dx = -Fl \langle x-0 \rangle^{-1} + F \langle x-0 \rangle^0 - F \langle x-l \rangle^0$

$M(x) = \int V(x) dx = -Fl \langle x-0 \rangle^0 + F \langle x-0 \rangle^1 - F \langle x-l \rangle^1$



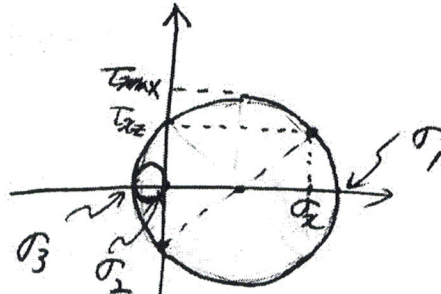
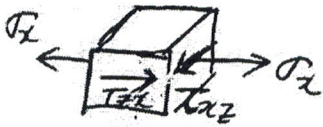
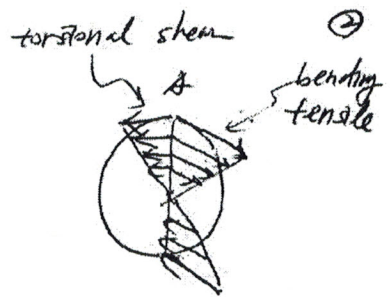
(c) Most highly stressed locations; A(0, r, 0), B(0, 0, r)

- For A, there are tensile stress σ_x due to bending and the shear stress due to torsion (τ_{xz}).

Area moment of inertia $I = \frac{\pi r^4}{4}$, Polar " $J = \frac{\pi r^4}{2}$

$$\sigma_x = -\frac{Mr}{I} = \frac{Fl \cdot x}{\frac{\pi r^4}{4}} = \frac{4Fl}{\pi r^3} = 31.8 \text{ MPa}$$

$$\tau_{xz} = \frac{Tr}{J} = \frac{(Fa)x}{\frac{\pi r^4}{2}} = \frac{2Fa}{\pi r^3} = 14.3 \text{ MPa}$$



$$T_{max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xz}^2} = 21.38 \text{ MPa}, \quad \sigma_1 = \frac{\sigma_x}{2} + T_{max} = 37.28 \text{ MPa}$$

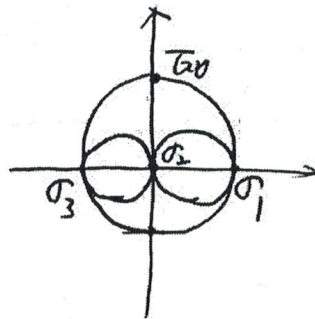
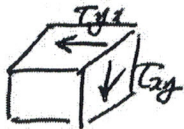
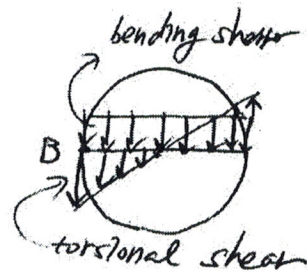
$$\sigma_2 = 0$$

$$\sigma_3 = \frac{\sigma_x}{2} - T_{max} = -5.48 \text{ MPa}$$

- For point B, it's a pure shear.

$$\tau_{bend} = \frac{Vr}{3I} = \frac{Fx}{\frac{3\pi r^4}{4}} = \frac{4F}{3\pi r^2} = 1.06 \text{ MPa}$$

$$\tau_{xy} = \tau_{tor} + \tau_{bend} = 15.36 \text{ MPa}$$



$$T_{max} = \tau_{xy} = \sigma_1 = -\sigma_3$$

$$\sigma_2 = 0$$

(d) Point A, Von Mises criterion gives

$$\sigma' = \sqrt{\sigma_1^2 + \sigma_3^2 - \sigma_1 \sigma_3} = 37.68 \text{ MPa} \Rightarrow N = 4.78$$

Maximum shear stress criterion gives

$$\sigma' = 2T_{max} = 42.76 \text{ MPa} \Rightarrow N = 4.21$$

For point B, Von Mises $\sigma' = \sqrt{\sigma_1^2 + \sigma_3^2 - \sigma_1\sigma_3} = \sqrt{3}\sigma_1 = 26.6 \text{ MPa}$

$\Rightarrow N = 6.77$

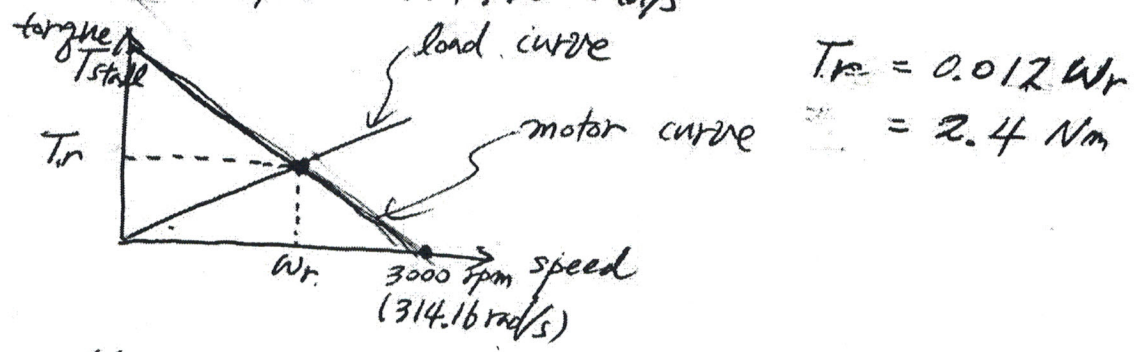
Maximum shear $\sigma' = 2\tau_{max} = 30.72 \text{ MPa} \Rightarrow N = 5.86$

(e) Shear function and bending moment function do not change. Slender section will have much smaller I since $I \propto r^4$. This makes it highly likely that the maximum stressed location occurs on the step ($x = l - l_1$). This will change the answer to (c) and (d).

3. (a) We need at least $mg \sin 10^\circ$ to overcome the gravity. Thus, the torque needed at the front axle is computed from.

$mg \sin 10^\circ = F_f + F_r \Rightarrow T_{fmin} = r mg \sin 10^\circ - T_r = 31.1 \text{ Nm}$

(b) $36 \text{ km/h} = 10 \text{ m/s} = v$, wheel speed $\omega_w = \frac{v}{r} = \frac{100}{3} \text{ rad/s}$
 motor speed $\omega_m = N \omega_w = 6 \times \frac{100}{3} = 200 \text{ rad/s} = \omega_r$
 $3000 \text{ rpm} = 314.16 \text{ rad/s}$



Motor power = $T_r \cdot \omega_r = 480 \text{ W} \Rightarrow$ choose 500 W

(c) For 500 W motor, ω_r and T_r will be a little bit larger than the above values determined for 480 W . We can find these from the load curve.

④

$$T_{load} = 0.012 W_{load}, \quad P_{load} = T_{load} \times W_{load} = 0.012 W_{load}^2$$

Equate $P_{load} = P_{rated}$, and $W_{load} = W_r$. Then,

$$W_r = \sqrt{\frac{P_{rated}}{0.012}} = \sqrt{\frac{500}{0.012}} = 204 \text{ rad/s} \Rightarrow T_r = 2.45 \text{ Nm}$$

The motor curve is given by

$$T - T_r = \frac{-T_r}{314.16 - W_r} (W - W_r) \quad \text{--- ①}$$

T_{stall} is when $W = 0$, thus,

$$T_{stall} = T_r \left(1 + \frac{200}{314.16 - 200} \right) = \underline{6.74 \text{ Nm}}$$

To find k and R , use the motor circuit equation.

$$4\mathcal{P} = Ri + kW \Rightarrow 4\mathcal{P} = R \frac{T}{k} + kW \quad \text{--- ②}$$

$$T_{stall} = \frac{4\mathcal{P}}{R} k, \quad W_{rated} = \frac{4\mathcal{P}}{k} = 314.16$$

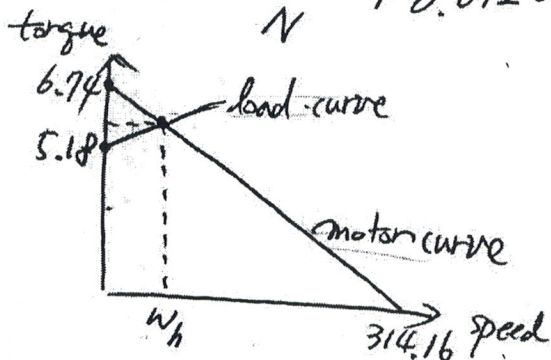
$$\Rightarrow k = 0.153 \text{ and } R = 1.09 \Omega$$

Thus, the current at the rated speed is given by

$$i = \frac{1}{R} (4\mathcal{P} - k \cdot 204) = 15.4 \text{ A}$$

(d) The load curve on the hill is (from the answer to (a))

$$T_{load} = \frac{T_{fmin}}{N} + 0.012 W = \frac{31.1}{6} + 0.012 W = 0.012 W + 5.18 \quad \text{--- ③}$$



Solve ① (or ②) and ③ for W .

$$\text{Then, } W_h = 46.5 \text{ rad/s}$$

$$\text{The axle speed} = \frac{W_h}{N} = 7.75 \text{ rad/s}$$

$$\Rightarrow \text{bicycle speed} = 7.75 \times r$$

$$= 2.325 \text{ m/s}$$

$$= \underline{\underline{8.37 \text{ km/h}}}$$