ECO2144B Fall 2011

Solution of Assignment 1: Brief Answers

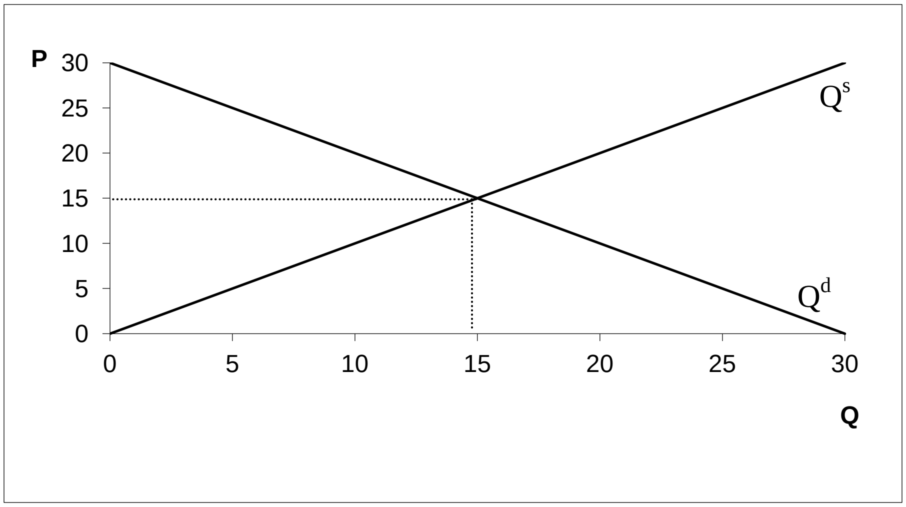
1.12 Suppose the supply curve for wool is given by $Q^s = P$, where Q^s is the quantity offered for sale when the price is P. Also suppose the demand curve for wool is given by $Q^d = 10 - P + I$, where Q^d is the quantity of wool demanded when the price is P and the level of income is I. Assume I is an exogenous variable.

a) Suppose the level of income is $I = 20$. Graph the supply and demand relationships, and indicate the equilibrium levels of price and quantity on your graph.

b) Explain why the market for wool would not be in equilibrium if the price of wool were 18.

c) Explain why the market for wool would not be in equilibrium if the price of wool were 14.

a) Assuming $I = 20$ we have $Q^s = P$ and $Q^d = 30 - P$. Graphing these yields:



The equilibrium occurs at $P = 15$, $Q = 15$.

b) At a price of 18, $Q^s > Q^d$ implying an excess supply of wool. Because sellers will not be able to sell all of their wool at this price, they will need to reduce price to attract buyers. At the lower price, the suppliers will offer a lower quantity of output for sale, and consumers will want to purchase more.

c) At a price of 14, $Q^d > Q^s$, implying an excess demand for wool. Buyers will begin to bid up the price of wool until the new equilibrium is reached. At the higher price, the suppliers will offer a higher quantity of output for sale, and consumers will want to purchase less.

1.13 Consider the market for wool described by the supply and demand equations in Problem 1.12. Suppose income rises from I_1 = 20 to I_2 = 24.
a) Using comparative statics analysis, find the impact of the change in income on the equilibrium price of wool.
b) Using comparative statics analysis, find the impact of the change in income on the equilibrium quantity of wool.

a) With $I_1 = 20$, we had $Q^s = P$ and $Q^d = 30 - P$, which implied an equilibrium price of 15.
With $I_2 = 24$, we have $Q^s = P$ and $Q^d = 34 - P$. Finding the point where $Q^s = Q^d$ yields

$$Q^s = Q^d$$
$$P = 34 - P$$
$$2P = 34$$
$$P = 17$$

Thus, a change in income of $\Delta I = 4$ yields a change in price of $\Delta P = 2$.

b) Plugging the result from part a) into the equation for Q^s reveals the new equilibrium quantity is $Q = 17$. Thus, a change in income of $\Delta I = 4$ yields a change in quantity of $\Delta Q = 2$.

2.13 Consider a linear demand curve, Q = 350 − 7P.
a) Derive the inverse demand curve corresponding to this demand curve.
b) What is the choke price?
c) What is the price elasticity of demand at P = 50?

a)
$$Q = 350 - 7P$$
$$7P = 350 - Q$$
$$P = 50 - \tfrac{1}{7}Q$$

b) The choke price occurs at the point where $Q = 0$. Setting $Q = 0$ in the inverse demand equation above yields $P = 50$.

b) At $P = 50$, the choke price, the elasticity will approach negative infinity.

2.14 Suppose that the quantity of steel demanded in France is given by $Q_s = 100 - 2P_s + 0.5Y + 0.2P_A$, where Q_s is the quantity of steel demanded per year, P_s is the market price of steel, Y is real GDP in France, and P_A is the market price of aluminum. In 2011, Ps = 10, Y = 40, and P_A = 100. How much steel will be demanded in 2011? What is the price elasticity of demand, given market conditions in 2011?

We are given that Y = 40, and P_A = 100, and so substituting these values into the equation that determines the quantity demanded gives us

$$Q_S = 100 - 2P_S + 0.5(40) + 0.2(100)$$

or

$$Q_S = 140 - 2P_S.$$

This is the equation for the demand curve for steel in France. When the price of steel is 10, the quantity of steel demanded is thus 120.

From equation (2.4) in the text, the price elasticity of demand for steel when the price is 10 is given by

$$\epsilon_{Q,P} = \frac{\Delta \boldsymbol{Q}}{\Delta \boldsymbol{P}}\frac{\boldsymbol{P}}{\boldsymbol{Q}} = \frac{\boldsymbol{dQ}}{\boldsymbol{dP}}\frac{\boldsymbol{P}}{\boldsymbol{Q}} = -2\frac{10}{120} = -0.167.$$

The price elasticity of demand for steel is -0.167, given market conditions in 2011.

2.17 Consider the following demand and supply relationships in the market for golf balls: $Q^d = 90 - 2P - 2T$ and $Q^s = -9 + 5P - 2.5R$, where T is the price of titanium, a metal used to make golf clubs, and R is the price of rubber.
a) If R = 2 and T = 10, calculate the equilibrium price and quantity of golf balls.
b) At the equilibrium values, calculate the price elasticity of demand and the price elasticity of supply.
c) At the equilibrium values, calculate the cross-price elasticity of demand for golf balls with respect to the price of titanium. What does the sign of this elasticity tell you about whether golf balls and titanium are substitutes or complements?

a) Substituting the values of R and T, we get

$$Demand: Q^d = 70 - 2P$$
$$Supply: Q^s = -14 + 5P$$

In equilibrium, $70 - 2P = -14 + 5P$, which implies that P = 12. Substituting this value back, Q = 46.

b)

Elasticity of Demand = $\frac{\Delta Q^d}{\Delta P}\frac{P}{Q^d} = \frac{dQ^d}{dP}\frac{P}{Q^d}$=–2(12/46), or –0.52.

Elasticity of Supply =$\frac{\Delta Q^s}{\Delta P}\frac{P}{Q^s} = \frac{dQ^s}{dP}\frac{P}{Q^s}$ = 5(12/46) = 1.30.

c)

Cross-Price Elasticity of Demand of golf balls with respect of price of titanium

$$= \frac{\Delta Q^d_{\text{golf}}}{\Delta T}\frac{T}{Q^d_{\text{golf}}} = \frac{dQ^d_{\text{golf}}}{dT}\frac{T}{Q^d_{\text{golf}}}.$$

Given $Q^d = 90 - 2P - 2T$, $\frac{dQ^d_{\text{golf}}}{dT}$=-2

As P=12, T=10, then

$$\varepsilon_{golf,titanium} = -2(\frac{10}{46}) = -0.43.$$

The negative sign indicates that titanium and golf balls are complements, i.e., when the price of titanium goes up the demand for golf balls decreases.

2.21. Suppose that the market for air travel between Chicago and Dallas is served by just two airlines, United and American. An economist has studied this market and has estimated that the demand curves for round-trip tickets for each airline are as follows: $Q^d_U = 10{,}000 - 100P_U + 99P_A$ (United's demand) $Q^d_A = 10{,}000 - 100P_A + 99P_U$ (American's demand) where P_U is the price charged by United, and P_A is the price charged by American.

a) Suppose that both American and United charge a price of \$300 each for a round-trip ticket between Chicago and Dallas. What is the price elasticity of demand for United flights between Chicago and Dallas?

b) What is the market-level price elasticity of demand for air travel between Chicago and Dallas when both airlines charge a price of \$300? (*Hint:* Because United and American are the only two airlines serving the Chicago–Dallas market, what is the equation for the total demand for air travel between Chicago and Dallas, assuming that the airlines charge the same price?)

a)

$$Q^d_U = 10000 - 100(300) + 99(300)$$

$$Q^d_U = 9700$$

Using $P_U = 300$ and $Q^d_U = 9700$ gives

$$\varepsilon_{Q,P} = -100\left(\frac{300}{9700}\right) = -3.09$$

b)

Market demand is given by $Q^d = Q^d_U + Q^d_A$. Assuming the airlines charge the same price we have

$$Q^d = 10000 - 100P_U + 99P_A + 10000 - 100P_A + 99P_U$$

$$Q^d = 20000 - 100P + 99P - 100P + 99P$$

$$Q^d = 20000 - 2P$$

When $P = 300$, $Q^d = 19400$. This implies an elasticity equal to

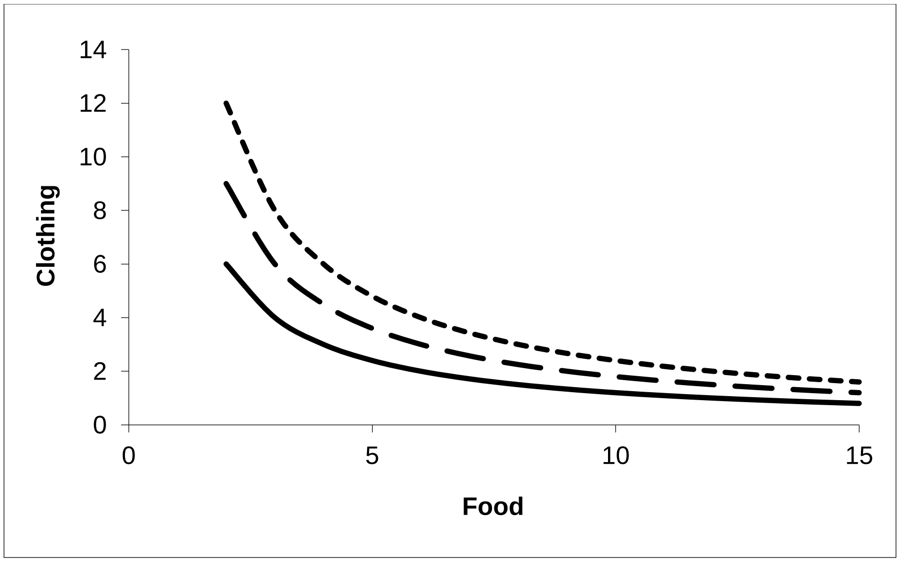
$$\varepsilon_{Q,P} = -2\left(\frac{300}{19400}\right) = -.0309$$

3.4 Consider the utility function $U(x, y) = y\sqrt{x}$ with the marginal utilities $MU_x = y/(2\sqrt{x})$ and $MU_y = \sqrt{x}$.
a) Does the consumer believe that more is better for each good?
b) Do the consumer's preferences exhibit a diminishing marginal utility of *x*? Is the marginal utility of *y* diminishing?

3.4 a) Since *U* increases whenever *x* or *y* increases, more of each good is better. This is also confirmed by noting that MU_x and MU_y are both positive for any positive values of x and y
.
b) Since $MU_x = y/2\sqrt{x}$, as x increases (holding y constant), MU_x falls. Therefore the marginal utility of *x* is diminishing. However, $MU_y = \sqrt{x}$. As *y* increases, MU_y does not change. Therefore the preferences exhibit a constant, not diminishing, marginal utility of *y*.

3.10 The utility that Julie receives by consuming food *F* and clothing *C* is given by *U*(*F*, *C*) = *FC*. For this utility function, the marginal utilities are $MU_F = C$ and $MU_C = F$.
a) On a graph with *F* on the horizontal axis and *C* on the vertical axis, draw indifference curves for *U* = 12, *U* = 18, and *U* = 24.
b) Do the shapes of these indifference curves suggest that Julie has a diminishing marginal rate of substitution of food for clothing? Explain.
c) Using the marginal utilities, show that the $MRS_{F,C} = C/F$. What is the slope of the indifference curve *U* = 12 at the basket with 2 units of food and 6 units of clothing? What is the slope at the basket with 4 units of food and 3 units of clothing? Do the slopes of the indifference curves indicate that Julie has a diminishing marginal rate of substitution of food for clothing? (Make sure your answers to parts (b) and (c) are consistent!)

a)



b) Yes, since the indifference curves are bowed in toward the origin we know that $MRS_{F,C}$ declines as *F* increases and *C* decreases along an indifference curve.

c) $$MRS_{F,C} = \frac{MU_F}{MU_C} = \frac{C}{F}$$

When $F = 2$ and $C = 6$, $MRS_{F,C} = 3$. The slope of the indifference curve is –3. When $F = 4$ and $C = 3$, $MRS_{F,C} = 0.75$, so the slope of the indifference curve is –0.75. Since the marginal rate of substitution falls as *F* increases and *C* decreases, she has a diminishing marginal rate of substitution.

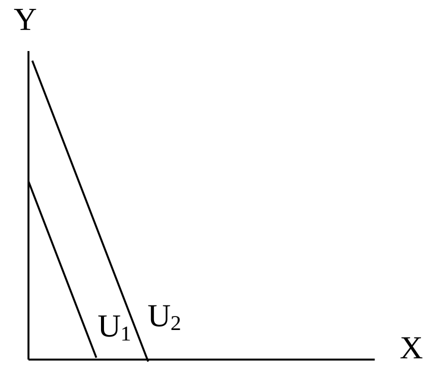
3.15 Consider the utility function $U(x, y) = 3x + y$, with $MU_x = 3$ and $MU_y = 1$.
a) Is the assumption that more is better satisfied for both goods?
b) Does the marginal utility of *x* diminish, remain constant, or increase as the consumer buys more *x*? Explain.
c) What is $MRS_{x,y}$?
d) Is $MRS_{x,y}$ diminishing, constant, or increasing as the consumer substitutes *x* for *y* along an indifference curve?
e) On a graph with *x* on the horizontal axis and *y* on the vertical axis, draw a typical indifference curve (it need not be exactly to scale, but it needs to reflect accurately whether there is a diminishing $MRS_{x,y}$). Also indicate on your graph whether the indifference curve will intersect either or both axes. Label the curve U_1.
f) On the same graph draw a second indifference curve U_2, with $U_2 > U_1$.

a) Yes, the "more is better" assumption is satisfied for both goods since both marginal utilities are always positive.
b) The marginal utility of *x* remains constant at 3 for all values of *x*.

c) $MRS_{x,y} = 3$, which is $\boldsymbol{MU_{x/}\ MU_{y=3/1=3}}$

d) The $MRS_{x,y}$ remains constant moving along the indifference curve.

e & f) See figure below



4.3 Julie has preferences for food *F* and clothing *C*. Her utility function was *U*(*F*, *C*) = *FC*. Her marginal utilities were *MUF* = *C* and *MUC* = *F*. You were asked to draw the indifference curves *U* = 12, *U* = 18, and *U* = 24, and to show that she had a diminishing marginal rate of substitution of food for clothing. Suppose that food costs $1 a unit and that clothing costs $2 a unit. Julie has $12 to spend on food and clothing.

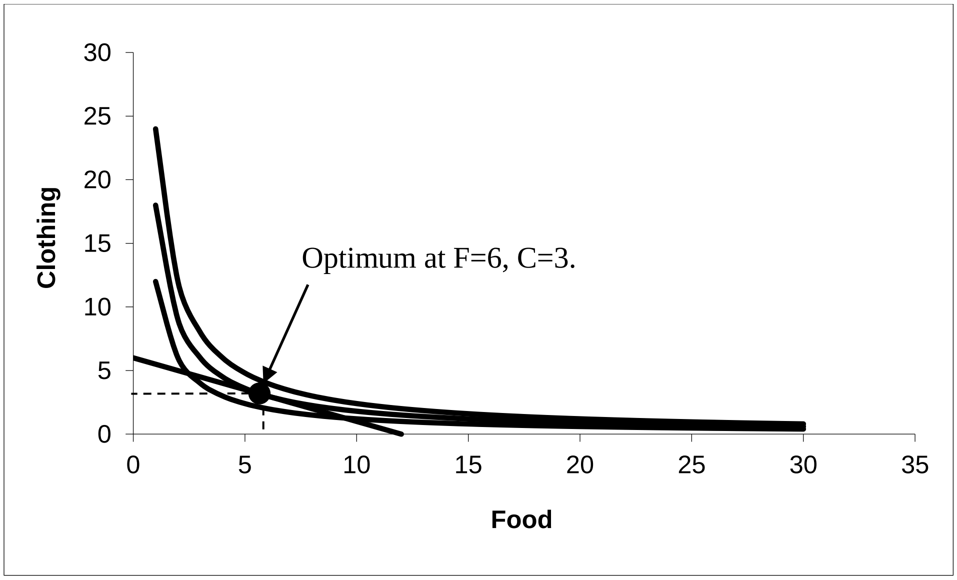
a) Using a graph (and no algebra), find the optimal (utility-maximizing) choice of food and clothing. Let the amount of food be on the horizontal axis and the amount of clothing be on the vertical axis.

b) Using algebra (the tangency condition and the budget line), find the optimal choice of food and clothing.

c) What is the marginal rate of substitution of food for clothing at her optimal basket? Show this graphically and algebraically.

d) Suppose Julie decides to buy 4 units of food and 4 units of clothing with her $12 budget (instead of the optimal basket). Would her marginal utility per dollar spent on food be greater than or less than her marginal utility per dollar spent on clothing? What does this tell you about how she should substitute food for clothing if she wanted to increase her utility without spending any more money?

a)



b) The tangency condition implies that

$$\frac{MU_F}{MU_C} = \frac{P_F}{P_C}$$

Plugging in the known information results in

$$\frac{C}{F} = \frac{1}{2}$$
$$2C = F$$

Substituting this result into the budget line, $F + 2C = 12$, yields

$$2C + 2C = 12$$
$$4C = 12$$
$$C = 3$$

Finally, plugging this result back into the tangency condition implies $F = 6$. At the optimum the consumer choose 6 units of food and 3 units of clothing.

c) At the optimum, $MRS_{F,C} = C/F = 3/6 = 1/2$. Note that this is equal to the ratio of the price of food to the price of clothing. The equality of the price ration and $MRS_{F,C}$ is seen in the graph above as the tangency between the budget line and the indifference curve for $U = 18$.

d) If the consumer purchases 4 units of food and 4 units of clothing, then

$$\frac{MU_F}{P_F} = \frac{4}{1} = 4 > \frac{MU_C}{P_C} = \frac{4}{2} = 2.$$

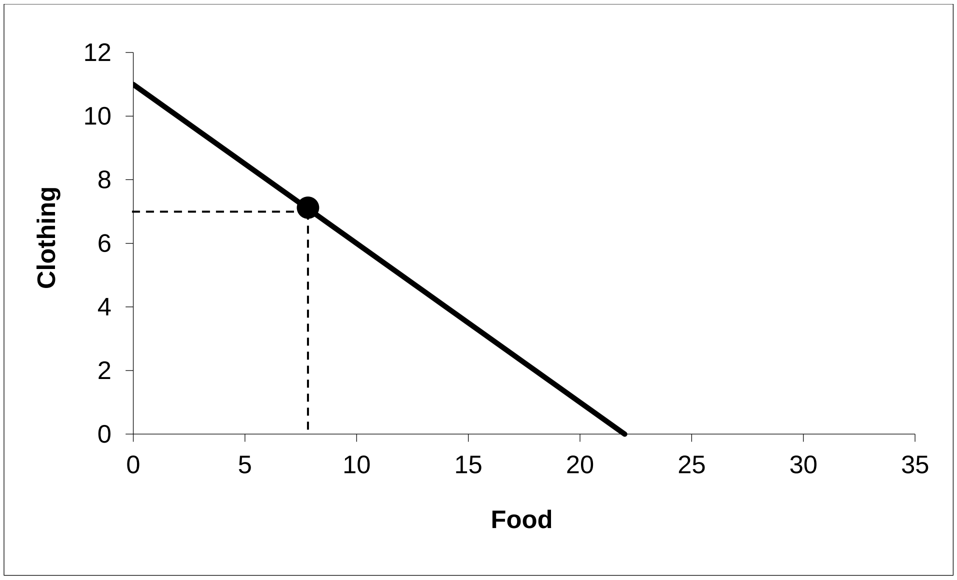
This implies that the consumer could reallocate spending by purchasing more food and less clothing to increase total utility. In fact, at the basket (4, 4) total utility is 16 and the consumer spent $12. By giving up one unit of clothing the consumer saves $2 which can than be used to purchase two units of food (they each cost $1). This will result in a new basket (6,3), total utility of 18, and spending of $12. By reallocating spending toward the good with the higher "bang for the buck" the consumer increased total utility while remaining within the budget constraint.

4.4 The utility that Ann receives by consuming food *F* and clothing *C* is given by *U*(*F*, *C*) = *FC* + *F*. The marginal utilities of food and clothing are *MUF* = *C* + 1 and *MUC* = *F*. Food costs $1 a unit, and clothing costs $2 a unit. Ann's income is $22.
a) Ann is currently spending all of her income. She is buying 8 units of food. How many units of clothing is she consuming?
b) Graph her budget line. Place the number of units of clothing on the vertical axis and the number of units of food on the horizontal axis. Plot her current consumption basket.
c) Draw the indifference curve associated with a utility level of 36 and the indifference curve associated with a utility level of 72. Are the indifference curves bowed in toward the origin?
d) Using a graph (and no algebra), find the utility maximizing choice of food and clothing.
e) Using algebra, find the utility-maximizing choice of food and clothing.
f) What is the marginal rate of substitution of food for clothing when utility is maximized? Show this graphically and algebraically.
g) Does Ann have a diminishing marginal rate of substitution of food for clothing? Show this graphically and algebraically.

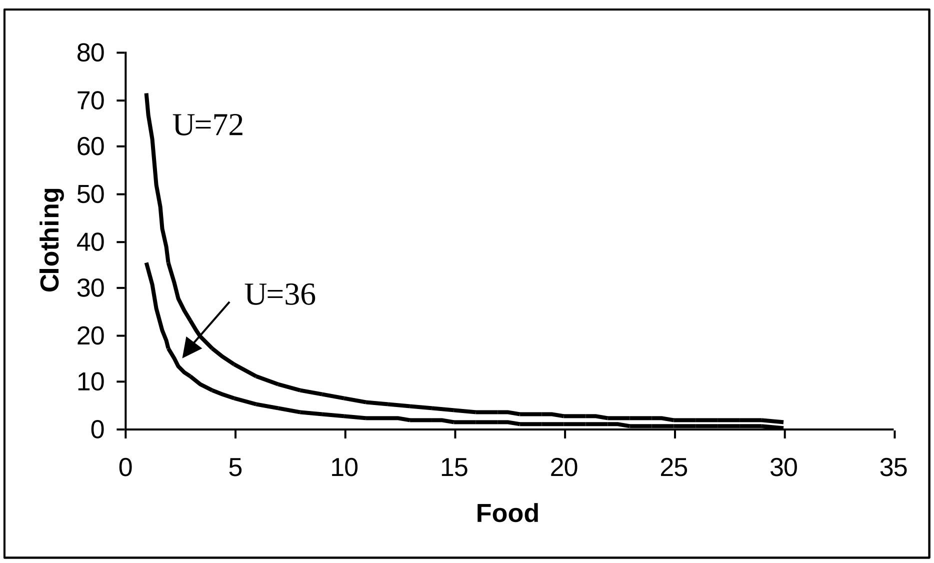
a) If Ann is spending all of her income then

$$F + 2C = 22$$
$$8 + 2C = 22$$
$$2C = 14$$
$$C = 7$$

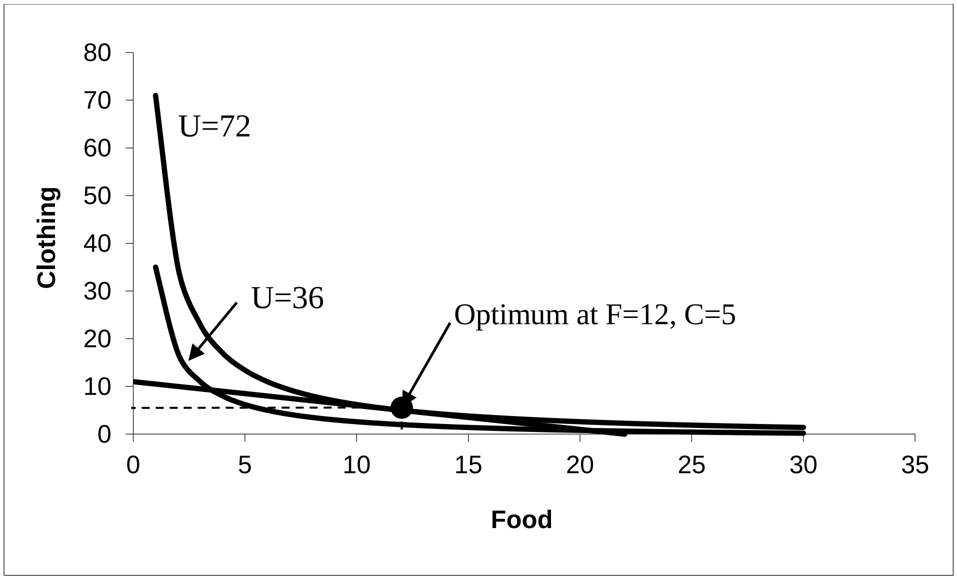
b)



c) Yes, the indifference curves are bowed in toward the origin. Also, note that they intersect the *F*-axis.



d)



e) The tangency condition requires that

$$\frac{MU_F}{MU_C} = \frac{P_F}{P_C}$$

Plugging in the known information yields

$$\frac{C+1}{F} = \frac{1}{2}$$
$$2C + 2 = F$$

Substituting this result into the budget line, $F + 2C = 22$ results in

$$(2C+2) + 2C = 22$$
$$4C = 20$$
$$C = 5$$

Finally, plugging this result back into the tangency condition implies that $F = 2(5) + 2 = 12$. At the optimum the consumer chooses 5 units of clothing and 12 units of food.

f) $MRS_{F,C} = \frac{C+1}{F} = \frac{5+1}{12} = \frac{1}{2}$ The marginal rate of substitution is equal to the price ratio.

g) Yes, the indifference curves do exhibit diminishing $MRS_{F,C}$. We can see this in the graph in part c) because the indifference curves are bowed in toward the origin. Algebraically, $MRS_{F,C} = {}^{C+1}\!/_{F}$. As F increases and C decreases along the indifference curve, $MRS_{F,C}$ diminishes.