CARLETON UNIVERSITY

DEFERRED
EXAMINATION
FEBRUARY 2005

Course: MATH 1004
STUDENTS MUST USE **SCANTRON SHEETS** AS SUPPLIED– USE **SOFT PENCIL** ONLY

DURATION: 3 HOURS

SCANTRON SHEETS REQUIRED

Department Name and Course Number: School of Mathematics and Statistics, MATH 1004
Course Instructor(s): Dr. S. Dubé, Dr. A.B. Mingarelli, Dr. M. Sadeghi

AUTHORIZED MEMORANDA

NON-PROGRAMMABLE CALCULATOR PERMITTED.

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1. Please count your pages now. **This examination has pages numbered from 1 - 19**, excluding this page. If you feel there is a page missing please report this to your Proctor.

2. **The examination is out of a total of 100** and consists of 30 multiple choice questions of which the first 25 are valued at 3 marks each, while the remaining ones are each worth 5 marks. **Please darken only one answer** as there is only one answer to any given question. Circling two or more answers to any question invalidates that question (*i.e.*, you get 0 marks for that question). If you do not believe any of the answers to a given question, please provide justification on the reverse side of the paper. Additional pages are added for your convenience including rough work.

3. Please **note your Instructor's name** and **Section** below: Use a soft pencil for inserting your replies onto the Scantron sheets. Please ensure that you erase completely if modifying an answer.

Multiple-Choice Questions
Please circle only one answer.

1. **[3 marks]** Let $f(x) = \ln(\sqrt{\sin x})$. Evaluate $f'(\pi/4)$. In other words, find the derivative of f at $x = \pi/4$.

   (a) $f'(\frac{\pi}{4}) = -1$
   (b) $f'(\frac{\pi}{4}) = \frac{1}{2}$
   (c) $f'(\frac{\pi}{4}) = \frac{\sqrt{2}}{3}$
   (d) $f'(\frac{\pi}{4}) = 0$

2. **[3 marks]** Let $f(x) = 1 + x^x$. Evaluate $f'(1)$. In other words, find the derivative of f at $x = 1$.

   (a) $f'(1) = 0$
   (b) $f'(1) = -1$
   (c) $f'(1) = 1$
   (d) $f'(1) = 2$

3. **[3 marks]** Let $f(x) = x|x|$. Calculate

$$L = \lim_{h \to 0} \frac{f(h) - f(0)}{h}.$$

   (a) $L = 0$
   (b) $L = 1$
   (c) $L = -1$
   (d) The limit does not exist

4. **[3 marks]** Evaluate the following limit: $L = \lim_{x \to \infty} \frac{x^2 + 2e^x}{3x^2 + 4e^x}$.

   (a) $L = 0$
   (b) $L = 1$
   (c) $L = \frac{1}{3}$
   (d) $L = \frac{1}{2}$

5. **[3 marks]** Let $f(x) = (3.2)^{x^2}$. Evaluate $f'(x)$. In other words, find the derivative of f at x.

   (a) $f'(x) = 2x \ln(3.2)\,(3.2)^{x^2}$
   (b) $f'(x) = (3.2)^{x^2}$
   (c) $f'(x) = 2x\,(3.2)^{x^2}$
   (d) $f'(x) = x^2\,(3.2)^{x^2-1}$

6. **[3 marks]** A differentiable function f has the property that $f(\sqrt{2}) = \pi$, $f'(\sqrt{2}) = \frac{\pi}{3}$ and $f'(\pi) = \frac{\pi}{6}$ and $f''(\sqrt{2}) = \pi\sqrt{3}$. What is the value of the derivative of $f(f(x))$ at $x = \sqrt{2}$?

   (a) $\frac{\pi}{2}$
   (b) π
   (c) 0
   (d) $\frac{\pi^2}{18}$

7. **[3 marks]** Simplify the expression $f(x) = \tan(\text{Arcsin}\,\sqrt{1-x^2})$ given that $0 \le x \le 1$.

   (a) $f(x) = x$
   (b) $f(x) = \sqrt{1-x^2}$
   (c) $f(x) = x\sqrt{1-x^2}$
   (d) $f(x) = \frac{\sqrt{1-x^2}}{x}$

8. **[3 marks]** Evaluate the limit: $L = \lim_{x\to 1^+} \frac{\sin(\sqrt{x-1})}{\sqrt{x^2-1}}$.

   (a) $L = \frac{\sqrt{2}}{2}$
   (b) $L = -1$
   (c) $L = \frac{1}{2}$
   (d) $L = 9$

9. **[3 marks]** Let v be given implicitly as a differentiable function of u by $v^2\cos(u)+u^2-1=0$. Calculate the value of the derivative $\dfrac{dv}{du}$ at the point (u,v) where $u=0$, $v=1$:

   (a) 1,
   (b) -1,
   (c) 2,
   (d) 0

10. **[3 marks]** Evaluate $L=\lim\limits_{x\to\infty}\left\{\sqrt{1+x^2}-\sqrt{x^2-1}\right\}$ using any method.

    (a) The limit does not exist
    (b) $L=1$
    (c) $L=0$
    (d) $L=-1$

11. **[3 marks]** Evaluate

$$I=\lim_{x\to 0}\frac{d}{dx}\int_{2x}^{4x}\frac{\sin u}{2u}\,du$$

    (a) $I=2$
    (b) $I=\dfrac{1}{2}$
    (c) The limit does not exist
    (d) $I=1$

12. **[3 marks]** Find the critical points of the function f defined by $f(x)=\dfrac{x}{1+x^2}$.

    (a) $x=0,\ x=1,\ x=2$

    (b) $x=0,\ x=1,\ x=2,\ x=-2$

    (c) $x=1,\ x=-1$

    (d) $x=0,\ x=2,\ x=-2$.

13. **[3 marks]** Determine the largest interval in which the function f whose values are given by $f(x) = x^4 - 2x^3 + 6$ is concave up.

    (a) $\{x : x < \frac{1}{2}\}$

    (b) $\{x : 1 < x\} \cup \{x : x < 0\}$

    (c) $\{x : 0 < x < 1\}$

    (d) $\{x : 2 < x\}$

14. **[3 marks]** Find all the asymptotes of the function defined by $p(x) = \dfrac{(x+1)^2}{2(x-1)(x+2)}$.

    (a) Vertical asymptotes at $x = -1, 1, 2$. No horizontal asymptotes.
    (b) Vertical asymptotes at $x = 1$ and $x = -2$. Horizontal asymptote given by $y = 1/2$.
    (c) Vertical asymptotes at $x = -2$ and $x = 1$. Horizontal asymptote given by $y = 1$.
    (d) Vertical asymptotes at $x = 1$ and $x = -2$. Horizontal asymptote at $y = -1$.

15. **[3 marks]** For what values of x is the function $f(x) = \dfrac{3}{1+x^2}$ decreasing?

    (a) $x > 0$
    (b) $x > 1$
    (c) $-\infty < x < \infty$
    (d) $x < 0$

16. **[3 marks]** Which of the following functions has a point of inflection at $x = 0$?

    (a) $f(x) = x^3 + 2x - 1$
    (b) $f(x) = -x^2 + 8x$
    (c) $f(x) = x^4 + x^2 + 1$
    (d) $f(x) = x - 2$

17. **[3 marks]** Find ALL the horizontal asymptotes of the graph of the function defined by $f(x) = \dfrac{4x}{\sqrt{5+x^2}}$

(a) $y = 4,\ y = 5$ only
(b) $y = 4,\ y = -4$ only
(c) $y = 1,\ y = 0$ only
(d) $y = 5$ only

18. **[3 marks]** An antiderivative of $f(x) = \sec^2(x+2)$ is given by

(a) $\tan(x+2) + 2$
(b) $\tan^2(x) + 2$
(c) $(\sin x)^2$
(d) $\tan x$

19. **[3 marks]** Evaluate the definite integral $\displaystyle\int_{\pi^2}^{16} \frac{\cos(\sqrt{x})}{3\sqrt{x}}\, dx$

(a) 0
(b) $\dfrac{2\sin 4}{3}$
(c) $\dfrac{1}{3}$
(d) $\dfrac{\sin 4}{4}$

20. **[3 marks]** The value of $\displaystyle\int t\,(\ln t)^2\, dt$ is given by

(a) $\dfrac{1}{2}t^2\,(\ln t)^2 + 2t\ +\ C$
(b) $\dfrac{1}{2}t^2\,(\ln t)^2 + \dfrac{t^2}{4}\ +\ C$
(c) $\dfrac{1}{2}t^2\,(\ln t)^2 - \dfrac{t^2 \ln t}{2}\ +\ \dfrac{t^2}{4} + C$
(d) $\dfrac{1}{3}t(\ln t)^3\ +\ C$

21. **[3 marks]** The improper integral $\int_0^\infty x^2 e^{-x^3}\, dx$ is given by

(a) $\frac{1}{3}$
(b) 1
(c) $\frac{\ln 2}{3}$
(d) $\frac{1}{3}\ln 3$

22. **[3 marks]** Evaluate the definite integral $\int_0^{\pi/4} \sin(2x)\cos(2x)\, dx$.

(a) 0
(b) 1/4
(c) 1
(d) -2

23. **[3 marks]** The general form of the partial fraction decomposition of the rational function

$$\frac{x^2}{(x^2+1)(x-1)^2}$$

is given by

(a) $\frac{x^2}{(x^2+1)} + \frac{1}{(x-1)^2}$
(b) $\frac{Ax+B}{x^2+1} - \frac{C}{(x+1)^2}$
(c) $\frac{A}{x^2+1} - \frac{B}{(x-1)^2}$
(d) $\frac{Ax+B}{x^2+1} + \frac{C}{(x-1)} + \frac{D}{(x-1)^2}$

where A, B, C, D are suitable constants.

24. **[3 marks]** Evaluate the definite trigonometric integral

$$\int_0^{\pi/2} \sin^2 x\, \cos^3 x\, dx.$$

(a) $\frac{2}{15}$
(b) 0
(c) $\frac{1}{2}$
(d) $\frac{1}{5}$

25. **[3 marks]** Find an expression for the area between the curves defined by $y = x + 2$ and $y = x^2$.

(a) $\int_2^{-1} (x^2 - x)\, dx$

(b) $\int_0^4 (x^2 - x - 2)\, dx$

(c) $\int_0^1 (x + 2 + x^2)\, dx$

(d) $\int_{-1}^2 (x + 2 - x^2)\, dx$

26. **[5 marks]** Find an expression for the volume of the solid of revolution obtained by rotating the region bounded by the curves $y = \sqrt{x}$ and $y = x$ about the y-axis.

(a) $\int_0^1 (y - y^2)\, dy$

(b) $\int_0^2 \pi\,(x - x^2)\, dx$

(c) $\int_0^1 \pi\,(y^2 - y^4)\, dy$

(d) $\int_0^1 (y - \sqrt{y})\, dy$

27. **[5 marks]** Solve the initial value problem for the differential equation

$$e^{-x}\frac{dy}{dx} = xy^2, \quad y(0) = 1.$$

(a) $\dfrac{1}{y(x)} = -e^x + xe^x$

(b) $\dfrac{1}{y(x)} = e^x - xe^x$

(c) $y(x) = \dfrac{1}{1-x}$

(d) $y(x) = \dfrac{e^x}{x+1}$

28. **[5 marks]** Evaluate the indefinite integral $\int x^2 \cos 3x\, dx$.

(a) $x^2 \sin 3x - 2x \cos 3x + 2 \sin 3x + C$

(b) $x^2 \cos 3x + \dfrac{2x}{3} \sin 3x - \dfrac{3}{5} \cos 3x + C$

(c) $\dfrac{1}{3}x^2 \sin 3x + \dfrac{2x}{9} \cos 3x - \dfrac{2}{27} \sin 3x + C$

(d) $\dfrac{1}{3}x^2 \sin 3x + \dfrac{2x}{5} \cos 3x - \dfrac{1}{10} \sin 3x + C$

29. **[5 marks]** Evaluate the definite integral $\int_0^1 \frac{x}{\sqrt{4+x^2}}\,dx$.

(a) $\sqrt{5}-2$

(b) $\frac{\sqrt{5}-1}{2}$

(c) $\frac{\sqrt{5}}{5}$

(d) 6

30. **[5 marks]** Evaluate the improper integral

$$\int_{-\infty}^{\infty} \frac{dx}{9+x^2}.$$

(a) 0

(b) $\frac{\pi}{3}$

(c) $\frac{\pi}{6}$

(d) π

END

Pages 9-19 were left blank for rough work. Therefore, there was no point in scanning them.

- Greg H.