Concordia University
Math 209 Mid Term Review Sec 3.1-4.5

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Date: 05/15/2013

Show all work.

1. Find the value of the following limit algebraically: $\lim\limits_{x\to -2} \frac{x^2-5x-14}{x+2}$
2. Consider $f(x) = 5x^2 + 1$:
   a) Determine $f'(x)$ using the formula: $f'(x) = \lim\limits_{h\to 0} \frac{f(x+h)-f(x)}{h}$.
   b) Determine $f'(-2)$.
   c) Determine the equation of the tangent to $f(x)$ at the point where $x = -2$.
3. Find $f'(x)$ for the following (Do not simplify answers):
   a) $f(x) = 4x^3 + 7x - 8 + \sqrt[4]{x}$
   b) $f(x) = \frac{4x-5}{x^2+3x}$
4. Find the derivatives of the following functions (Do not simplify answers):
   a) $f(x) = (e^x + x)^3(2 + \ln x)^4$
   b) $f(x) = [e^{12x^3-9x-15} + 4\ln(x^3 + 3x - 1)]^{10}$
5. Consider the Revenue function $R(x) = 200x - \frac{x^2}{30}$ and the Cost function $C(x) = 72000 + 60x$, where x is the number of articles, $R(x)$ and $C(x)$ are in dollars. Use differentials to approximate the change in profit as production increases from 1500 to 1510 articles.
6. a) Given the Revenue function $R(x) = \frac{x^3}{3} + 200x$ where x is the number of articles sold and $R(x)$ is the total Revenue produced in dollars, find the average rate of change in the Revenue as x goes from 20 to 21. Describe in words what this answer represents.
   b) For the Revenue function in a) find $R'(x)$ at a sales level of 20 articles. Describe in words what this answer represents.
   c) For the Revenue function in a) Use $R'(20)$ and $R(20)$ to approximate $R(21)$
   d) For the Revenue function in a) find the marginal average Revenue at a sales level of 20 articles. Describe in words what this answer represents.
7. At what nominal (yearly) rate compounded continuously must money be invested to double in ten years ?
8. The resale value R (in dollars) of a company car after t years is estimated to be given by $R(t) = 20000(0.86)^t$. What is the rate of depreciation after one year ? What is this rate after two years and after three years ?
9. Consider the price p and demand x are related by the equation $x = 300 - (p - 15)^2$.
   a) Find the rate of change of the demand compared to price if the price is as given and then explain in words what each answer means:(i) $p = 1\$$ (ii) $p = 16\$$.
   b) Find the rate of change of price compared to number of articles if the price is as given and then explain in words what each answer means:(i) $p = 1\$$ (ii) $p = 16\$$. (round the answers to the nearest cent)
10. Consider x and y are related by the equation $2x + xy - y^2 - e^x = 10$. Find the rate of change of y compared to x if $x = 0$ and $y = 2$. Which variable is changing faster at the given point, x or y ?

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Answers:

1. -9, **2. a)** $10x$ **b)** -20 **c)** $y = -20x - 19$, **3. a)** $12x^2 + 7 + \frac{1}{4}x^{-\frac{3}{4}}$. **b)** $\frac{(x^2+3x)(4)-(4x-5)(2x+3)}{(x^2+3x)^2}$, **4. a)** $f(x) = (e^x + x)^3 4(2 + \ln x)^3 \frac{1}{x} + (2 + \ln x)^4 3(e^x + x)^2(e^x + 1)$. **b)** $10[e^{12x^3-9x-15} + 4\ln(x^3 + 3x - 1)]^9[e^{12x^3-9x-15}(36x^2 - 9) + \frac{4(3x^2+3)}{x^3+3x-1}]$, **5.** 400\$, **6. a)** 620.33\$, **the exact Revenue from the 21st article sold. b)** 600\$, **the marginal revenue when** $x = 20$ **or the approximate Revenue from the 21st article sold. c)** $R(20) + R'(20) = 7266.66$\$ **d)** 13.33\$ **the increase in Revenue attributed to each article sold by selling the 21st article., 7.** 6.93%, **8.** $2594\$/year, 2231\$/year, 1919\$/year$., **9 a)(i) Demand increasing** 28 **articles per** 1\$ **increase in price (at this point price increase is not lowering the demand). (ii) Demand decreasing** 2 **articles per** 1\$ **increase in price (at this point price increase is lowering the demand). 9 b) (i) Price increases** 0.04\$ **per** 1 **article increase in demand (at this point demand increase is causing the price to rise) (ii) Price decreases** 0.50\$ **per** 1 **article increase in demand (at this point demand increase is not causing the price to rise). 10.** $\frac{dy}{dx} = \frac{e^x-2-y}{x-2y}$, **at the given point** $\frac{dy}{dx} = \frac{3}{4}$ **(x is changing faster than y)**