

M. Golding is a manufacturer of toy sailboats used by children to play in the bathtub. He is wondering how to improve the quality of his forecasts. A management student, who knows about a lot of things, including forecasting, proposed three forecasting methods. The results of student's analysis are given in the following table:

Month	Demand	Moving Average	Exponential Smoothing	Linear Regression
		N = 3	$\alpha = 0.3$	a = 1367 b = 20.5
1	1514		1514	1387.5
2	1850		1514.0	1408
3	1712		1614.8	1428.5
4	1185	1692.0	1644.0	1449
5	1250	1582.3	1506.3	1469.5
6	1283	1382.3	1429.4	1490
7	1711	1239.3	1385.5	1510.5
8	1511	1414.7	1483.1	1531
9	1425	1501.7	1491.5	1551.5
10	750	1549.0	1471.5	1572
11	875	1228.7	1255.1	1592.5
12	304	1016.7	1141.1	1613
13	2277	643.0	889.9	1633.5
14	2790	1152.0	1306.1	1654
15	1925	1790.3	1751.2	1674.5
16	1546	2330.7	1803.4	1695
17	1563	2087.0	1726.2	1715.5
18	1641	1678.0	1677.2	1736
19	2419	1583.3	1666.3	1756.5
20	1855	1874.3	1892.1	1777
21	2034	1971.7	1881.0	1797.5
22	1055	2102.7	1926.9	1818
23	1333	1648.0	1665.3	1838.5
24	405	1474.0	1565.6	1859
25	3080	931.0	1217.4	1879.5
26	3496	1606.0	1776.2	1900
27	2746	2327.0	2292.1	1920.5
28	1981	3107.3	2428.3	1941
29	2017	2741.0	2294.1	1961.5
30	2046	2248.0	2211.0	1982
31	2999	2014.7	2161.5	2002.5
32	2311	2354.0	2412.7	2023
33	2617	2452.0	2382.2	2043.5
34	1044	2642.3	2452.7	2064
35	1851	1990.7	2030.1	2084.5
36	505	1837.3	1976.3	2105.0

The coefficient of determination, R^2 for the linear regression over the 36 months was 0.082

- [3] (a) Make a forecast for month 37 with the three models Show all your calculations and fill in the blanks:

Moving Average

$F_{37} =$

Exponential Smoothing

$F_{37} =$

(b) The student calculated the Sum of the Absolute Errors ($\sum E_t$) for the 35 months inclusive:

	Moving Average	Exponential Smoothing	Linear Regression
Sum of the Absolute Errors (month 1 to 35)	20,990.67	18,149.89	17,757.00

Taking into consideration the forecast of period 36 showing in the table on page 2, calculate the Mean Absolute Deviation (MAD) for two of the three forecasting methods. Show all your calculations.

Moving Average: MAD =

Linear Regression: MAD =

[2] (c) The student calculated the Sum of the Squared Errors ($\sum E_t^2$) for the 36 months inclusive:

	Exponential Smoothing	Linear Regression
Sum of the Squared Errors (month 1 to 36)	20,841,369.42	18,426,019.50

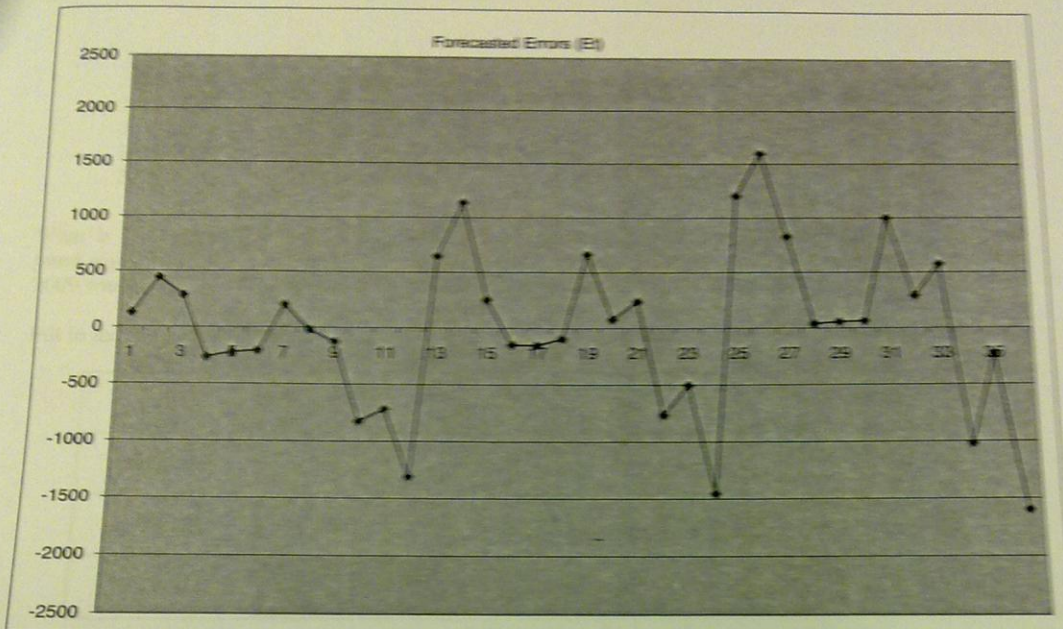
Calculate the standard deviation of the errors for two of the three forecasting methods, using the above data. Show all your calculations.

Exponential Smoothing: $\sigma =$

Linear Regression: $\sigma =$

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Golding decides to track the evolution of the errors from the regression model through time (see graph below).



- (d) Based on the graph and the measures of accuracy/adequacy for linear regression method, what are your conclusions and recommendations? (i.e. is linear regression a good method to use in this case? What forecasting method should Mr. Golding use? Should other methods be considered for this forecast? If yes, indicate which one(s) and why, but do not do any calculations. Justify each of your answers briefly.

Problem 2: Forecasting: General Independent Questions (16 marks)

(a) The president of Province University wants to forecast student enrollments for next academic year (2010-2011) based on the following historical data:

Year	Enrollments
2005-2006	20,000
2006-2007	20,500
2007-2008	21,000
2008-2009	23,100
2009-2010	25,000

What is the forecast for 2010-2011 using double exponential smoothing (exponential smoothing with trend adjustment) if $\alpha = 0.5$ and $\beta = 0.1$? Assume that the exponentially smoothed forecast, \hat{F}_t , for 2008-2009 was 21,425 and the exponentially smoothed trend estimate, \hat{T}_t , for 2008-2009 was 1,575.

Fill in the following table:

$\alpha = 0.5$ $\beta = 0.1$

Year	A_t	F_t	\hat{F}_t	\hat{T}_t
2008-2009	23,100			
2009-2010	25,000			
2010-2011				

Show your calculations below:

(b) In January 2010, Mr. Honey, a life insurance broker, wished to forecast the annual number of new life-insurance contracts for his company for 2010 using an exponential smoothing model with $\alpha = 0.2$.

Unfortunately, a clerk committed an error in computing the 2010 forecast: he used 1500 as the number of new contracts in 2008, instead of 5100 contracts in that year. Given this error, should the 2010 forecast be increased or decreased and by how much? Give the details of your analysis.

(c) Suppose you use the multiplicative decision model (with linear regression) with 4 seasons (i.e. $N = 4$) on a stable time series, centered around an average value of 1,300. What would you expect the values of a (i.e. the y-intercept estimate), and b (i.e. the slope estimate), and S_1, S_2, S_3 and S_4 (i.e. the seasonal factors) to be? Justify your answer. **Hint:** plot the graph of a stable times series.

- [4] (d) The Holiday Hotel chain owns four hotels in Ottawa region. The chain collected the number of room occupancy per day in each of its hotel for the past five years. Using these data, the chain calculated the average number of room occupancy for each quarter of the year. The table below summarizes the time series data.

Average number of room occupancy per quarter

Season	Year				
	1	2	3	4	5
I	1996	2036	2182	2227	2288
II	2320	2414	2475	2552	2612
III	2844	2917	2933	3041	3102
IV	2248	2264	2338	2394	2499

The multiplicative decomposition model is used to forecast the quarter demand. There are four seasons per year and linear regression has been used on the de-seasonalized demand to estimate the trend component. The following results have been obtained using this model:

Year	Season	Period t	Demand (A _t)	Trend (T _t)	Forecast (F _t)	Error (E _t)
1	I	1	1996	2314.81	2023.14394	-27.14394
	II	2	2320	2332.57	2332.57	-12.57
	III	3	2844	2350.33	2803.94369	40.05631
	IV	4	2248	2368.09	2209.42797	38.57203
2	I	5	2036	2385.85	2085.2329	-49.2329
	II	6	2414	2403.62	2403.62	10.38
	III	7	2917	2421.38	2888.70634	28.29366
	IV	8	2264	2439.14	2275.71762	-11.71762
3	I	9	2182	2456.90	2147.3306	34.6694
	II	10	2475	2474.66	2474.66	0.34
	III	11	2933	2492.43	2973.46899	-40.46899
	IV	12	2338	2510.19	2342.00727	-4.00727
4	I	13	2227	2527.95	2209.4283	17.5717
	II	14	2552	2545.71	2545.71	6.29
	III	15	3041	2563.47	3058.21971	-17.21971
	IV	16	2394	2581.24	2408.29692	-14.29692
5	I	17	2288	2599.00	2271.526	16.474
	II	18	2612	2616.76	2616.76	-4.76
	III	19	3102	2634.52	3142.98236	-40.98236
	IV	20	2499	2652.28	2474.57724	24.42276

Using this multiplicative decomposition model, calculate the forecast for the four quarters of year 6. **Show your work below the table.**

Year	Season	Period	Forecast
6	I	21	
	II	22	
	III	23	
	IV	24	

Problem 3: Aggregate Planning (13 marks)

Forecast demands for the Tony television sets over the next six months appear below.

Month	Forecasted Demand
January	150,000
February	130,000
March	240,000
April	180,000
May	190,000
June	120,000

There are currently (end of December) 180 workers employed at the plant. Ending inventory in December is expected to be 1,000 units, and the firm would like to have 2,000 units on hand at the end of June. The number of units produced by one worker in one day is 50 units. The employee works 20 days per month, 8 hours a day, at an hourly rate of \$16 per hour at regular time; and overtime costs \$24 per hour. It costs \$4,000 to hire an employee and \$3,500 to lay off one, the cost of holding one unit in inventory for one month is \$2, and backorder cost is \$5/unit.

- (a) Given the beginning inventory of 1,000 units, determine the quantity to produce each month and the minimum number of employees required each month if Tony wishes to meet the forecasted demand and ending inventory requirements of 2,000 units on hand at the end of June by varying the workforce level only (i.e. No stockouts or back orders, no ending inventory from January to May, no overtime and idle time is allowed). Show your calculations.

- (b) Given the beginning inventory of 1,000 units and assuming that Tony wishes to maintain a safety stock corresponding to 10% exactly of the forecasted demand at the end of each month (this requirement is also for the month of June instead of an inventory of 2,000 units), determine ONLY the quantity to produce each month if the company wishes to meet the forecasted demand and ending inventory requirements by varying only the workforce level (i.e. a chase strategy). Show your calculations. **This question is independent of part (a).**

Month	Forecasted Demand
January	150,000
February	130,000
March	240,000
April	180,000
May	190,000
June	120,000

- [2] (c) Tony is considering a **level strategy using regular time only**, and absorbing fluctuations in demand with inventories. Given the beginning inventory of 1,000 units, what is the **minimum workforce level** that will be required at the beginning of January that will guaranty no stockouts or backlogs during any of the next six months? Since they don't wish to use any overtime, they will hire or fire as many employees as necessary at the beginning of January. Show your calculations. **This question is independent from parts (a) and (b).**

- [2] (d) Forecasted demand for the last six months of the year appears below. The firm chooses to implement a level strategy: employees will produce at full capacity each month. The following table shows the end of month inventory or backorders resulting from this production plan. What is the total cost of this plan (for these six months)? Show your calculations. **This question is independent from parts (a), (b) and (c).**

Month	Forecasted Demand	Final Inventory	Backorders
July	180000	0	9,000
August	150000	11,000	0
September	170000	11,000	0
October	220000	0	39,000
November	160000	0	29,000
December	140000	1,000	0

(e) Find the preference thresholds for the following option.

- 2] (i) **Overtime versus hiring and firing:** What is the **smallest number of units** an employee can build using overtime production before it becomes advantageous to hire another employee and then fire that employee later when demands drop? Show your calculations.

- (ii) **Idle time versus building extra inventory:** What is the maximum time an employee can be kept building inventory to satisfy future demand before it becomes more advantageous to keep that employee idle? Show your calculations.