

Solution To Test 1 (Version A)

MAT1300-3X, Summer 2013

Total = 20 marks

1. (2 marks) (a) Find the slope of a line going through two points $(-2, -5)$ and $(3, 5)$.
 (b) Find the equation of this line.

Solution. (a) The slope of the line is $m = \frac{5 - (-5)}{3 - (-2)} = 2$.

(b) The equation of this line has the form $y = 2x + b$. Since $5 = 2 * 3 + b$. $b = -1$. The equation is $y = 2x - 1$.

2. (2 marks) Find the intersection point(s) of the graphs of functions $y = 2x^2 + 3x$, and $y = 4x + 1$.

Solution. Let $2x^2 + 3x = 4x + 1$. $2x^2 - x - 1 = 0$. $x = \frac{1 \pm \sqrt{1+8}}{4} = \frac{1 \pm 3}{4} = 1, -\frac{1}{2}$. The intersection points are $(1, 5)$, and $\left(-\frac{1}{2}, -1\right)$.

3. (4 marks) (a) Find the inverse function of the function $y = f(x)$, where $f(x) = \frac{3x-2}{-x+1}$.
 (2 marks)

(b) What is the domain of function $y = f(x)$? What is the domain of the inverse $y = f^{-1}(x)$? What is the range of function $y = f(x)$? What is the range of the inverse $y = f^{-1}(x)$? (2 marks)

Solution. (a) Solve the equation $y = \frac{3x-2}{-x+1}$ for x : $-xy + y = 3x - 2$. $(y+3)x = y+2$. $x = \frac{y+2}{y+3}$.

Interchanging x and y , the inverse function is $y = \frac{x+2}{x+3}$.

(b) The domain of $f(x)$ is $x \neq 1$, and the range of $f^{-1}(x)$ is $y \neq 1$.

The domain of $f^{-1}(x)$ is $x \neq -3$, and the range of $f(x)$ is $y \neq -3$.

4. (2 marks) Find the limit $\lim_{x \rightarrow 1} \frac{\sqrt{x}-1}{x-1}$.

Solution. $\lim_{x \rightarrow 1} \frac{\sqrt{x}-1}{x-1} = \lim_{x \rightarrow 1} \frac{(\sqrt{x}-1)(\sqrt{x}+1)}{(x-1)(\sqrt{x}+1)} = \lim_{x \rightarrow 1} \frac{x-1}{(x-1)(\sqrt{x}+1)} = \lim_{x \rightarrow 1} \frac{1}{\sqrt{x}+1} = \frac{1}{2}$.

5. (3 marks) (a) Find the derivative of the function $y = \frac{e^x}{x^2}$. (2 marks)

(b) At which value x does the graph of this function have a horizontal tangent line?

Solution. $y' = \frac{x^2 e^x - 2x e^x}{x^4} = \frac{(x-2)e^x}{x^3}$. When $x = 2$, the derivative of this function is 0, and its graph has a horizontal tangent line.

6. (2 marks) Solve the equation $3^{2x-1} = 5$, and express your answer with natural logarithm.

Solution. $2x - 1 = \log_3 5 = \frac{\ln 5}{\ln 3}$. $x = \frac{1}{2} \left(\frac{\ln 5}{\ln 3} + 1 \right)$.

7. (2 marks) Find the derivative of the function $y = x(\ln x - 1)$. Simplify your result.

Solution. $y' = \ln x - 1 + x \frac{1}{x} = \ln x$.

8. (3 marks) The revenue function of a product is given by $R(x) = 5000x - x^2$, and the cost function of this product is given by $C(x) = 2500x + 10000$.

(a) Find the profit function $P(x)$.

(b) Find the marginal profit function.

(c) If the current production level is $x = 1000$, should the company increase or decrease the production level x to increase the profit? Explain.

Solution. (a) The profit function is $P(x) = R(x) - C(x) = 2500x - x^2 - 10000$.

(b) The marginal profit function is $P'(x) = 2500 - 2x$.

(c) Since the marginal profit function is positive at $x = 1000$, increasing x will increase the profit.