MATH 1007 (Foroozan) Final Exam Review Packet for Test on 5/4/11 (SAMPLE)

Chapter 1: Two Candidates

1. Simple Majority Method:
- Selects a winner: The candidate that gets more than half of the total vote
- If each candidate gets exactly half of the voters, then it is considered a tie
2. Super Majority Method:
- Let p be a number ½ ≤ 1
- The supermajority method with parameter p is a method/function that selects a winner(s) based on the candidates who get a function of p of all votes
- A candidate who gets at least pt votes where t is a total number of votes.
- If no candidate achieves this, then it is considered a tie
- This is called a quota method and the q = $\lceil pt \rceil$; the smallest whole number greater than or equal to p
- If p = ½ : Simple Majority
- If p = 1: Unanimity
3. Status Quo method
- Candidate A and Candidate B
- One of the candidates is called a 'status quo candidate' and the other is called the 'challenger'
- Base method:
  - If either candidate wins under the base method, that candidate wins based on the status quo method
  - If there is a tie in the base method, then the status quo candidate is declared the winner by status quo method
4. Weighted voting method
- Suppose there are n voters: 1, 2, 3,…., n
  - Each voter is given a weight wi to the voter i
  - t = w1 +w2 + w3 + …. + wn
- A candidate who gets more than half of the votes is declared the winner
5. Monarchy Method
- One of the candidates is a monarch that wins no matter how anyone else votes
6. Dictatorship Method
- One of the voters is a dictator. The candidate he/she selects will be declared the winner
7. All-tie method
- The election is a tie no matter how the voters vote

Criteria:

Anonymity Criteria
- A method satisfies anonymity criteria if it treats all the voters equally
- A method that violates anonymity is the weighted method or dictatorship
- A method that satisfies anonymity is a simple majority method or monarchy
- A method is anonymous if and only if its outcome depends on the tabulated profile

Neutrality criteria
- A method is neutral if it treats all candidates equally
- A method that violates neutrality is the status quo method, and monarchy method
- A method that satisfies neutrality is the dictatorship, simple and supermajority methods

Monotonity Criteria
- Suppose a method chooses A as a winner. If after a change in which some voters witch their votes from B to A, A will still be chosen as a winner if the method satisfies monotonicity

8. Parity Method

- Then the method must declare B as a winner
- It treats all candidates equally

4. Monotonicity criteria
   - A method is monotone if the following conditions hold:
     - Suppose under a certain profile, A wins the method
     - Imagine a 2nd profile which is identical to the first profile except 'at least' one vote who raises candidate A's votes
     - Then A must still win the second profile as well

Before:

A	C	B	C	**C**	A	B
B	A	C	B	**B**	B	A
C	B	A	A	**A**	C	C

The method chooses A as a winner

After:

A	C	B	C	**C**	A	B
B	A	C	B	**A**	B	A
C	B	A	A	**B**	C	C

If the method satisfies monotonicity, then A is still the winner

5. The Pareto Criteria
   - A method satisfies pareto criteria if every voter prefers one candidate to another, say A over B, the method does not select B as a winner, regardless if A wins or not

A	C	A	C	A	C	A
B	A	C	A	C	A	B
C	B	B	B	B	B	C

If the method satisfies pareto method, then B cannot be selected as winner regardless if A wins or not

6. Condorcet criteria
   - Condorcet candidate: A candidate is called a Condorcet candidate if that candidate can beat any other candidate in a head-to-head match
   - Not every profile has a Condorcet candidate

- A method satisfies the Condorcet criteria if this method faces a profile with a Condorcet candidate and declares this candidate a unique winner

7. Anti-condorcet criteria
   - Anti-condorcet candidate: The anti-condorcet candidate loses to every other candidate in a head-to-head match
   - A method that satisfies anti-condorcet criteria if it faces an anti-condorcet candidate, this candidate cannot be selected as a winner (tie/unique)

Independence

- "Consistency"
- Independence of irrelevant alternative
- Say A is Al Gore; B is Bush

Before:

A	C	B	C	C	A	**B**

B	B	B	C	C
C	C	C	A	A

- A is the majority candidate
- A is also a Condorcet candidate
- B wins by borda count method

- Borda count method violates independence creiteria

C	A	A	B	B
A	B	B	A	A
B	C	C	C	C

- A wins
- B loses

C	A	A	B	B
A	B	B	C	C
B	C	C	A	A

- B wins
- A loses

<u>Black Method</u>

- This is a method that selects the Condorcet candidate as a winner, if there is one. Otherwise, it selects the borda count winner.
- Proposition:
    - The black method satisfies majority, Condorcet, anti-Condorcet, monotonicity, and Pareto, but not independence
        - Suppose there is majority candidate. This candidate is also the Condorcet candidate. The Black method selects this candidate as the unique winner
        - Suppose there is an anti-Condorcet candidate. This candidate cannot be the Condorcet candidate. If another candidate is a Condorcet candidate, the black method selects him/her as a winner but not the anti-Condorcet candidate. If there is no Condorcet candidate, then the Black method uses Borda Count to select the winner. We know that Borda count satisfies anti-condorcet criteria, so the anti-Condorcet candidate is not a winner
        - By definition, the Condorcet candidate is selected as a winner using Black method. So it obviously satisfies Condorcet criteria
        - If A is over B by all voters, then candidate B cannot be the Condorcet candidate. Moreover, has a lower borda score than candidate A, so candidate B cannot be a winner by Black method. Therefore, Black method satisfies Pareto
        - According to Alan Taylor's theorm, there is no method that can satisfy both independence and Condorcet criterion. Since the black method does satisfy Condorcet criterion, it cannot satisfy the Independence criterion.
- Proposition:

        - Goal: To get $A_k /h \sim P_k / P$
    - P(Total population) /h (Total number of seats)
    - P/h: The average size of a congressional district
    - Example: US
    - For the state of Maryland
        - $P_{Maryland} / A_{Maryland} = 5,307,886 / 8 \approx 663,486$
        - Therefore, Maryland is underrepresented
        - So, P_k / A_k is the average size of a congressional district in the Kth state
        - Goal: $P/h \sim P_k / A_k$
    - The real number
    - H (P_k/P) is called the fair share or standard quota
    - Q_k – Notation for standard quota for state K
    - Standard quota is denoted by "sd"
    - The "sd" of Maryland is 8.2
        - $q_k = P_k/P$
        - $P_k/(P/h) = h \times (P_k/P)$
        - P/h - This is called the standard divisor
- <u>Hamilton's method:</u>
    - Assign every state its lower quota, then assign the seats that remain to the state (at most one per state) in decreasing order of the size of the fractional parts of the standard quota
- The apportionment method is set to satisfy the house monotonicity criteria
- If an increase in 'h' (and all other parameters are fixed), it can never result in any decrease in A_k
    - Hamilton violates population monotonicity

Divisor Method

- Jefferson Method: If h is fixed, then we say p/h is the standard divisor. Modify the divisor (d), compute the modified quota P_k/d
    - Round each number down to obtain A_k
    - Add up the number of seats $a_1, a_2, a_3 \ldots a_n$.
    - If this is h, then you are done
    - Otherwise, modify d and repeat the process till $a_1 + a_2 + a_3 \ldots + a_n = h$
    - One objection to Jefferson's method: It has the tendency to favor the larger state
- Quota criterion: A method assigns to each state its lower quota or its upper quota
- Criterion upper quota: The method must assign to every state, a number of seats greater than its upper quota
    - Jefferson's violates upper quota but satisfies the lower quotas
- Criterion lower quota: The method must not assign to every state a number of seats less than its lower quota
    - Since the modified divisor for Jefferson apportionment is less (or equal) than the standard quota, it will be larger than (or equal) to the standard quota
    - Jefferson's violates upper quota but satisfies the lower quotas
    - Jefferson's apportionment: Rounding down modifies quota will still be larger than rounding down standard quota (lower quota)

<u>Adam's Method</u>

- Choose a modified divisior and compute the modifies quota Pk/d, round each number up to obtain $a_1, a_2, a_3 \ldots a_n$.
- Adam's method tends to favor smaller states
- Hamilton's method violates upper quota but Adam's method violates lower quota

- Similarly, for each column, write the largest entry below the column
- These numbers represent the security numbers for the corresponding row and columns
- Next, draw an arrow next to the row in which the value of the security level is greatest and the column for which the value of the security level is the smallest

2	2	→ 2
1	3	1
2	3	

R = 2

C = 2

	Rock	Paper	Scissors	
Rock	0	-1	1	-1 ←
Paper	1	0	-1	-1 ←
Scissors	-1	1	0	-1 ←
	+1	+1	+1	

- Best Response
  - Given a particular strategy choice by an opponent, the strategy that maximizes a player's payoff against that strategy is called the best response
- The Flow Diagram
  - In each column of the matrix game, draw a vertical arrow pointing to the largest entry
  - Similarly, in each row, draw horizontal arrows pointing to the smallest entry

2	2
1	3

R = 2

R = -1

	Rock	Paper	Scissors
Rock	0 →	-1 ←	1
Paper	1	0	-1
Scissors	-1	1	0

R≠C

C=1

- Saddle Point
  - An outcome such that the strategy for each player is a best response to the strategy of an opponent
  - In other words, an outcome is a saddle point if and only if, in the flow diagram all the arrows point into it. Equivalently, the outcome (k,l) is a saddle point if the corresponding entry $U_{k,l}$ is the smallest entry in its row and the largest entry in the column
- Theorem 13.7