Problem 2a. [2pts] Find the limit:

$$\lim_{x\to\infty} \frac{e^{3x}-e^{-3x}}{e^{3x}+e^{-3x}}$$

Sol^n:

1 $\left\{ \lim_{x\to\infty} \dfrac{e^{3x}-e^{-3x}}{e^{3x}+e^{-3x}} = \lim_{x\to\infty} \dfrac{1-e^{-6x}}{1+e^{-6x}} \right.$ [dividing by e^{3x}]

1 $\left\{ = \lim_{x\to\infty} \dfrac{1-\frac{1}{e^{6x}}}{1+1/e^{6x}} = \dfrac{1-0}{1+0} \right.$ [Since $\lim_{x\to\infty} \frac{1}{e^{6x}} \to 0$]

$= 1$

Problem 2b. [2pts] Differentiate:

$$f(x) = (x^3+2x)7^x$$

1 $\left\{ \begin{array}{l} \Rightarrow \ln f(x) = \ln (x^3+2x)\,7^x \\ \Rightarrow \ln f(x) = \ln(x^3+2x) + x\ln 7 \end{array} \right.$

Diff. w.r.to x

1/2 $\left\{ \Rightarrow \dfrac{f'(x)}{f(x)} = \dfrac{3x^2+2}{x^3+2x} + \ln 7 \right.$

1/2 $\left\{ \begin{array}{l} \Rightarrow f'(x) = f(x)\left[\dfrac{3x^2+2}{x^3+2x} + \ln 7\right] \\ \Rightarrow f'(x) = (x^3+2x)7^x\left[\dfrac{3x^2+2}{x^3+2x} + \ln 7\right] \end{array} \right.$

$= 7^x\left[(3x^2+2) + \ln 7\,(x^3+2x)\right]$

Another way:

$$f'(x) = \underbrace{(3x^2+2)\,7^x}_{1} + \underbrace{7^x \ln 7\,(3x^3+2x)}_{1}$$

[One part correct + another part partly correct $\longrightarrow$ 1 + 1/2 = 1½]

Problem 3a. [2pt] Integrate:

$$\int_0^{1/2} \frac{(-3)x}{1+x^2} dx$$

$= -\frac{3}{2} \int_0^{1/2} \frac{2x}{1+x^2} \, dx$ ½ for setting up the integral correctly

$= -\frac{3}{2} \ln(1+x^2) \Big|_0^{1/2}$ 1 mark for integrating correctly

$= -\frac{3}{2} \left[\ln\left(1+\left(\frac{1}{2}\right)^2\right) - \ln(1+0^2) \right]$

$= -\frac{3}{2} \ln(1.25)$ ½ mark for the correct solution

≈ -0.3347

Problem 3b. [2pt] Find the values of $a \in \mathbb{R}$, $a \neq 0$, such that

$$\int_1^e \frac{\ln x}{ax} dx = a$$

Let $u = \ln x$ $\quad u(e) = \ln(e) = 1$

$du = \frac{1}{x} dx \quad u(1) = \ln(1) = 0$

$a = \frac{1}{a} \int_0^1 u \, du.$ ½ mark for setting up the integral

$= \frac{1}{a} \left[\frac{u^2}{2} \right]_0^1$ 1 mark for integrating correctly

$= \frac{1}{2a} [1^2 - 0^2]$

$= \frac{1}{2a}$

$\therefore a = \frac{1}{2a}.$

$a^2 = \frac{1}{2}.$ ½ mark for the correct solution.

$a = \pm \frac{1}{\sqrt{2}}$ or $a = \pm \frac{\sqrt{2}}{2}.$

7

Problem 1 (Blue)

(yellow) **Problem 4.** [4pts] Determine the domain [1pt], range [1pt] and find the inverse [2pts] of the function $f: A \to B$ given by:

$$f(x) = \frac{4x-1}{x+10}$$

DOMAIN: Numerator is fine, no $\sqrt{\ }$, or $\ln(\ )$....

so need to make sure denom. $\neq 0$:

$\boxed{(-1/2) \text{ for } \geq 0 \text{ instead of } \neq 0}$

$x+10 \neq 0 \Rightarrow x \neq -10.$

So $A = \{x \in \mathbb{R} \mid x \neq -10\}$

$\boxed{(-1/2) \text{ for wrong brackets}}$ $A = (-\infty, -10) \cup (-10, \infty)$

$A = \mathbb{R} \setminus \{-10\}$

$A = \mathbb{R} - \{-10\}$

All okay ways of writing it.

RANGE: Notice $\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 4$, so $f(x)$ has H.A.

at $y=4$, so $y \neq 4$. Also notice: $y = \frac{4x-1}{x+10} \Rightarrow y(x+10) = 4x-1$

$\Rightarrow yx + 10y = 4x - 1 \Rightarrow 10y + 1 = 4x - yx \Rightarrow 10y + 1 = x(4-y) \Rightarrow x = \frac{10y+1}{4-y}$

$\boxed{(-1/2) \text{ or } (-1) \text{ for } \geq 0 \text{ (depending on other mistakes)}}$

Either way, $y \neq 4$. So $y \neq 4$.

$B = \{y \in \mathbb{R} \mid y \neq 4\}$

$\boxed{(-1/2) \text{ for wrong brackets}}$ $B = (-\infty, 4) \cup (4, \infty)$

$B = \mathbb{R} \setminus \{4\}$

$B = \mathbb{R} - \{4\}$

All acceptable.

-1/2 for recognizing the range ~~imposed~~ as a condition imposed by the inverse, or for remembering to look for H.A.s, but finding the wrong one

INVERSE: As above, $y = \frac{4x-1}{x+10}$. Switch x & y & solve for y:

$x = \frac{4y-1}{y+10} \Rightarrow x(y+10) = 4y - 1 \Rightarrow 10x + 1 = 4y - xy \Rightarrow 10x + 1 = y(4-x)$

$\Rightarrow y = \frac{10x+1}{4-x}$. So $f^{-1}(x) = \frac{10x+1}{4-x}$ (or $\frac{-10x-1}{x-4}$, or x & y can be exchanged)

Problem 5a. [2pts] Using logarithmic properties, simplify the expression

$$\frac{1}{3}\ln(x+2)^3 + \frac{1}{2}\ln x - \ln(x^2+3x+2)^2$$

$$\Rightarrow \ln(x+2) + \ln x^{\frac{1}{2}} - \ln(x^2+3x+2)^2$$

$$\Rightarrow \ln(x^{\frac{1}{2}}(x+2)) - \ln[(x+2)^2(x+1)^2]$$

$$\Rightarrow \ln\left[\frac{x^{\frac{1}{2}}\cancel{(x+2)}}{(x+2)^{\cancel{2}}(x+1)^2}\right]$$

$$\Rightarrow \ln\left[\frac{x^{\frac{1}{2}}}{(x+2)(x+1)^2}\right]$$

OR

$$\ln\left[\frac{\sqrt{x}}{(x+2)(x+1)^2}\right]$$

Comments:

① If instead you got to "$\frac{1}{2}\ln x - 2\ln(x+1) - \ln(x+2)$" $\left(-\frac{1}{2}\right)$ and didn't put it together

② FACTOR! Things cancel out $\left(-\frac{1}{2}\right)$

③ $\ln a + \ln b = \ln(ab)$ NOT $\ln a + \ln b = \ln(a+b)$

④ CANNOT cancel "ln"s

⑤ Keep numbers as exponents to apply the other rules properly

Problem 5b. [2pts] A population of cells develops with a constant relative growth rate of 0.8 per member per day. On day 0, the population consists of two members (two cells). Find the population size after 6 days.

Relative Growth Rate: $\frac{1}{P}\frac{dP}{dt} = 0.8 = K$

Initial Population: $P(0) = 2$

Final Time: $t = 6$

We know the solution must take the form:

$$P(t) = P(0)e^{Kt}$$

where $K = 0.8$, $t = 6$, $P(0) = 2$

$$\Rightarrow P(6) = 2e^{0.8(6)} = 2e^{4.8} \approx 243$$

Comments:

① If you used the correct formula but made an error in the calculation, $\left(-\frac{1}{2}\right)$ or (-1) depending on the error.

② If the formula was a little wrong $\left(-\frac{1}{2}\right)$

③ If the formula was completely wrong (-1)

④ P(6) and P(t) are functions, not multiplication