Université d'Ottawa · University of Ottawa

Faculté des sciences
Mathématiques et de statistique

Faculty of Science
Mathematics and Statistics

Final Exam for MAT2377
Probability and Statistics for Engineers.

Time : 3 hours **Professor : M. Zarepour & G. Lamothe**

Name : ______________________________

Student Number : ______________________________

Calculators are permitted. It is an open book exam.
There are 4 short answer questions and 12 multiple choice questions.
The exam will be marked on a total of 28 points.

Submit your answers for the multiple choice questions in the following table.

Question	Answer	Question	Answer
1		7	
2		8	
3		9	
4		10	
5		11	
6		12	

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Short Answer Questions

[4] 1. Let X be a random variable with the probability mass function

$$f(x) = c(1 + |x - 4|), \text{ for } x = 3, 4, 5$$

and 0 otherwise.

(a) Find the value for c.

(b) Find $P(X = 4|X \geq 4)$.

(c) Compute the expected value of X.

(d) Compute $P(X \geq \mu)$, where $\mu = E[X]$.

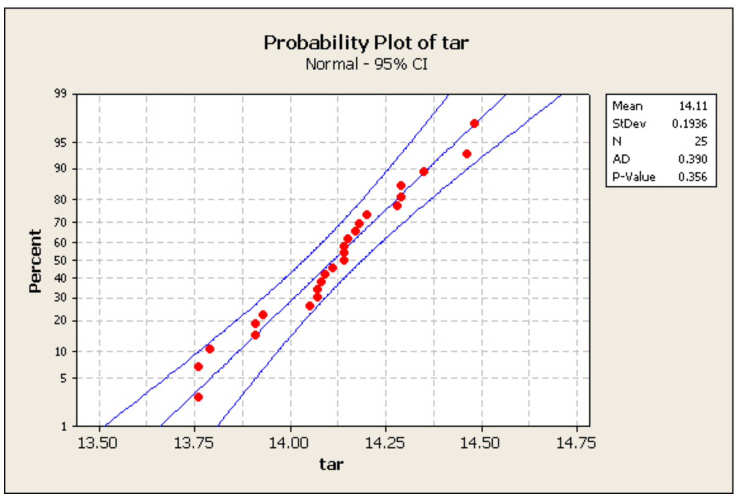
[**4**] 2. We obtained twenty five measurements of the tar content of a certain kind of cigarette. The manufacturer claims that $\mu = 14$ mg. Using Minitab, we produced the following descriptive statistics.

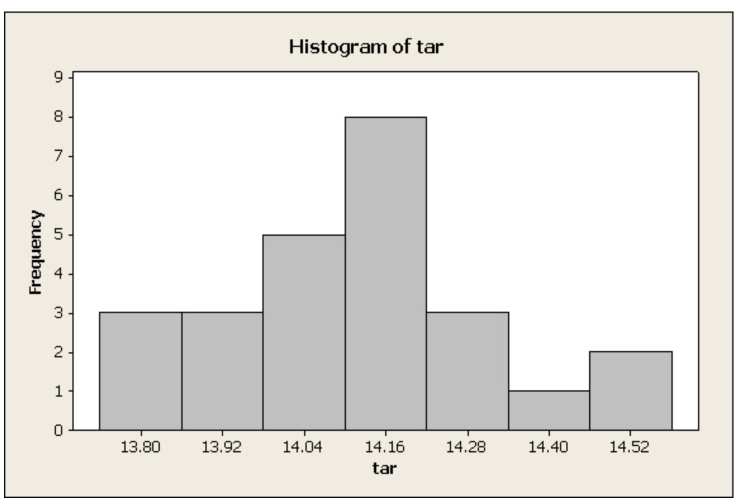
Descriptive Statistics: tar

Variable	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3
tar	25	0	14.112	0.0387	0.194	13.760	13.990	14.140	14.240

Variable	Maximum
tar	14.480

Below are the normal probability plot and the histogram for the 25 measurements.





(a) Based on the above histogram and normal probability plot, would it appear reasonable to assume that the tar content is normally distributed ? Discuss.

(b) Do we have sufficient evidence at $\alpha = 5\%$ to conclude that the true mean tar content is larger than 14 mg ?

(c) Construct a 95% confidence interval for the mean tar content.

(Question 2 cont.)

[**4**] 3. Crystalline forms of certain chemical compounds are used in various electronic devices. It is often more desirable to have large crystals rather than small ones. In a laboratory study, 14 crystals of the same initial size were allowed to grow for certain periods of time. The following data gives the weight y of the crystal (in grams) and the period x of time (in hours) which was used for each crystal.

Time	Weight	Time	Weight
2	0.08	16	8.4
4	1.12	18	8.81
6	4.43	20	10.81
8	4.98	22	11.16
10	4.92	24	10.12
12	7.18	26	13.12
14	5.57	28	15.04

For these data, we have :

$$\bar{x} = 15, \quad \bar{y} = 7.55, \quad \sum_{i=1}^{14}(x_i - \bar{x})^2 = 910, \quad \sum_{i=1}^{14}(x_i - \bar{x})(y_i - \bar{y}) = 458.12$$

$$\sum_{i=1}^{14}(y_i - \bar{y})^2 = 244.16$$

The time and weight are stored in columns C1, respectively C2. Below is the result of the linear regression analysis produced by Minitab :

Regression Analysis: C2 versus C1

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The regression equation is
C2 = 0.001 + 0.503 C1


Predictor       Coef   SE Coef      T      P
Constant      0.0014    0.5994   0.00  0.998
C1           0.50343   0.03520  14.30  0.000


S = 1.06177   R-Sq = 94.5%   R-Sq(adj) = 94.0%


Analysis of Variance

Source          DF      SS      MS       F      P
Regression       1  230.63  230.63  204.58  0.000
Residual Error  12   13.53    1.13
Total           13  244.16
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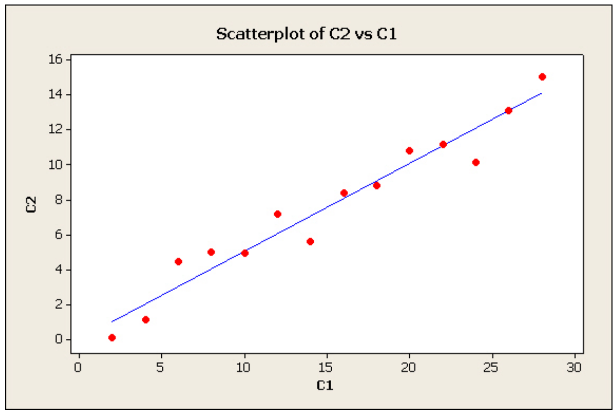
(a) Below is a scatter plot of y against x. Does it appear to be reasonable to use the simple linear regression model $y = \beta_0 + \beta_1 x + \varepsilon$? Discuss.



(b) Write down the estimated regression line and use it to estimate the mean weight in grams for a period of $x = 5$ hours.

(c) Compute a 95% confidence interval for the mean weight for a period of $x = 5$ hours.

(d) Find the coefficient of determination and interpret within the context of the problem.

(Question 3 cont.)

[**4**] 4. A process for producing vinyl covering has been stabilized for a long period of time and the surface hardness measurement has a normal distribution with mean $\mu = 4.5$ and standard deviation $\sigma = 1.5$. A second shift has been hired and trained and their production needs to be monitored. Consider testing the hypothesis $H_0 : \mu = 4.5$ against $H_1 : \mu \neq 4.5$. A random sample of hardness measurements were made on $n = 25$ vinyl specimens produced by the second shift. Assume that hardness is normally distributed with $\sigma = 1.5$.

(a) Using a significance level of $\alpha = 5\%$, compute the probability of committing an error of type II, if the true mean is $\mu = 4.0$.

(b) The $n = 25$ observations yielded the sample mean $\overline{x} = 3.9$. Compute the P-value and give the conclusion of the test at $\alpha = 5\%$.

Multiple Choice Questions

Submit your answers for the multiple choice questions in the table found on the front page.

[**1**] 1. In a box of 12 light bulbs, there are 3 defective items. An inspector inspects 3 light bulbs selected at random and without replacement. Find the probability that there are exactly 2 defective light bulbs in his sample.

(A) 0.0791 (B) 0.0066 (C) 0.026 (D) 0.7 (E) 0.122

[**1**] 2. It is known that the manufacturing time (in hours) of a certain product is normally distributed with mean μ and variance $\sigma^2 = 0.25$. What sample size is required so that we have 90% confidence that the maximum error of the estimate of μ is 0.05?

(A) 384 (B) 385 (C) 271 (D) 280 (E) 250

[**1**] 3. An operator receives on the average 20 calls per hour in accordance with a Poisson process. What is the probability that she waits more than 12 minutes before receiving the first call?

(A) 0.9816 (B) 0.9084 (C) 0.0916 (D) 0.0183 (E) 0.49.

[**1**] 4. If X and Y are two random variable such that

$$E(X) = E(Y) = 0, E(X^2) = E(Y^2) = 1$$

and $E((X - Y)^2) = 4$. Then the correlation coefficient between X and Y is

(A) 0 (B) 0.5 (C) 1 (D) -0.5 (E) -1

[**1**] 5. The probability that a machine produces a defective item is 0.01. Each item is checked as it is produced. Assume that these are independent trials. Compute the probability that at least 50 items must be checked to find one that is defective.

(A) 0.56 (B) 0.82 (C) 0.64 (D) 0.61 (E) 0.29

[**1**] 6. If the cumulative distribution function of a random variable X is given by

$$F(x) = \begin{cases} 1 - \frac{4}{x^2}, & \text{for } x > 2 \\ 0, & \text{for } x \le 2. \end{cases}$$

Compute the probability that $P(4 < X < 5)$.

(A) 0.09 (B) 0.91 (C) 0.25 (D) 0.75 (E) 0.22

[**1**] 7. If the amount of cosmic radiation to which a person is exposed while flying by plane across Canada is a normal random variable with $\mu = 4.35$ mrem and $\sigma = 0.59$ mrem, find the probability that a person on such flights will be exposed to at least 5.50 mrem.

(A) 0.9744 (B) 0.2561 (C) 0.5000 (D) 0.3576 (E) 0.0256

[**1**] 8. Fifteen bearings made by a certain process have a mean diameter of 0.506 cm with a standard deviation of 0.004 cm. Compute the standard error of the estimate of the mean.

(A) 0.004 (B) 0.001 (C) 0.506 (D) 0.0003 (E) 0.015

[**1**] 9. Refer to Question 8. Assume that the diameter of a bearing is normally distributed. Compute a 95% confidence interval for the mean diameter.

(A) [0.504,0.508] (B) [0.502,0.510] (C) [0.500,0.512] (D) [0.505,0.507]
(E) [0.498,0.514]

[**1**] 10. Let $X_1, \ldots, X_{20}$ be a random sample from a normal population with mean $\mu = 5$ and variance $\sigma^2 = 1.5$. Let $\overline{X}$ be the sample mean. Find c such that

$$P\left(\frac{\overline{X} - 5}{\sigma/\sqrt{20}} < c\right) = 0.90.$$

(A) 1.645 (B) 1.96 (C) -1.96 (D) -1.28 (E) 1.28

[**1**] 11. If $P(A) = 0.8$, $P(C) = 0.35$, and $P(A \cap C) = 0.28$, are the events A and C independent ?

(A) yes (B) no (C) insufficient information is given

[**1**] 12. An agricultural cooperative claims that 90% of the watermelons shipped out are ripe and ready to eat. Find the probability that among 18 watermelons shipped out at least 17 are ripe and ready to eat.

(A) 0.550 (B) 0.450 (C) 0.001 (D) 0.300 (E) 0.734