NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

6 marks B24. Determine whether the series $\sum_{n=1}^{\infty}(-1)^{n+1}\cdot\frac{n+1}{n^2+1}$ converges absolutely, conditionally, or diverges. Justify your answer.

Let $A_n = \left|(-1)^{n+1}\frac{n+1}{n^2+1}\right| = \frac{n+1}{n^2+1}$

and $B_n = \frac{1}{n}$.

$$\lim_{n\to\infty}\frac{A_n}{B_n} = \lim_{n\to\infty}\frac{\frac{n+1}{n^2+1}}{1/n} = \lim_{n\to\infty}\frac{n^2+n}{n^2+1}$$

$$= \lim_{n\to\infty}\frac{1+1/n}{1+1/n^2} = \frac{1+0}{1+0} = 1$$

$\sum B_n$ diverges (Harmonic series)

Thus $\sum A_n$ diverges by the Limit Comparison Test

Hence $\sum(-1)^{n+1}\frac{n+1}{n^2+1}$ is not absolutely convergent

Let $b_n = \frac{n+1}{n^2+1}$

$$b_n' = \frac{(n^2+1)(1)-(n+1)(2n)}{(n^2+1)^2} = \frac{n^2+1-2n^2-2n}{(n^2+1)^2}$$

$$= \frac{-n^2-2n+1}{(n^2+1)^2} < 0$$

so $\{b_n\}$ is decreasing.

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty}\frac{n+1}{n^2+1} = \lim_{n\to\infty}\frac{\frac{1}{n}+\frac{1}{n^2}}{1+1/n^2} = \frac{0+0}{1+0} = 0$$

Hence $\sum(-1)^{n+1} b_n$ converges by the Alternating Series Test.

Thus, $\sum(-1)^{n+1}\frac{n+1}{n^2+1}$ is conditionally convergent.

NOTE: THE MULTIPLE CHOICE QUESTIONS ARE TO BE ANSWERED IN THE BOOKLET. CIRCLE YOUR SELECTED ANSWER.

2 marks A7. Evaluate the indefinite integral $\int \frac{x}{1-x^5}\,dx$ as a power series.

$$\frac{1}{1-x^5} = \sum_{n=0}^{\infty}(x^5)^n = \sum_{n=0}^{\infty} x^{5n}$$

$$\text{so } \frac{x}{1-x^5} = x\sum_{n=0}^{\infty} x^{5n} = \sum_{n=0}^{\infty} x^{5n+1}$$

$$\text{so } \int \frac{x}{1-x^5}\,dx = \int \sum_{n=0}^{\infty} x^{5n+1}\,dx = \sum_{n=0}^{\infty}\int x^{5n+1}\,dx = \sum_{n=0}^{\infty}\frac{x^{5n+2}}{5n+2} + C$$

A: $\sum_{n=0}^{\infty}\frac{x^{5n+2}}{5n+2}+C$	B: $\sum_{n=0}^{\infty}\frac{x^{5n+1}}{5n}+C$	C: $\sum_{n=0}^{\infty}\frac{x^{5n+3}}{5n+2}+C$	D: $\sum_{n=0}^{\infty}\frac{x^{5n+1}}{5n+1}+C$
E: $\sum_{n=0}^{\infty}(5n+2)\,x^{5n+2}+C$			

2 marks A8. Which of the following statements most accurately describes the series $\sum_{n=0}^{\infty}\frac{1}{2^n n!}$?

$$\sum_{n=0}^{\infty}\frac{1}{2^n n!} = \sum_{n=0}^{\infty}\frac{(1/2)^n}{n!} = e^{1/2} = \sqrt{e}$$

A: divergent	B: conditionally convergent	C: absolutely convergent
D: absolutely convergent and the sum is $\sqrt{e}$	E: absolutely convergent and the sum is e^2	

2 marks A9. What is the coefficient of x^{2012} in the Maclaurin series of the function $f(x) = \sin(-x)$?

f is an odd function so the coefficient of x^{2012} is 0 since 2012 is even

A: 0	B: $\frac{1}{2012!}$	C: $\frac{-1}{2012!}$	D: 2012!	E: 1

NOTE: THE MULTIPLE CHOICE QUESTIONS ARE TO BE ANSWERED IN THE BOOKLET. CIRCLE YOUR SELECTED ANSWER.

2 marks A13. Given that $x = e^t$ and $y = te^{-t}$, find the formula for $\frac{d^2y}{dx^2}$.

$$\frac{dx}{dt} = e^t; \quad \frac{dy}{dt} = t(-e^{-t}) + e^{-t} = e^{-t}(1-t)$$

$$\frac{d^2x}{dt^2} = e^t; \quad \frac{d^2y}{dt^2} = e^{-t}(-1) + (1-t)(-e^{-t}) = (t-2)e^{-t}$$

$$\text{so } \frac{d^2y}{dx^2} = \frac{\frac{dx}{dt}\left(\frac{d^2y}{dt^2}\right) - \frac{d^2x}{dt^2}\frac{dy}{dt}}{(dx/dt)^3} = \frac{e^t(t-2)e^{-t} - (1-t)e^{-t}e^t}{(e^t)^3} = \frac{t-2-1+t}{e^{3t}} = \frac{2t-3}{e^{3t}}$$

A: $(1-t)e^{-2t}$	B: $(1-t)e^{-t}$	C: $(2t-3)e^{-3t}$	D: te^{-2t}	E: $\frac{e^{-t}}{2t-3}$

2 marks A14. A curve C is given by parametric equations $x = 2t^3$ and $y = 1 + 4t - t^2$. Find the slope of the tangent to C at the point $(x_0, y_0) = (-2, -4)$.

when $x = -2$, $t = -1$ $\quad (-2 = 2t^3)$

$$\frac{dx}{dt} = 6t^2; \quad \frac{dy}{dt} = 4 - 2t$$

$$m = \left.\frac{dy/dt}{dx/dt}\right|_{t=-1} = \left.\frac{4-2t}{6t^2}\right|_{t=-1} = \frac{6}{6} = 1$$

A: -1	B: $-\frac{1}{2}$	C: 0	D: $\frac{1}{2}$	E: 1

2 marks A15. A curve C is given by parametric equations $x = t^2 - t - 6$ and $y = 2t$. How many times does C cross the y-axis as t varies from -5 to 5?

y-axis is $x = 0$

Solve $0 = t^2 - t - 6$

$= (t-3)(t+2)$

$\Rightarrow t = 3, -2$

A: never	B: once	C: two times	D: three times	E: four times

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

5 marks B27. Let C be the curve given by equations $x = e^t + e^{-t}$ and $y = 3 - 2t$. Find the length of C for $0 \le t \le \ln 3$.

$$L = \int_0^{\ln 3} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$\frac{dx}{dt} = e^t - e^{-t} \; ; \; \frac{dy}{dt} = -2$$

$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = \left(e^t - e^{-t}\right)^2 + (-2)^2$$
$$= e^{2t} - 2e^t e^{-t} + e^{-2t} + 4$$
$$= e^{2t} - 2 + e^{-2t} + 4$$
$$= e^{2t} + 2 + e^{-2t}$$
$$= \left(e^t + e^{-t}\right)^2$$

Thus, $L = \int_0^{\ln 3} \sqrt{(e^t + e^{-t})^2}\, dt$

$$= \int_0^{\ln 3} (e^t + e^{-t})\, dt$$
$$= e^t - e^{-t} \Big|_0^{\ln 3}$$
$$= e^{\ln 3} - e^{-\ln 3} - (e^0 - e^0)$$
$$= 3 - \frac{1}{3} - (1 - 1)$$
$$= \frac{8}{3}$$

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

5 marks B29. Find the solution of the differential equation $y' = \frac{\ln x}{xy}$ satisfying the initial condition $y(1) = 2$. (You may leave your solution in the implicit form.)

The equation is separable.

$$y\,dy = \frac{\ln x}{x}\,dx$$

$$\int y\,dy = \int \frac{\ln x}{x}\,dx \qquad \begin{array}{l} u = \ln x \\ du = \frac{1}{x}\,dx \end{array}$$

$$= \int u\,du$$

$$= \frac{u^2}{2} + c$$

so $\frac{y^2}{2} = \frac{1}{2}(\ln x)^2 + C$

so $y^2 = (\ln x)^2 + 2C = (\ln x)^2 + C_1$

$y(1) = 2 \Rightarrow 2^2 = (\ln 1)^2 + C_1$

$= 0^2 + C_1$

$= C_1$

so $C_1 = 4$

Thus, $y^2 = (\ln x)^2 + 4$

PART A (44 marks) (2 marks for each problem)

NOTE: THE MULTIPLE CHOICE QUESTIONS ARE TO BE ANSWERED IN THE BOOKLET. CIRCLE YOUR SELECTED ANSWER.

2 marks A1. Evaluate $\int_1^2 x \ln x \, dx$.

$\int_1^2 x \ln x \, dx =$ $\quad\left| \begin{array}{l} u = \ln x, \; dv = x\,dx \\ du = \frac{1}{x}dx, \; v = x^2/2 \end{array}\right.$

$\frac{x^2}{2}\ln x \Big|_1^2 - \int_1^2 \frac{x^2/2}{x}dx$

$= \frac{x^2}{2}\ln x\Big|_1^2 - \frac{1}{2}\int_1^2 x\,dx = \frac{4}{2}\ln 2 - \frac{1}{2}\ln 1 - \frac{1}{2}\frac{x^2}{2}\Big|_1^2$

$= 2\ln 2 - \frac{1}{4}(4-1) = 2\ln 2 - \frac{3}{4}$

A: $\ln 2 - 2$	B: $\ln 2 - 1$	C: $2\ln 2$	D: $\ln 2 + 2$	E: $2\ln 2 - \frac{3}{4}$

2 marks A2. Evaluate $\int_0^{\pi/4} \sec^4\theta \, d\theta = \int_0^{\pi/4}(1+\tan^2\theta)\sec^2\theta\,d\theta$ $\quad\left| \begin{array}{l} u = \tan\theta \\ du = \sec^2\theta\,d\theta \end{array}\right.$

$\theta = 0 \Rightarrow u = \tan 0 = 0$

$\theta = \pi/4 \Rightarrow u = \tan \pi/4 = 1$

$= \int_0^1 (1+u^2)\,du$

$= u + \frac{u^3}{3}\Big|_0^1$

$= 1 + 1/3 - 0$

$= 4/3$

A: 4	B: 3	C: $\frac{2\sqrt{2}}{3}$	D: $\frac{4}{3}$	E: $\frac{4\sqrt{2}}{3}$

2 marks A3. Which of the following statements most accurately describes the improper integral $\int_1^\infty \frac{1}{2x^2+x}dx$?

$\frac{1}{2x^2+x} = \frac{1}{x(2x+1)} = \frac{A}{x} + \frac{B}{2x+1} = \frac{A(2x+1)+Bx}{x(2x+1)}$

$\Rightarrow \quad 1 = A(2x+1) + Bx$

$x=0 \Rightarrow 1 = A$

$x = -1/2 \Rightarrow 1 = -\frac{1}{2}B \Rightarrow B = -2$ see below

A: divergent	B: convergent to $\ln(1/3)$	C: convergent to $\ln(3/2)$	D: convergent to $\ln 3$
E: convergent to $3\ln 3$			

Thus $\int \frac{1}{2x^2+x}dx = \int \frac{1}{x} - \frac{2}{2x+1}\,dx = \ln|x| - \ln|2x+1| + C$

$= \ln\left|\frac{x}{2x+1}\right| + C$

Hence, $\int_1^\infty \frac{1}{2x^2+x}dx = \lim_{t\to\infty}\int_1^t \frac{1}{2x^2+x}dx$

$= \lim_{t\to\infty} \ln\left|\frac{x}{2x+1}\right|\Big|_1^t$

$= \lim_{t\to\infty} \ln\left|\frac{t}{2t+1}\right| - \ln\left|\frac{1}{3}\right|$

$= \ln\frac{1}{2} - \ln 1/3 = \ln\frac{1/2}{1/3}$

$= \ln 3/2$

PART B (44 marks)

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

6 marks B23. Evaluate $\int \frac{\sqrt{x^2-1}}{x}\,dx$.

$$\int \frac{\sqrt{x^2-1}}{x}\,dx \qquad \Big| \text{ Let } x = \sec\theta, \quad dx = \sec\theta\tan\theta\,d\theta$$

$$= \int \frac{\sqrt{\sec^2\theta - 1}}{\sec\theta}\sec\theta\tan\theta\,d\theta$$

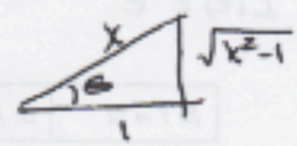
$$= \int \sqrt{\tan^2\theta}\,\tan\theta\,d\theta$$

$$= \int \tan^2\theta\,d\theta$$

$$= \int (\sec^2\theta - 1)\,d\theta$$

$$= \tan\theta - \theta + C$$

$$= \sqrt{x^2-1} - \cos^{-1}\frac{1}{x} + C$$



NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

5 marks B25. Find a power series representation for $f(x) = x^2 \ln(1-x)$ and determine its radius of convergence.

Recall $\ln(1-x) = -\sum_{n=1}^{\infty} \frac{x^n}{n}$ for $[-1, 1)$ ⊛

with radius of convergence $R = 1$

Then $f(x) = x^2 \ln(1-x)$

$$= x^2\left(-\sum_{n=1}^{\infty} \frac{x^n}{n}\right)$$

$$= -\sum_{n=1}^{\infty} \frac{x^{n+2}}{n}.$$

with radius of convergence $R = 1$.

⊛ $\ln(1-x) = -\int \frac{1}{1-x}\,dx$

$$= -\int \sum_{n=0}^{\infty} x^n\,dx$$

$$= -\sum_{n=0}^{\infty} \int x^n\,dx$$

$$= -\sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1} + c$$

$$= -\sum_{n=1}^{\infty} \frac{x^n}{n} + c$$

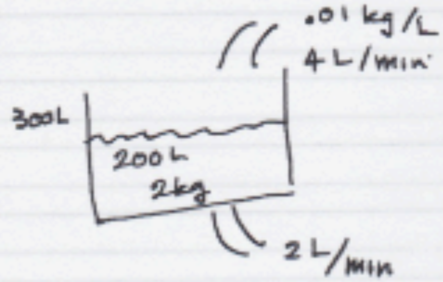
$\frac{1}{1-x}$ has radius of convergence 1

so $\ln(1-x)$ also has radius of convergence 1

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

5 marks B30. A 300 litre tank initially contains 200 litres of brine (salt water) with 2 kg of dissolved salt. Brine that contains 0.01 kg of salt per litre enters the tank at the rate of 4 litres per minute. The solution is kept thoroughly mixed and drains from the tank at the rate of 2 litres per minute. How much salt is in the tank when it overflows?

Let $y(t)$ be the number of kg of salt in the tank at time t



$y(0) = 2$

Rate In $\;.01\frac{kg}{L} \cdot \frac{4L}{min}$

$= .04 \text{ kg/min}$

Rate Out $\;\frac{y(t)\text{ kg}}{(200+2t)\text{ L}} \cdot \frac{2L}{min}$

$= \frac{2y}{2t+200} \text{ kg/min} = \frac{y}{t+100}\frac{kg}{min}.$

Thus $\frac{dy}{dt} = .04 - \frac{y}{t+100}$

or $\frac{dy}{dt} + \frac{1}{t+100}y = \frac{4}{100} = \frac{1}{25}$

$p(t) = \frac{1}{t+100}$ so $\int p(t)dt = \ln|t+100|$

so $I(t) = e^{\ln|t+100|} = t+100$

$\Rightarrow (t+100)\frac{dy}{dt} + y = \frac{1}{25}(t+100)$

$\rightarrow \frac{d}{dt}\left[(t+100)y\right] = \frac{1}{25}(t+100)$

$\rightarrow (t+100)y = \int \frac{1}{25}(t+100)dt = \frac{1}{25}\left(\frac{t+100}{2}\right)^2 + C$

$\Rightarrow y = \frac{1}{50}(t+100) + \frac{C}{t+100} = \frac{t}{50} + 2 + \frac{C}{t+100}$

$\left.\begin{array}{l} y(0) = 2 \\ y(0) = 2 + \frac{C}{100} \end{array}\right\} \Rightarrow C = 0$

so $y(t) = \frac{t}{50} + 2.$

The tank overflows at $t = 50$

$y(50) = \frac{50}{50} + 2 = 3$

The tank has 3 kg of salt in it when it overflows

NOTE: THE MULTIPLE CHOICE QUESTIONS ARE TO BE ANSWERED IN THE BOOKLET. CIRCLE YOUR SELECTED ANSWER.

2 marks A10. Given $f(x) = \frac{x^2}{1-x^2}$, find the value of $f^{(2012)}(0)$.

$$\frac{x^2}{1-x^2} = x^2 \frac{1}{1-x^2} = x^2 \sum_{n=0}^{\infty} (x^2)^n = x^2 \sum_{n=0}^{\infty} x^{2n} = \sum_{n=0}^{\infty} x^{2n+2}$$

$$\text{so } \sum_{n=0}^{\infty} x^{2n+2} = \sum \frac{f^{(n)}(0)}{n!} x^n$$

$$\Rightarrow \frac{f^{(2012)}(0)}{(2012)!} = 1 \quad \text{so } f^{(2012)}(0) = (2012)!$$

A: $-2012!$	B: $\frac{-1}{2012!}$	C: 0	D: $\frac{1}{2012!}$	E: 2012!

2 marks A11. Each of the following functions, except for one, has a Maclaurin series representation for all $x \neq 0$ in some interval $(-R, R)$. Which is the one exception?

$$\text{Recall } \cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$$

$$\text{so } \frac{\cos x}{x} = \frac{1}{x} - \frac{x}{2!} + \cdots$$

which is not a Maclaurin series

A: $\frac{\cos x}{x}$	B: $\frac{\sin x}{x}$	C: $\frac{\tan x}{x}$	D: $\frac{1-e^x}{x}$	E: $\frac{\arcsin x}{x}$

2 marks A12. What is the Taylor series of $f(x) = \sin x$ centered at $a = \frac{\pi}{2}$?

$$\sin x = \cos\left(x - \tfrac{\pi}{2}\right) \left[\cos x \cos \tfrac{\pi}{2} + \sin x \sin \tfrac{\pi}{2} = \sin x\right]$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n \left(x - \frac{\pi}{2}\right)^{2n}}{(2n)!}$$

A: $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$	B: $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \left(x - \frac{\pi}{2}\right)^{2n+1}$	C: $\sum_{n=0}^{\infty} \frac{1}{n!} x^n$
D: $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \left(x - \frac{\pi}{2}\right)^{2n}$	E: does not exist	

NOTE: THE MULTIPLE CHOICE QUESTIONS ARE TO BE ANSWERED IN THE BOOKLET. CIRCLE YOUR SELECTED ANSWER.

2 marks A16. How many petals are there on the rose $r = 3\cos(4\theta)$?

A: 2	B: 3	C: 4	D: 6	E: 8

2 marks A17. What is the length of the polar curve $r = \cos\theta$ for $0 \le \theta \le \frac{\pi}{6}$?

$$L = \int_0^{\pi/6} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$
$$= \int_0^{\pi/6} \sqrt{(\cos\theta)^2 + (-\sin\theta)^2}\, d\theta$$
$$= \int_0^{\pi/6} \sqrt{\cos^2\theta + \sin^2\theta}\, d\theta$$
$$= \int_0^{\pi/6} d\theta = \theta\Big|_0^{\pi/6} = \pi/6$$

A: $\frac{\pi}{6}$	B: $\frac{\pi}{3}$	C: $\frac{\pi}{2}$	D: $1+\frac{\pi}{4}$	E: π

2 marks A18. A polar curve C is given by equation $r = 2\sin(2\theta)$. What is the slope of the tangent to C at $\theta_0 = \frac{\pi}{4}$?

$x = r\cos\theta$, $y = r\sin\theta$ so

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta} = \frac{4\cos 2\theta \sin\theta + 2\sin 2\theta\cos\theta}{4\cos 2\theta\cos\theta - 2\sin 2\theta \sin\theta}$$

$$m = \frac{dy}{dx}\Big|_{\theta=\pi/4} = \frac{4\cos\frac{\pi}{2}\sin\frac{\pi}{4} + 2\sin\frac{\pi}{2}\cos\frac{\pi}{4}}{4\cos\frac{\pi}{2}\cos\frac{\pi}{4} - 2\sin\frac{\pi}{2}\sin\frac{\pi}{4}} = \frac{0 + 2(1)(\sqrt{2}/2)}{0 - 2(1)(\sqrt{2}/2)} = \frac{\sqrt{2}}{-\sqrt{2}} = -1$$

A: −2	B: −1	C: 0	D: 1	E: 2

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

5 marks B26. Find the first three non-zero terms of the Maclaurin series of $f(x) = \sqrt[3]{8+x}$. Justify your answer.

$$f(x) = (8+x)^{1/3} = \left[8\left(1+\frac{x}{8}\right)\right]^{1/3} = 8^{1/3}\left(1+\frac{x}{8}\right)^{1/3} = 2\left(1+\frac{x}{8}\right)^{1/3}$$

$$= 2\sum_{n=0}^{\infty}\binom{1/3}{n}\left(\frac{x}{8}\right)^n$$

For $n=0$, $\dfrac{2\binom{1/3}{0}}{8^0} = \dfrac{2(1)}{1} = 2$

For $n=1$, $\dfrac{2\binom{1/3}{1}}{8^1} = \dfrac{2(1/3)}{8} = \dfrac{1}{12}$

For $n=2$, $\dfrac{2\binom{1/3}{2}}{8^2} = \dfrac{2\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{64} = \dfrac{-4/9}{64} = -\dfrac{4}{9\cdot 64}$

$$= -\frac{1}{9\cdot 16} = -\frac{1}{144}$$

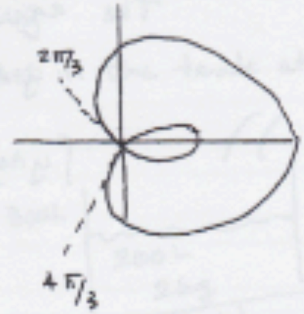
The first 3 non-zero terms are

$$2 + \frac{1}{12}x - \frac{1}{144}x^2$$

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

6 marks B28. Find the area of the region bounded by the inner loop of the limaçon $r = \frac{1}{2} + \cos\theta$.

$$r = 0 \Rightarrow \tfrac{1}{2} + \cos\theta = 0$$
$$\cos\theta = -\tfrac{1}{2}$$
$$\theta = \tfrac{2\pi}{3}, \tfrac{4\pi}{3}$$



$$A = \int_{2\pi/3}^{4\pi/3} \tfrac{1}{2} r^2 \, d\theta$$

$$= 2\int_{2\pi/3}^{\pi} \tfrac{1}{2} r^2 \, d\theta \text{ by symmetry}$$

$$= \int_{2\pi/3}^{\pi} \left(\tfrac{1}{2} + \cos\theta\right)^2 d\theta$$

$$= \int_{2\pi/3}^{\pi} \tfrac{1}{4} + \cos\theta + \cos^2\theta \, d\theta$$

$$= \int_{2\pi/3}^{\pi} \tfrac{1}{4} + \cos\theta + \frac{1+\cos 2\theta}{2} \, d\theta$$

$$= \int_{2\pi/3}^{\pi} \tfrac{3}{4} + \cos\theta + \tfrac{1}{2}\cos 2\theta \, d\theta$$

$$= \left[\tfrac{3}{4}\theta + \sin\theta + \tfrac{1}{2}\frac{\sin 2\theta}{2}\right]_{2\pi/3}^{\pi}$$

$$= \left(\tfrac{3}{4}\pi + \sin\pi + \frac{\sin 2\pi}{4}\right) - \left(\tfrac{3}{4}\cdot\tfrac{2\pi}{3} + \sin\tfrac{2\pi}{3} + \frac{\sin 4\pi/3}{4}\right)$$

$$= \frac{3\pi}{4} + 0 + 0 - \left(\pi/2 + \frac{\sqrt{3}}{2} + \frac{-\sqrt{3}/2}{4}\right)$$

$$= \frac{3\pi}{4} - \pi/2 - \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{8}$$

$$= \frac{\pi}{4} - \frac{3\sqrt{3}}{8}$$

NOTE: THE MULTIPLE CHOICE QUESTIONS ARE TO BE ANSWERED IN THE BOOKLET. CIRCLE YOUR SELECTED ANSWER.

2 marks A4. The sequence $a_n = \tan\left(\frac{2n\pi}{1+8n}\right)$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \tan\left(\frac{2n\pi}{1+8n}\right) = \lim_{n\to\infty} \tan\left(\frac{2\pi}{\frac{1}{n}+8}\right)$$

$$= \tan\left(\frac{2\pi}{0+8}\right) = \tan \pi/4 = 1$$

A: diverges	B: converges to 1	C: converges to $\frac{\pi}{4}$	D: converges to 0	E: converges to -1

2 marks A5. Which of the three series

(i) $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n+1}$ (ii) $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{n^2-1}}$ (iii) $\sum_{n=1}^{\infty} \frac{n^{10}}{(-10)^{n+1}}$

converge?

(i) diverges by comparison with $\sum \frac{1}{\sqrt{n}}$ (use Limit Comp. Test)

(ii) converges " " " $\sum \frac{1}{n\sqrt{n^2}} = \sum \frac{1}{n^2}$ "

(iii) converges by the Ratio Test

A: only (i) and (ii) converge	B: only (i) and (iii) converge	C: only (ii) and (iii) converge
D: only (ii) converges	E: all three converge	

2 marks A6. Which function is represented by the power series $\sum_{n=0}^{\infty} \frac{2}{3^{n+1}} x^n$ when $|x| < 3$?

$$\sum_{n=0}^{\infty} \frac{2x^n}{3^{n+1}} = \frac{2}{3}\sum_{n=0}^{\infty} \frac{x^n}{3^n} = \frac{2}{3}\sum\left(\frac{x}{3}\right)^n$$

$$= \frac{2}{3}\left(\frac{1}{1-x/3}\right) = \frac{2}{3-x}$$

A: $\frac{2}{1-3x}$	B: $\frac{2}{3+x}$	C: $\frac{2}{x-3}$	D: $\frac{2}{3-x}$	E: $\frac{1}{1-x}$

NOTE: THE MULTIPLE CHOICE QUESTIONS ARE TO BE ANSWERED IN THE BOOKLET. CIRCLE YOUR SELECTED ANSWER.

2 marks A19. Which of the differential equations

(i) $y' + xy^2 = \sqrt{x}$ (ii) $y \sin x = x^2 y' - x$ (iii) $y' = \frac{1}{x} + \frac{1}{y}$

are linear?

(i) no (ii) yes (iii) no

A: only (i) is linear	B: only (ii) is linear	C: only (i) and (ii) are linear
D: only (ii) and (iii) are linear	E: only (iii) is linear	

2 marks A20. Find the integrating factor $I(t)$ for the following linear differential equation

$$ty' + 2y = t - 1 + \frac{1}{t}. \Rightarrow y' + \frac{2}{t}y = 1 - \frac{1}{t} + \frac{1}{t^2}$$

$$p(t) = \frac{2}{t}$$

$$I(t) = e^{\int 2/t \, dt} = e^{2\ln t} = (e^{\ln t})^2 = t^2$$

A: $I = 2t$	B: $I = e^{2t}$	C: $I = t^2$	D: $I = e^{2t+1}$	E: $I = e^{t^2}$

2 marks A21. A general solution of the equation $xy' + y = \sqrt{x}$ is $\Rightarrow y' + \frac{1}{x}y = \frac{1}{\sqrt{x}}$

$$\Rightarrow p(x) = \frac{1}{x} \Rightarrow I(x) = e^{\int \frac{1}{x} dx} = e^{\ln x} = x \Rightarrow xy' + y = \sqrt{x}$$

$$\Rightarrow \frac{d}{dx}(xy) = x^{1/2} \Rightarrow xy = \int x^{1/2}\,dx = \frac{x^{3/2}}{3/2} + C \Rightarrow y = \frac{2}{3}x^{1/2} + \frac{C}{x}$$

A: $\sqrt{x} + C$	B: $\frac{2}{3}\sqrt{x} + \frac{C}{x}$	C: $\frac{2\sqrt{x}+C}{3x}$	D: $\sqrt{x} + \frac{C}{3x}$	E: 0

2 marks A22. Precisely one of the following lines IS NOT an axis of symmetry of the curve $r = 3\cos(2\theta)$. Which one?

A: $y = 0$	B: $x = 0$	C: $y = x$	D: $y = -x$	E: $y = 2x$