STAT 2507 C & BIT 2000 A
Solutions to Midterm Test (Version 2)

1. **Answer: (d).** The sample variance is the sum of **non-negative** terms. Therefore, if the sample variance is zero, then **all** the terms are zero. Whence

$$x_1 - \bar{x} = x_2 - \bar{x} = \ldots = x_5 - \bar{x},$$

which immediately implies $x_1 = x_2 = \ldots = x_5$.

2. **Answer: (b).** We have

$$\mathbf{P}(4\text{"fives" in 4 throwings}) = (1/6)^4 = 0.00077,$$
$$\mathbf{P}(\text{at least 5 "fives" in 6 throwings})$$
$$= \mathbf{P}(5 \text{ "fives" in 6 throwings}) + \mathbf{P}(6 \text{ "fives" in 6 throwings})$$
$$= C_5^6(1/6)^5(5/6) + (1/6)^6 = 0.00066,$$
$$\mathbf{P}(3 \text{ "fives" followed by no "fives" in 9 throwings}) = (1/6)^3(5/6)^9 = 0.000897.$$

3. **Answer: (c).** Range and median will not change, hence the answer.

4. **Answer: (b).** By the property of a discrete probability distribution with pmf $p(x)$, one always has $\sum_x p(x) = 1$.

5. **Answer: (d).** In (a)–(c), the z-scores do not exceed 2, whereas in (d) the z-score is 4.

6. **Answer: (b).** Let O_i, $i = 1, 2$, be the event that the ith selected person has the blood type O. Then the required probability is

$$\mathbf{P}(O_1 \cup O_2) = \mathbf{P}(O_1) + \mathbf{P}(O_2) - \mathbf{P}(O_1 \cap O_1) = 0.4 + 0.4 - (0.4)^2 = 0.64.$$

7. **Answer: (c).** Let X be the number of defective parts among the 4 selected. Then X has a hypergeometric distribution with parameters $N = 25$, $M = 5$, $n = 4$. The required probability is then $\mathbf{P}(X = 1) = C_1^5 C_3^{20}/C_4^{25}$.

8. **Answer: (d).** By definition of the binomial experiment.

9. **Answer: (a).** Let A_i, $i = 1, 2, 3, 4$, be the event that the ith selected battery will work. Then by the multiplication rule, we have

$$\mathbf{P}(\text{first four batteries work}) = \mathbf{P}(A_1)\mathbf{P}(A_2|A_1)\mathbf{P}(A_3|A_1 \cap A_2)\mathbf{P}(A_4|A_1 \cap A_2 \cap A_3)$$
$$= \frac{7}{12} \times \frac{6}{11} \times \frac{5}{10} \times \frac{4}{9} = 0.071.$$

10. **Answer: (c).** By assumption, $\mu - 2\sigma = 1$ and $\mu + 2\sigma = 5$. Thus we have two linear equations with two unknowns. Solving these equations we get $\mu = 3$ and $\sigma = 1$. Hence $\mu - \sigma = 2$ and $\mu + \sigma = 4$.

11. **Answer: (c).** Let X be the assembly time. By assumption, $X \sim N(55, 4^2)$. Therefore, using the normal table, we get that the probability of interest is

$$\mathbf{P}(X < 60) = \mathbf{P}\left(\frac{X-55}{4} < \frac{60-55}{4}\right) = \mathbf{P}(Z < 1.25) = 0.8944.$$

12. **Answer: (d).** Define the events

$$\begin{aligned} A &= \{\text{a passenger uses airport A}\}, \\ B &= \{\text{a passenger uses airport B}\}, \\ C &= \{\text{a passenger uses airport C}\}, \\ D &= \{\text{a passenger is found to carry a weapon}\}. \end{aligned}$$

By assumption,

$$\mathbf{P}(A) = 0.5, \quad \mathbf{P}(B) = 0.3, \quad \mathbf{P}(C) = 0.2,$$
$$\mathbf{P}(D|A) = 0.9, \quad \mathbf{P}(D|B) = 0.5, \quad \mathbf{P}(D|C) = 0.4.$$

Therefore, using the Bayes' rule,

$$\begin{aligned} \mathbf{P}(A|D) &= \frac{\mathbf{P}(D|A)\mathbf{P}(A)}{\mathbf{P}(D|A)\mathbf{P}(A) + \mathbf{P}(D|B)\mathbf{P}(B) + \mathbf{P}(D|C)\mathbf{P}(C)} \\ &= \frac{0.9 \times 0.5}{0.9 \times 0.5 + 0.5 \times 0.3 + 0.4 \times 0.2} = 0.66. \end{aligned}$$

13. **Answer: (c).** Let X be the number of people among 30 who have at least one credit card. Then by assumption, $X \sim \text{Bin}(30, 0.5)$. The probability of interest is $\mathbf{P}(X < 19) = \mathbf{P}(X \le 18)$. Since $np = nq = 30 \times 0.5 = 15 > 5$, the normal approximation to the binomial distribution will be adequate. That is, X is approximately distributed as a normal random variable with mean $\mu = np = 30 \times 0.5 = 15$ and variance $npq = 30 \times 0.5 \times 0.5 = 7.5$. Using the correction for continuity, we obtain from the normal table

$$\mathbf{P}(X \le 18) \approx \mathbf{P}\left(Z \le \frac{18.5 - 15}{\sqrt{7.5}}\right) = \mathbf{P}(Z \le 1.28) = 0.8997.$$

14. **Answer: (a).** Obviously.

15. **Answer: (d).** Clearly, (a) and (b) are not correct answers; (c) is also not correct because $(0.875)^2 = 0.766$ is too small to be the variance for the data in the histogram.

16. **Answer: (c).** Define the events

$$\begin{aligned} C &= \{\text{a customer has car insurance}\}, \\ H &= \{\text{a customer has homeowner insurance}\}. \end{aligned}$$

By assumption,

$$\mathbf{P}(C) = 0.5, \quad \mathbf{P}(H) = 0.4, \quad \mathbf{P}(C \cap H) = 0.25.$$

Therefore the required probability is

$$\begin{aligned} \mathbf{P}(C \cap H^c) + \mathbf{P}(H \cap C^c) &= [\mathbf{P}(C) - \mathbf{P}(C \cap H)] + [\mathbf{P}(H) - \mathbf{P}(C \cap H)] \\ &= (0.5 - 0.25) + (0.4 - 0.25) = 0.4. \end{aligned}$$

17. **Answer: (a).** The line that goes through the points $(2, 3)$ and $(3.5, b)$ with $b > 3.2$ has a positive slope, and we know that the correlation between two positively linearly related observations is one.

18. **Answer: (b).** The normal distribution has the properties (i) and (ii). It also has property (iv) because if $X \sim N(\mu, \sigma^2)$, then $\mathbf{P}(|X - \mu| < \sigma) = \mathbf{P}(|Z| < 1) = \mathbf{P}(Z \leq 1) = \mathbf{P}(Z \leq -1) = 0.6826$. It does not have property (iii) because mean of the normal distribution can assume any real value, and variance can assume any positive real value.

19. **Answer: (b).** By assumption,

$$r(s_y/s_x) = -2, \quad s_x = 1, \quad s_y = 4.$$

From this, $r = (-2)(s_x/s_y) = (-2) \times (1/4) = -0.5$. Therefore the slope of the least squares line $x = a + by$ will be

$$b = r(s_x/s_y) = (-0.5)(1/4) = -1/8.$$

20. **Answer: (c).** The data set consists of $n = 29$ observations. Hence the (sample) median is the 15th ordered observation, that is 48. The only boxplot with the median equal to 48 is (c).