École de gestion TELFER School of Management

LIÉE au LINKED with LEADERSHIP

ADM 2304
APPLIED STATISTICAL METHODS IN BUSINESS

3 March 2012
9:00 – 11:00

NAME (please print): ___________________________________

Student Number: __________________

SECTION REGISTERED (Circle one):

M N P Q R

Instructions

Length of Exam: 5 pages, plus 2+ pages of Minitab output (please return).
Please show all your work and explain your answers briefly where required.
You are encouraged to use the Minitab output as much as possible.
You are permitted to have a non-programmable calculator and a sheet (8.5 x 11 inch) of notes.
Statistical tables (normal, t and chi-square) are provided separately (please return).

Marks:	________	+	________	+	_______	+	________	=	___________
	9		14		11		11		45

Question 1 [9 marks]

A simple random sample of balanced mutual funds was taken. The relevant data on unit prices are presented in Appendix A. Do all tests at the .05 level of significance.

a. Test the hypothesis that the mean value of unit prices of balanced mutual funds is less than $15.00. Use the critical value approach to reach your conclusion.
[4]

b. Calculate the 95% asymmetrical or 1-sided confidence interval and explain how it is consistent with the conclusion you reached in part ‘a’ above.
[2]

c. Now test the hypothesis that the median value of unit prices of balanced mutual funds is less than $15.00. Perform a complete hypothesis test.
[2]

d. Which test is more appropriate? The one performed in part ‘a’ or the one performed in part ‘c’ above? Justify your answer.
[1]

Question 2. [14 marks]

A recently amalgamated hospital still runs two separate surgical departments on two campuses. Each surgical unit contains multiple operating rooms and performs over 10,000 surgeries a year, broken down into 900 different types of surgery. The administration wanted to know which unit was more efficient and hired two business students to examine the surgery times. The first student took two samples of 100 surgeries from each hospital taken at random from the total number of surgeries performed in the last year. The second student first divided the surgeries into the different surgical types and chose 100 types of surgery at random from the 900 possible options. For each of these 100 types of surgery, she randomly chose one surgery from each unit. Appendix B summarizes and plots the data. Perform all tests using the .05 level of significance and use the normal approximation to the t-distribution.

a. What conclusion should the first student reach? Perform the appropriate hypothesis test and comment on the assumptions of the test by referring to the appropriate graph(s).
[6]

Boxplot(s) ____________show(s) that this large sample(s) t-test is valid because it is reasonable to assume the population data are

________________________________.

b. What conclusion should the second student reach? Perform the appropriate hypothesis test and comment on the assumptions of the test by referring to the appropriate graph(s).
[6]

Boxplot(s) ___________show(s) that this large sample(s) t-test is valid because it is reasonable to assume the population data are

__________________________________.

c. Discuss which student's conclusion is more justified as well as the reasons behind any differences between the two students' conclusions.
[2]

Question 3. [11 marks]

All levels of governments are getting ready to wield the mighty budget-cutting axe. Different political parties usually tend to have their own political agenda and concentrate on specific issues. Moreover, in advance of the budget, parties conduct their own opinion polls to push their point of view to change the direction of the budget.

Party A found 280 respondents in a random sample of 400 who thought health-care was a very important issue. Party B found 300 in a random sample of 500 who considered the economy and job creation to be a very important issue.

a. Test the hypothesis that fewer than 75% of the population consider health-care to be a very important issue. Use the .05 level of significance.
[4]

b. If the margin of error were to be limited to ± 1%, what sample size should be used to estimate the proportion of people who consider health care to be a very important issue? (Assume a 95% confidence level.)
[2]

c. Now test the hypothesis that the proportion who believe health care to be a very important exceeds the proportion who believe the economy and job creation to be a very important by more than 5%. Perform the test by calculating the appropriate 95% asymmetric (one-sided) confidence interval for the true difference in proportions.
[5]

Question 4. [11 marks]

Two political polls were taken during mid-February to gauge the popularity of the four major Republican candidates for the presidency. The data are presented in the table below (ref. http://www.realclearpolitics.com/epolls/2012/president/us/republican_presidential_nomination-1452.html). Perform all tests at the .05 level of significance.

Poll	Santorum	Romney	Gingrich	Paul	**Total**
Associated Press	148	144	67	67	**426**
Rasmussen	351	243	135	90	**819**

a. Test whether there is a statistically significant difference between the distribution of support as estimated by the two polls. What is the approximate p-value? (Use the output in Appendix C, but write your intermediate calculations in the table above.)
[6]

The p-value is between ____________ and ______________.

b. A pundit divined that Santorum and Romney were tied with 35% of the vote and that Gingrich and Paul were tied with 15% of the vote. Test whether the results of the Rasmussen poll can be used to refute the pundit's contention.
[5]

Appendix A

Data Display

```
Unit_Price$
    5.95   16.68    9.57   13.03   12.53   22.66   14.99   17.04   10.51
   14.21   13.45   12.25   14.91   12.65    8.19   18.01   10.98    9.62
   12.34   14.71   13.77   14.53

Variable      N  N*    Mean  SE Mean  StDev  Minimum      Q1  Median      Q3
Unit_Price$  22   0  13.299    0.763  3.580    5.950  10.862  13.240  14.930

Variable     Maximum
Unit_Price$   22.660
```

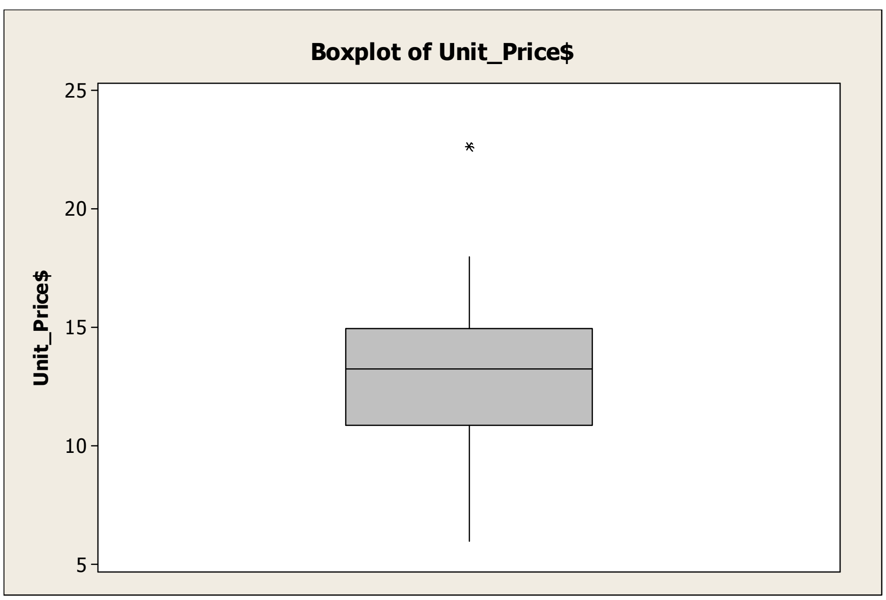
One-Sample T: Unit_Price$

```
Test of mu = 15 vs < 15
                                         95% Upper
Variable      N    Mean  StDev  SE Mean      Bound      T      P
Unit_Price$  22  13.299  3.580    0.763
```

Wilcoxon Signed Rank Test: Unit_Price$

```
Test of median = 15.00 versus median < 15.00

                 N for   Wilcoxon          Estimated
              N   Test  Statistic      P      Median
Unit_Price$  22     22       54.0  0.010       13.28
```



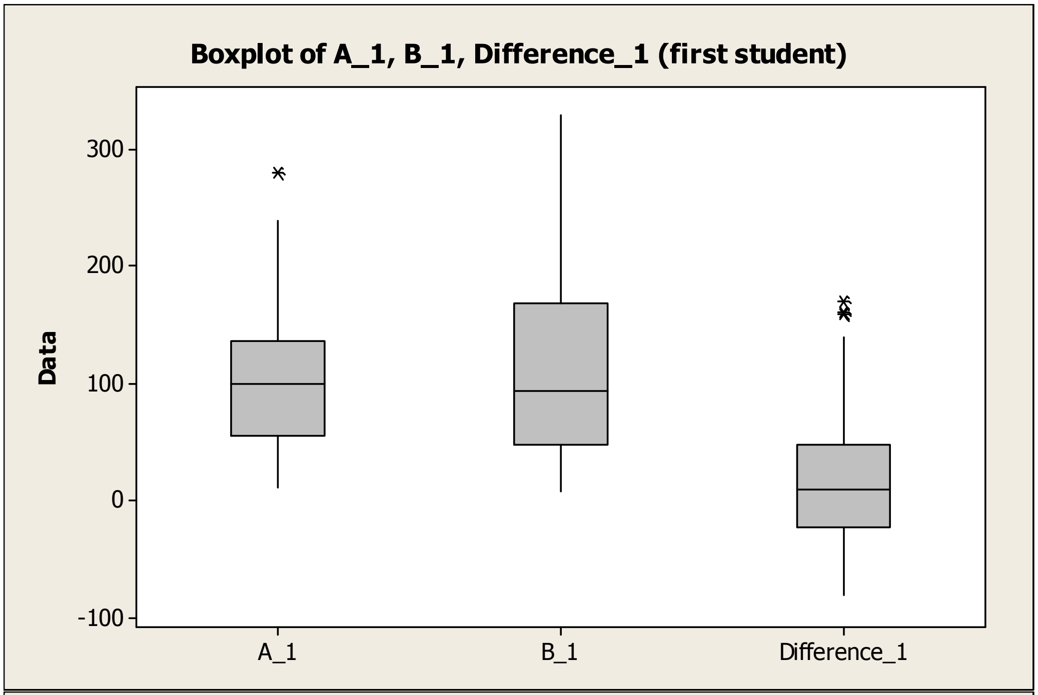
Appendix B

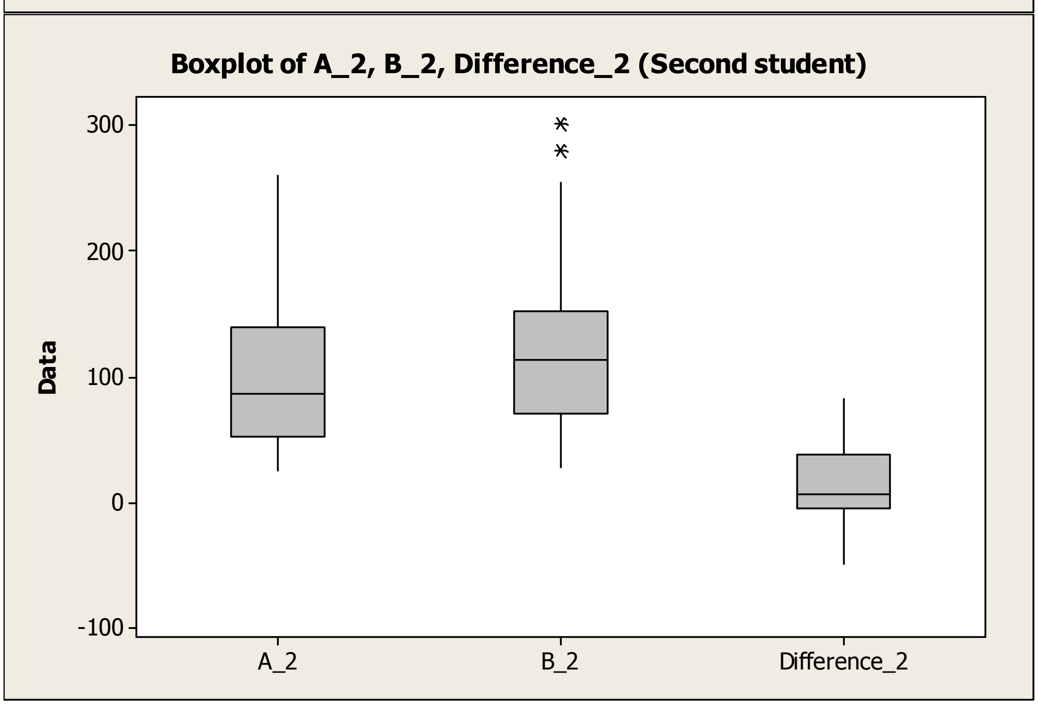
Summary data on times (in minutes) from first student's samples:

	N	Mean	StDev	SE Mean
A_1	100	103.64	60.91	6.09
B_1	100	120.57	85.15	8.51
Difference	100	-16.93	58.58	5.86

Summary data on times (in minutes) from second student's samples:

	N	Mean	StDev	SE Mean
A_2	100	103.43	60.64	6.06
B_2	100	119.87	65.34	6.53
Difference	100	-16.44	30.23	3.02





Appendix C.

```
Expected counts are printed below observed counts
Chi-Square contributions are printed below expected counts

           C1       C2       C3       C4  Total
    1     148      144       67       67    426
       170.74   132.42    69.12    53.72
        3.029    1.013    0.065    3.283

    2     351      243      135       90    819
       ______    ____     ____    103.28
       ______    ____     ____     1.707

Total     499      387      202      157   1245

Chi-Sq = _____, DF = _, P-Value = _____
```

Chi-square distribution

$\chi 2_{\alpha;\nu}$

$\alpha = P(\chi^2 \geq \chi^2_{\alpha;\nu})$

ν	0.200	0.100	0.050	0.045	0.040	0.035	0.030	0.025	0.020	0.015	0.010	0.005
1	1.64	2.71	3.84	4.02	4.22	4.45	4.71	5.02	5.41	5.92	6.63	7.88
2	3.22	4.61	5.99	6.20	6.44	6.70	7.01	7.38	7.82	8.40	9.21	10.60
3	4.64	6.25	7.81	8.05	8.31	8.61	8.95	9.35	9.84	10.47	11.34	12.84
4	5.99	7.78	9.49	9.74	10.03	10.35	10.71	11.14	11.67	12.34	13.28	14.86
5	7.29	9.24	11.07	11.34	11.64	11.98	12.37	12.83	13.39	14.10	15.09	16.75
6	8.56	10.64	12.59	12.88	13.20	13.56	13.97	14.45	15.03	15.78	16.81	18.55
7	9.80	12.02	14.07	14.37	14.70	15.08	15.51	16.01	16.62	17.40	18.48	20.28
8	11.03	13.36	15.51	15.82	16.17	16.56	17.01	17.53	18.17	18.97	20.09	21.95
9	12.24	14.68	16.92	17.25	17.61	18.01	18.48	19.02	19.68	20.51	21.67	23.59
10	13.44	15.99	18.31	18.65	19.02	19.44	19.92	20.48	21.16	22.02	23.21	25.19
11	14.63	17.28	19.68	20.02	20.41	20.85	21.34	21.92	22.62	23.50	24.72	26.76
12	15.81	18.55	21.03	21.39	21.79	22.23	22.74	23.34	24.05	24.96	26.22	28.30
13	16.98	19.81	22.36	22.73	23.14	23.60	24.12	24.74	25.47	26.40	27.69	29.82
14	18.15	21.06	23.68	24.07	24.49	24.96	25.49	26.12	26.87	27.83	29.14	31.32
15	19.31	22.31	25.00	25.39	25.82	26.30	26.85	27.49	28.26	29.23	30.58	32.80
16	20.47	23.54	26.30	26.70	27.14	27.63	28.19	28.85	29.63	30.63	32.00	34.27
17	21.61	24.77	27.59	27.99	28.44	28.95	29.52	30.19	31.00	32.01	33.41	35.72
18	22.76	25.99	28.87	29.29	29.75	30.26	30.84	31.53	32.35	33.38	34.81	37.16
19	23.90	27.20	30.14	30.57	31.04	31.56	32.16	32.85	33.69	34.74	36.19	38.58
20	25.04	28.41	31.41	31.84	32.32	32.85	33.46	34.17	35.02	36.09	37.57	40.00
21	26.17	29.62	32.67	33.11	33.60	34.14	34.76	35.48	36.34	37.43	38.93	41.40
22	27.30	30.81	33.92	34.37	34.87	35.42	36.05	36.78	37.66	38.77	40.29	42.80
23	28.43	32.01	35.17	35.63	36.13	36.69	37.33	38.08	38.97	40.09	41.64	44.18
24	29.55	33.20	36.42	36.88	37.39	37.96	38.61	39.36	40.27	41.41	42.98	45.56
25	30.68	34.38	37.65	38.12	38.64	39.22	39.88	40.65	41.57	42.73	44.31	46.93
26	31.79	35.56	38.89	39.36	39.89	40.48	41.15	41.92	42.86	44.03	45.64	48.29
27	32.91	36.74	40.11	40.60	41.13	41.73	42.41	43.19	44.14	45.33	46.96	49.64
28	34.03	37.92	41.34	41.83	42.37	42.97	43.66	44.46	45.42	46.63	48.28	50.99
29	35.14	39.09	42.56	43.05	43.60	44.22	44.91	45.72	46.69	47.91	49.59	52.34
30	36.25	40.26	43.77	44.28	44.83	45.45	46.16	46.98	47.96	49.20	50.89	53.67
35	41.78	46.06	49.80	50.34	50.93	51.59	52.34	53.20	54.24	55.55	57.34	60.27
40	47.27	51.81	55.76	56.32	56.95	57.64	58.43	59.34	60.44	61.81	63.69	66.77
50	58.16	63.17	67.50	68.12	68.80	69.56	70.42	71.42	72.61	74.11	76.15	79.49
100	111.67	118.50	124.34	125.17	126.08	127.09	128.24	129.56	131.14	133.12	135.81	140.17
150	164.35	172.58	179.58	180.57	181.65	182.86	184.22	185.80	187.68	190.03	193.21	198.36
200	216.61	226.02	233.99	235.12	236.35	237.72	239.27	241.06	243.19	245.85	249.45	255.26
224	241.59	251.52	259.91	261.10	262.39	263.84	265.47	267.35	269.58	272.38	276.16	282.27
250	268.60	279.05	287.88	289.12	290.49	292.00	293.71	295.69	298.04	300.97	304.94	311.35
500	526.40	540.93	553.13	554.84	556.71	558.80	561.14	563.85	567.07	571.08	576.49	585.21

Standard Normal Distribution

P(Z < z) **(z negative)**

Second decimal place in z

0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01	0.00	**z**
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-3.9
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	-3.8
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	-3.7
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	-3.6
0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	-3.5
0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	-3.4
0.0003	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0005	0.0005	0.0005	-3.3
0.0005	0.0005	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0007	0.0007	-3.2
0.0007	0.0007	0.0008	0.0008	0.0008	0.0008	0.0009	0.0009	0.0009	0.0010	-3.1
0.0010	0.0010	0.0011	0.0011	0.0011	0.0012	0.0012	0.0013	0.0013	0.0013	-3.0
0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0019	-2.9
0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026	-2.8
0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034	0.0035	-2.7
0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0045	0.0047	-2.6
0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0060	0.0062	-2.5
0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0082	-2.4
0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107	-2.3
0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139	-2.2
0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179	-2.1
0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228	-2.0
0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287	-1.9
0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359	-1.8
0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446	-1.7
0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548	-1.6
0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668	-1.5
0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808	-1.4
0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968	-1.3
0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151	-1.2
0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357	-1.1
0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587	-1.0
0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841	-0.9
0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090	0.2119	-0.8
0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2420	-0.7
0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743	-0.6
0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085	-0.5
0.3121	0.3156	0.3192	0.3228	0.3264	0.3300	0.3336	0.3372	0.3409	0.3446	-0.4
0.3483	0.3520	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3821	-0.3
0.3859	0.3897	0.3936	0.3974	0.4013	0.4052	0.4090	0.4129	0.4168	0.4207	-0.2
0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4602	-0.1
0.4641	0.4681	0.4721	0.4761	0.4801	0.4840	0.4880	0.4920	0.4960	0.5000	0.0

Standard Normal Distribution

P(Z < z) **(z positive)**

Second decimal place in z

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Student's t distribution

t_α

$\alpha = P(t > t_\alpha)$ = one-tail probability

df	0.100	0.050	0.025	0.010	0.009	0.008	0.005	0.001	0.0005	0.0004	0.0001
1	3.08	6.31	12.71	31.82	35.36	39.78	63.66	318.31	636.62	837.66	3183.10
2	1.89	2.92	4.30	6.96	7.35	7.81	9.92	22.33	31.60	36.25	70.70
3	1.64	2.35	3.18	4.54	4.72	4.93	5.84	10.21	12.92	14.18	22.20
4	1.53	2.13	2.78	3.75	3.87	4.01	4.60	7.17	8.61	9.25	13.03
5	1.48	2.02	2.57	3.36	3.46	3.57	4.03	5.89	6.87	7.29	9.68
6	1.44	1.94	2.45	3.14	3.23	3.32	3.71	5.21	5.96	6.28	8.02
7	1.41	1.89	2.36	3.00	3.07	3.16	3.50	4.79	5.41	5.67	7.06
8	1.40	1.86	2.31	2.90	2.97	3.04	3.36	4.50	5.04	5.26	6.44
9	1.38	1.83	2.26	2.82	2.89	2.96	3.25	4.30	4.78	4.98	6.01
10	1.37	1.81	2.23	2.76	2.83	2.89	3.17	4.14	4.59	4.77	5.69
11	1.36	1.80	2.20	2.72	2.78	2.84	3.11	4.02	4.44	4.60	5.45
12	1.36	1.78	2.18	2.68	2.74	2.80	3.05	3.93	4.32	4.47	5.26
13	1.35	1.77	2.16	2.65	2.71	2.77	3.01	3.85	4.22	4.37	5.11
14	1.35	1.76	2.14	2.62	2.68	2.74	2.98	3.79	4.14	4.28	4.99
15	1.34	1.75	2.13	2.60	2.66	2.71	2.95	3.73	4.07	4.21	4.88
16	1.34	1.75	2.12	2.58	2.64	2.69	2.92	3.69	4.01	4.15	4.79
17	1.33	1.74	2.11	2.57	2.62	2.67	2.90	3.65	3.97	4.09	4.71
18	1.33	1.73	2.10	2.55	2.60	2.66	2.88	3.61	3.92	4.04	4.65
19	1.33	1.73	2.09	2.54	2.59	2.64	2.86	3.58	3.88	4.00	4.59
20	1.33	1.72	2.09	2.53	2.58	2.63	2.85	3.55	3.85	3.97	4.54
21	1.32	1.72	2.08	2.52	2.57	2.62	2.83	3.53	3.82	3.93	4.49
22	1.32	1.72	2.07	2.51	2.56	2.61	2.82	3.50	3.79	3.91	4.45
23	1.32	1.71	2.07	2.50	2.55	2.60	2.81	3.48	3.77	3.88	4.42
24	1.32	1.71	2.06	2.49	2.54	2.59	2.80	3.47	3.75	3.85	4.38
25	1.32	1.71	2.06	2.49	2.53	2.58	2.79	3.45	3.73	3.83	4.35
26	1.31	1.71	2.06	2.48	2.53	2.58	2.78	3.43	3.71	3.81	4.32
27	1.31	1.70	2.05	2.47	2.52	2.57	2.77	3.42	3.69	3.79	4.30
28	1.31	1.70	2.05	2.47	2.51	2.56	2.76	3.41	3.67	3.78	4.28
29	1.31	1.70	2.05	2.46	2.51	2.56	2.76	3.40	3.66	3.76	4.25
30	1.31	1.70	2.04	2.46	2.50	2.55	2.75	3.39	3.65	3.75	4.23
31	1.31	1.70	2.04	2.45	2.50	2.55	2.74	3.37	3.63	3.73	4.22
32	1.31	1.69	2.04	2.45	2.49	2.54	2.74	3.37	3.62	3.72	4.20
33	1.31	1.69	2.03	2.44	2.49	2.54	2.73	3.36	3.61	3.71	4.18
34	1.31	1.69	2.03	2.44	2.49	2.54	2.73	3.35	3.60	3.70	4.17
35	1.31	1.69	2.03	2.44	2.48	2.53	2.72	3.34	3.59	3.69	4.15
36	1.31	1.69	2.03	2.43	2.48	2.53	2.72	3.33	3.58	3.68	4.14
37	1.30	1.69	2.03	2.43	2.48	2.52	2.72	3.33	3.57	3.67	4.13
38	1.30	1.69	2.02	2.43	2.47	2.52	2.71	3.32	3.57	3.66	4.12
39	1.30	1.68	2.02	2.43	2.47	2.52	2.71	3.31	3.56	3.65	4.10
40	1.30	1.68	2.02	2.42	2.47	2.52	2.70	3.31	3.55	3.65	4.09
50	1.30	1.68	2.01	2.40	2.45	2.49	2.68	3.26	3.50	3.59	4.01
60	1.30	1.67	2.00	2.39	2.43	2.48	2.66	3.23	3.46	3.55	3.96
100	1.29	1.66	1.98	2.36	2.41	2.45	2.63	3.17	3.39	3.47	3.86
1000	1.28	1.65	1.96	2.33	2.37	2.41	2.58	3.10	3.30	3.38	3.73