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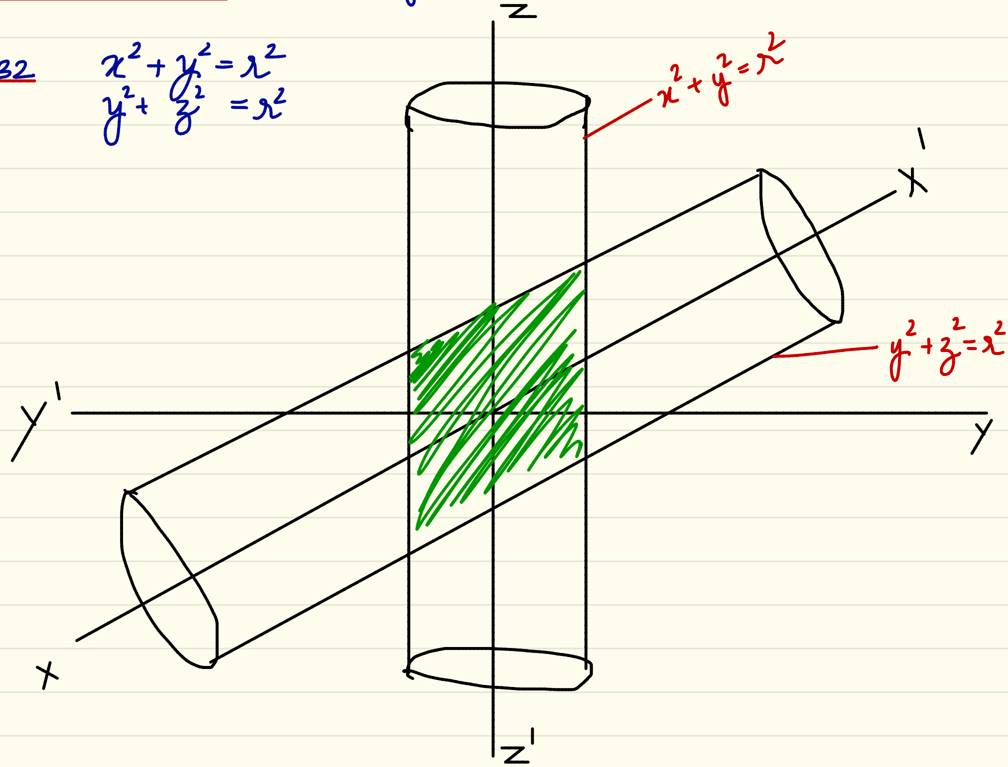
Math 253 – Written Homework Assignment

Section 15.3 – Double Integrals over General Areas

Q32 $x^2 + y^2 = r^2$
$y^2 + z^2 = r^2$



Take a slice of the solid, parallel to the x-y plane:-
Side $= 2x$, thickness $= dz$

Volume $= \int_{-r}^{r} 4x^2 \, dz$, But, on the intersection $x^2 = y^2$ &
$y^2 = r^2 - z^2$

$$\Longrightarrow V = \int_{-r}^{r} 4(r^2 - z^2) \, dz = \left[4r^2 z - \frac{z^3}{3} \right]_{-r}^{r} = \boxed{\frac{16r^3}{3}}$$

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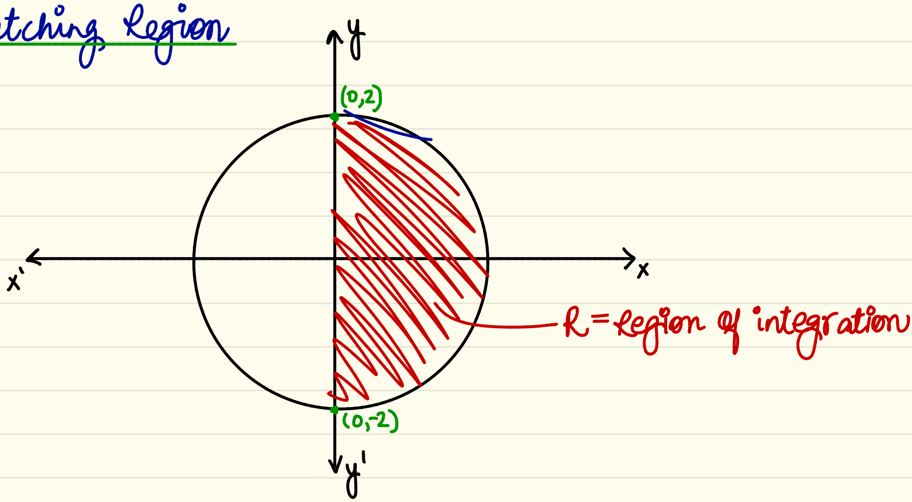
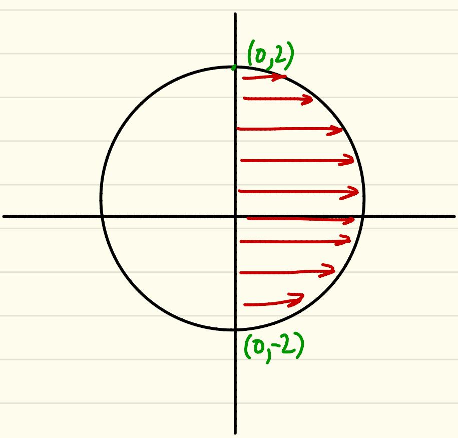
Q46 $\displaystyle\int_{-2}^{2}\int_{0}^{\sqrt{4-y^2}} f(x,y).\,dx.\,dy$

Sketching Region

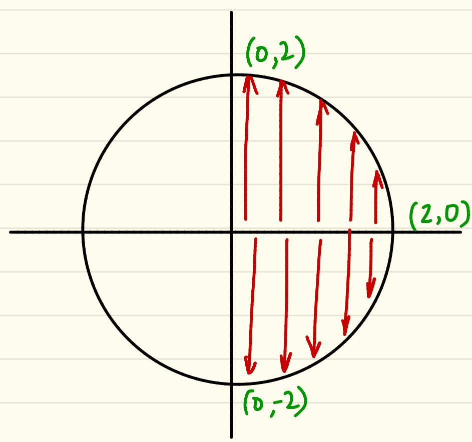


Changing order of Integration

Previously :-



Now :-



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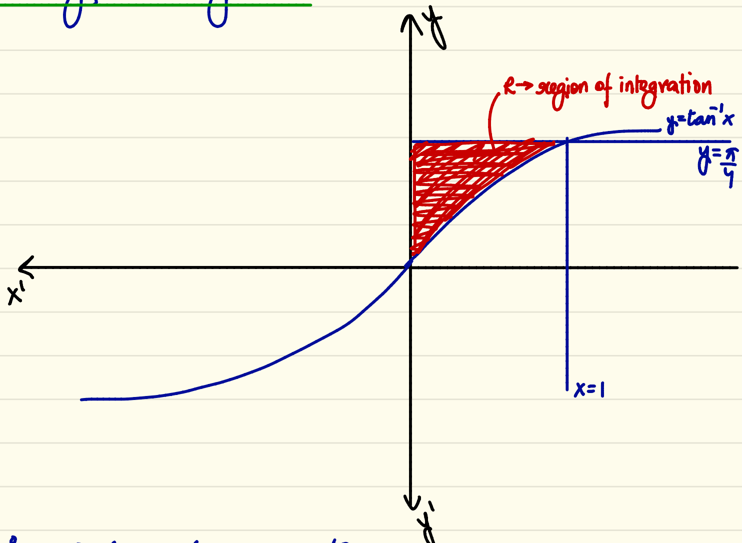
$$\int_0^2 \int_{\sqrt{4-x^2}}^2 f(x,y).\,dy.\,dx + \int_0^2 \int_{-2}^{\sqrt{4-x^2}} f(x,y).\,dy.\,dx \longrightarrow \text{Answer}$$

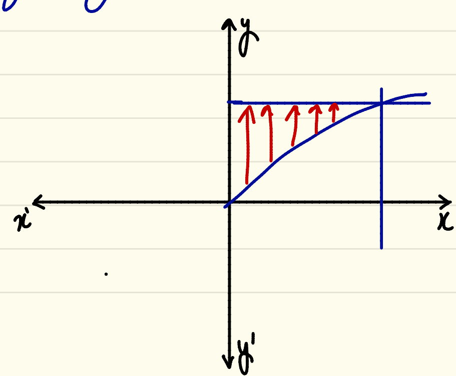
Q.48 $\displaystyle\int_0^1 \int_{\tan^{-1}x}^{\pi/4} f(x,y).\,dy.\,dx$

Sketching the Region



Changing Order of Integration

Previously:



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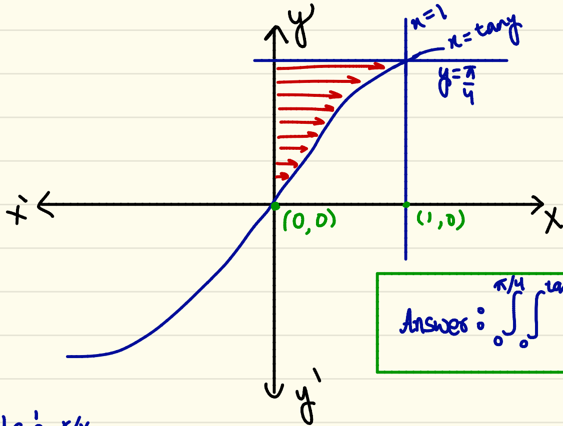
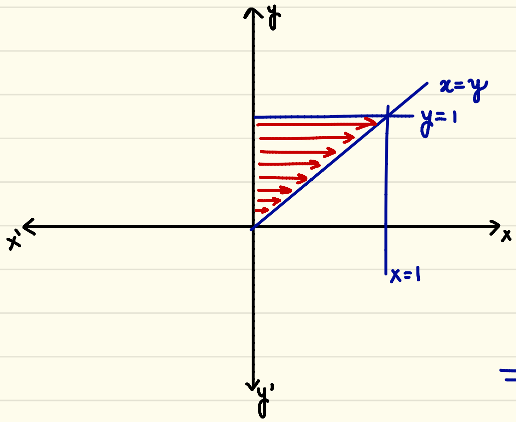
Now :-



$$\boxed{\text{Answer: } \int_0^{\pi/4}\int_0^{\tan y} f(x,y).dx.dy}$$

Q52 $\int_0^1\int_x^1 e^{x/y}.dy.dx$



Reversed order of integration gives :-

$$\int_0^1\int_0^y e^{x/y}.dx.dy$$

$$= \int_0^1 ye^1 - ye^0 \; dy$$

$$= \left[\frac{y^2}{2}e^1 - \frac{y^2}{2}\right]_0^1$$

$$= \boxed{\frac{y^2}{2}\left[e^1 - 1\right] = \frac{1}{2}\left[e^1 - 1\right]}$$

Section 15.4 – Double Integrals in Polar Coordinates

Q10 $\iint_R \frac{y^2}{x^2+y^2}.dA$ where, R is the region that lies between the cir s,
$x^2+y^2=a^2$
$x^2+y^2=b^2$

& $0<a<b$

The region R is the area of annulus between circles of radii a and b. Graphically, this can be expressed as:-

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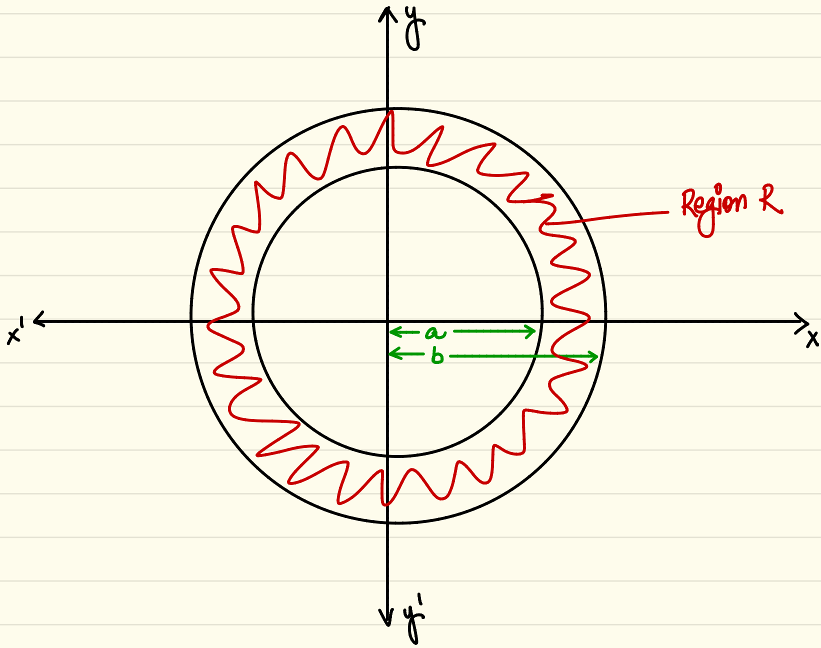

$\theta \rightarrow 0$ to 2π
$r \rightarrow a$ to b

$x = r\cos\theta$
$y = r\sin\theta$

$dA = r.d\theta.dr$

Then, $\iint_R \frac{y^2}{x^2+y^2}.dA = \int_0^{2\pi}\int_a^b \frac{r^2\sin^2\theta}{r^2(\cos^2\theta+\sin^2\theta)} \times r\,dr\,d\theta = \int_a^b r.dr . \int_0^{2\pi} \sin^2\theta.d\theta$

$$= \left[\frac{b^2}{2}-\frac{a^2}{2}\right] . \frac{1}{2}\int_0^{2\pi} 1-\cos2\theta.d\theta = \left[\frac{b^2-a^2}{2}\right]\left[\frac{1}{2}\right]\left[2\pi - \frac{\sin 2(2\pi)}{2} + \frac{\sin 0}{2}\right]$$

$$= \frac{b^2-a^2}{4}\left[2\pi - 0\right] = (b^2-a^2)\frac{\pi}{2}$$

$\Rightarrow$ $\boxed{\text{Answer} \rightarrow \frac{\pi}{2}(b^2-a^2)}$

Q26 Find Volume of the region :

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bounded by the paraboloids:

$$z = 3x^2 + 3y^2$$
&
$$z = 4 - x^2 - y^2$$

The paraboloids intersect when $3x^2 + 3y^2 = 4 - x^2 - y^2$
$\Rightarrow x^2 + y^2 = 1$
$\Rightarrow$ region bound is $x^2 + y^2 \leq 1$

$$V = \iint_R 4 - x^2 - y^2 - 3x^2 - 3y^2 \, . \, dA$$

$$V = \int_0^{2\pi}\int_0^1 4(1 - r^2(\sin^2\theta + \cos^2\theta))\, r \, . \, dr \, . \, d\theta$$

$$V = \int_0^{2\pi} d\theta \, . \int_0^1 4r - r^3 \, . \, dr$$

$$\boxed{V = 2\pi \longrightarrow \text{Answer}}$$

Q38 average distance from points in a disk Δ of radius a to the origin is given by :-

$$\frac{1}{\text{Area}(\Delta)} \iint_\Delta r \, . \, dA = \frac{1}{\pi a^2} \int_0^{2\pi}\int_0^a r^2 \, dr d\theta$$

$$= \frac{1}{\pi a^2}\left[\frac{2\pi \, . \, a^3}{3}\right]$$

$$\boxed{\text{Average Distance} = \frac{2}{3}a}$$