MAT 2377C
Midterm exam (practice)

October 2012 **Professor: Rafal Kulik**
Time: 70 minutes

Student Number: _____

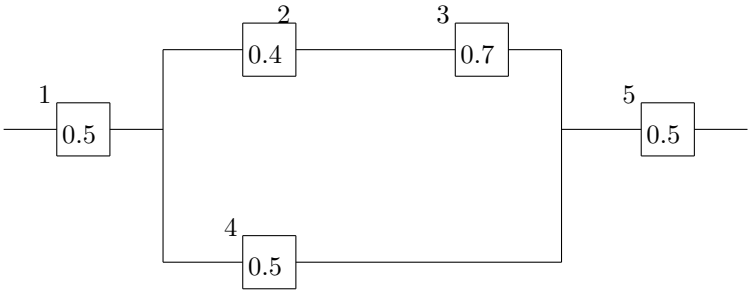
Family Name: _____

First Name: _____

This is a closed book examination.
Only non-programmable and non-graphic calculators are permitted.
Record your answer to each question in the table below.
Number of pages: **3** (including this one).
Number of questions: **10**.
NOTE: At the end of the examination, hand in only this page. You may keep the questionnaire.

Question	Answer	Question	Answer
1		13	
2		14	
3		15	
4		16	
5		17	
6		18	
7		19	
8		20	
9		21	
10		22	
11		23	
12		24	

Q1. Consider the following system with five components. We say that it is functional if there exists a path of functional components from left to right. The probability of each component functions is shown. Assume that the components function or fail independently. What is the probability that the system operates ?



(a) .84 (b) .16 (c) .035 (d) .50 (e) none of the preceding

Solution to Q1:

Call 'Box B' - components 2,3,4, 'Box C' - components 2,3.

$$\begin{aligned} P(\text{Box C operates}) &= P(\text{component 2 operates and component 3 operates}) \\ &= P(\text{component 2 operates})P(\text{component 3 operates}) = 0.4 \times 0.7 = 0.28. \end{aligned}$$

$$\begin{aligned} P(\text{Box B operates}) &= P(\text{Box C operates or component 4 operates}) \\ &= P(\text{Box C operates}) + P(\text{component 4 operates}) - \\ &\quad P(\text{Box C operates})P(\text{component 4 operates}) \\ &= 0.28 + 0.5 - 0.28 * 0.5 = 0.64. \end{aligned}$$

$$\begin{aligned} P(\text{system operates}) &= P(\text{component 1 and Box B and component 5 operate}) \\ &= P(\text{component 1 operates})P(\text{Box B operates})P(\text{component 5 operates}) \\ &= 0.5 * 0.64 * 0.5 = 0.16. \end{aligned}$$

Q2. A city has 1000 married couples with both husband and wife working. Each person was asked whether his or her salary exceeded \$30,000. The following information was obtained.

		Husband less than \$30,000	Husband more than \$30,000
Wife	less than \$30,000	430	410
Wife	more than \$30,000	60	100

What is the probability that a wife earns more than \$30,000 given that the husband earns more than \$30,000?

(a) 0.8059 (b) 0.1961 (c) 0.4300 (d) 0.5700 (e) none of the preceding

Solution to Q2:

Let A be the event that a wife earns more than \$30,000, B be the event that the husband earns more than \$30,000. To calculate $P(A|B)$. We have $P(A \cap B) = 100/1000 = 0.1$, $P(B) = 510/1000 = 0.51$

$$P(A|B) \quad = \quad P(A \cap B)/P(B) = 0.1/0.51 = 0.1961 \; .$$

Q3. In a group of twenty students, each student has a probability of 0.7 of passing an exam. What is the probability that the majority of them will pass an exam?

(a) 0.9829 (b) 0.0171 (c) 0.0480 (d) 0.9520

Solution to Q3:

X- number of students who pass an exam. $X \sim \mathcal{B}(20, 0.7)$. To compute $P(X \geq 11) = 1 - P(X \leq 10) = 1 - 0.0480 = 0.9520$.

Q4. A manufacturer produces bolts such that 75% of the bolts have a diameter between 3.02 and 3.07 cm. Suppose that the bolts are being selected at random one at a time. What is the probability that we will require to select 5 bolts to observe a bolt with a diameter between 3.02 and 3.07?

(a) 0.0791 (b) 0.0010 (c) 0.0029 (d) 0.2373 (e) none of the preceding

Solution to Q4:

success = we select a bolt with a diameter between 3.02 and 3.07 cm. If X is the number of steps required to observe the first bolt with a diameter between 3.02 and 3.07 cm, then X is Geometric with $p = 0.75$. Thus $P(X = 5) = (1 - p)^4 p = 0.0029$.

Q5. Consider a random variable X with the following cumulative distribution function:

$$F(x) = \begin{cases} 0 & \text{if } x \leq -1 \\ \frac{3}{4}x - \frac{1}{4}x^3 + \frac{1}{2} & \text{if } -1 < x < 1 \\ 1 & \text{if } x \geq 1 \end{cases}$$

Calculate the expected value and the variance of X.

(a) 0 and 3 (b) 1 and .2 (c) 1 and 3

(d) 0 and .2 (e) none of the preceding

Q6. Suppose that major cracks on a highway can be modeled as a Poisson process with 0.2 major cracks per mile on average. What is the probability that the first major crack occurs between 12 and 15 miles from the start of inspection?

(a) $\exp(-15) - \exp(-12)$ (b) $\exp(-12) - \exp(-15)$

(c) $\exp(-3) - \exp(-2.4)$ (d) $\exp(-2.4) - \exp(-3)$

(e) none of the preceding

Solution to Q6:

We have $\lambda = 0.2$. The distance to the first crack, X say, has exponential distribution with parameter λ. To compute $P(12 < X < 15) = P(X < 15) - P(X < 12) = \exp(-12\lambda) - \exp(-15\lambda) = \exp(-2.4) - \exp(-3)$.

Q7. In a data communication system, several messages that arrive at a node are bundled into a packet before they are transmitted over the network. Assume the messages arrive to the node according to a Poisson process with $\lambda = 30$ messages per minute. Five messages are used to form a packet. What is the mean time and the standard deviation time until a packet is formed?

(a) $\frac{1}{6}; \frac{\sqrt{5}}{30}$ (b) $\frac{1}{6}; \frac{5}{30}$ (c) $\frac{1}{6}; \frac{1}{6}$ (d) 30; 30 (e) none of the preceding

Solution to Q7:

Let X be the time between messages. Then has exponential distribution with $\lambda = 30$. Let Y be the time until a packet is formed. Then Y is Gamma with parameters $\lambda = 30$ and $r = 5$.

Q8. The cross-sectional area of plastic tubing for use in pulmonary resuscitators is normally distributed with $\mu = 12.5\,\mathsf{mm}^2$ and $\sigma = 0.2\,\mathsf{mm}^2$. When the area is less than $12\,\mathsf{mm}^2$ or greater than $13\,\mathsf{mm}^2$, the tube does not fit properly. What is the probability that a tube chosen randomly will fit properly?

(a) 0.953790 (b) 0.999790 (c) 0.98758 (d) 0.97450 (e) none of the preceding

Solution to Q8:

X - area, $X \sim \mathcal{N}(12.5, 0.2^2)$. To compute (use tables from the textbook)

$$P(12 < X < 13) = P\left(\frac{12 - 12.5}{0.2} < \frac{X - 12.5}{0.2} < \frac{13 - 12.5}{0.2}\right) = 0.98758$$

Q9. Let X be a normal random variable with mean 2 and variance 4. The value of t such that $P(X < t) = 0.9901$, is

(a) 2.33 (b) 6.66 (c) 4.66 (d) 0.9901 (e) none of the preceding

Solution to Q9:

Q10. Consider the following sample of size $n = 6$:

$$12,\ 10,\ 5,\ 25,\ 18,\ 10$$

Compute the sample mean, sample median and the sample standard deviation.

$$5,\ 10,\ 10,\ 12,\ 18,\ 25$$

(a) $\overline{x} = 13.33$, median $= 11$ and $s = 7.09$ (b) $\overline{x} = 13.667$, median $= 11$ and $s = 50.26$

(c) $\overline{x} = 13.33$, median $= 20$ and $s^2 = 50.26$ (d) $\overline{x} = 11.0$, median $= 20$ and $s = 5.439$

(e) none of the preceding.

This is the last question

Solutions to multiple choice questions:

Q1 $\longrightarrow$ b
Q2 $\longrightarrow$ b
Q3 $\longrightarrow$ d
Q4 $\longrightarrow$ c
Q5 $\longrightarrow$ d
Q6 $\longrightarrow$ d
Q7 $\longrightarrow$ a
Q8 $\longrightarrow$ c
Q9 $\longrightarrow$ b
Q10 $\longrightarrow$ a