Tutorial 4 Solutions MATH 1104A, Fall 2010.

1. $a : (F), \quad b : (T)$.

2. It is clear that
$$B = A^{-1}.\begin{bmatrix} 4 & 5 & 3 \\ 5 & 6 & 4 \end{bmatrix}$$
where
$$A^{-1} = \frac{1}{(-1)3-(-2)2}\begin{bmatrix} 3 & -2 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 2 & -1 \end{bmatrix}.$$
Hence,
$$B = \begin{bmatrix} 3 & -2 \\ 2 & -1 \end{bmatrix}.\begin{bmatrix} 4 & 5 & 3 \\ 5 & 6 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 2 \end{bmatrix}.$$

3. By means of elementary row operations, we have to reduce $[A|I_3]_{3\times 6}$ and $[B|I_4]_{4\times 8}$ to $[I_3|A^{-1}]_{3\times 6}$ and $[I_4|B^{-1}]_{4\times 8}$ respectively. So, we get

   (a)
$$(A^{-1})^T = \begin{bmatrix} \frac{1}{10} & -\frac{1}{5} & -\frac{9}{10} \\ \frac{2}{5} & \frac{1}{5} & \frac{7}{5} \\ -\frac{1}{10} & \frac{1}{5} & -\frac{1}{10} \end{bmatrix}$$

   (b)
$$(B^{-1})^T = \begin{bmatrix} 8 & 3 & -1 & 0 \\ -7 & -2 & 1 & 0 \\ 9 & 0 & -2 & 1 \\ -16 & -3 & 3 & -1 \end{bmatrix}.$$

4. (a)
$$A^{-1} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}.$$
   Also we have $A^{-1} = E_4E_3E_2E_1$ and $A = E_1^{-1}E_2^{-1}E_3^{-1}E_4^{-1}$ where
$$E_1 = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix},$$
   and
$$E_3 = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \quad E_4 = \begin{bmatrix} 1 & \frac{-3}{2} \\ 0 & 1 \end{bmatrix}.$$

(b)

$$B^{-1} = \begin{bmatrix} 5 & 2 & -2 \\ 2 & 2 & -1 \\ -2 & -1 & 1 \end{bmatrix}.$$

Also we have $B^{-1} = E_4 E_3 E_2 E_1$ and $B = E_1^{-1} E_2^{-1} E_3^{-1} E_4^{-1}$ where

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix},$$

and

$$E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}, \quad E_4 = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

5. In this case $n = 2$ and $m = 5$. Also we have

$$Y = \begin{bmatrix} 3 \\ 1 \\ 1 \\ 2 \\ 4 \end{bmatrix}, \quad U = \begin{bmatrix} 1 & -3 & 9 \\ 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 3 & 9 \end{bmatrix}.$$

Hence

$$U^T U = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -3 & -1 & 0 & 1 & 3 \\ 9 & 1 & 0 & 1 & 9 \end{bmatrix} \begin{bmatrix} 1 & -3 & 9 \\ 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 3 & 9 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 20 \\ 0 & 20 & 0 \\ 20 & 0 & 164 \end{bmatrix}$$

and

$$U^T Y = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -3 & -1 & 0 & 1 & 3 \\ 9 & 1 & 0 & 1 & 9 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 1 \\ 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 11 \\ 4 \\ 66 \end{bmatrix}.$$

The set of normal equations for $\vec{b}$ are

$$\begin{bmatrix} 5 & 0 & 20 \\ 0 & 20 & 0 \\ 20 & 0 & 164 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 11 \\ 4 \\ 66 \end{bmatrix}$$

whence

$$\begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 1.15 \\ 0.2 \\ 0.26 \end{bmatrix}.$$

This means that the least squares approximating quadratic for these data is $y = 1.15 + 0.20x + 0.26x^2$.