Tutorial 1 Solutions 1104D, Fall 2011

1. We have to solve the following set of equations to find the intersection points of those two lines:

$$\begin{cases} 3+t & = & 4+2s \\ 1-2t & = & 6+3s \\ 3+3t & = & 1+s \end{cases}$$

hence we have $s = t = -1$ and the only intersection point is $(2, 3, 0)$.

2. (a) We have $\|\vec{u}\| = \sqrt{5}$, $\|\vec{v}\| = \sqrt{18}$, $\|\vec{u}+\vec{v}\| = 5$ and $\|\vec{u}-\vec{v}\| = \sqrt{21}$.

   (b) We have $\vec{u}.\vec{v} = 1$, thus the angle between $\vec{u}$ and $\vec{v}$ is θ where

$$\cos\theta = \frac{1}{\sqrt{5}\sqrt{18}} = \frac{1}{\sqrt{90}}.$$

   (c) Based on our vectors, we get $(\vec{u}+\vec{v}).(\vec{u}-\vec{v}) = (3.1+0.2+4.(-4)) = -13$ but $\|\vec{u}\|^2 + \|\vec{v}\|^2 = 5 + 18 = 23$. So

$$(\vec{u}+\vec{v}).(\vec{u}-\vec{v}) \neq \|\vec{u}\|^2 + \|\vec{v}\|^2,$$

   and

$$(\vec{u}+\vec{v}).(\vec{u}-\vec{v}) = \|\vec{u}\|^2 - \|\vec{v}\|^2.$$

   Also $\|\vec{u}\|^2 - \|\vec{v}\|^2 = 5 - 18 = -13$ and $\|\vec{u}+\vec{v}\|^2 + \|\vec{u}-\vec{v}\|^2 = 25 + 21 = 46$, which means

$$\|\vec{u}+\vec{v}\|^2 + \|\vec{u}-\vec{v}\|^2 \neq \|\vec{u}\|^2 - \|\vec{v}\|^2.$$

3. We have

$$proj_{\vec{v}}\vec{u} = \frac{\vec{u}.\vec{v}}{\|\vec{v}\|^2}\vec{v} = \frac{2.3+1.6-1.3}{3^2+6^2+3^2}\vec{v} = \frac{1}{6}\begin{bmatrix} 3 \\ 6 \\ 3 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1 \\ 1/2 \end{bmatrix}.$$