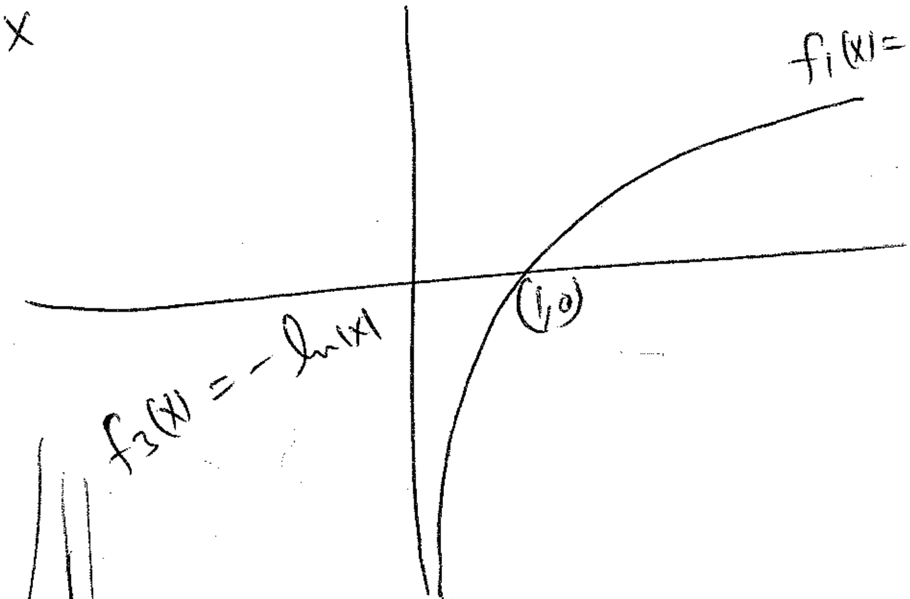
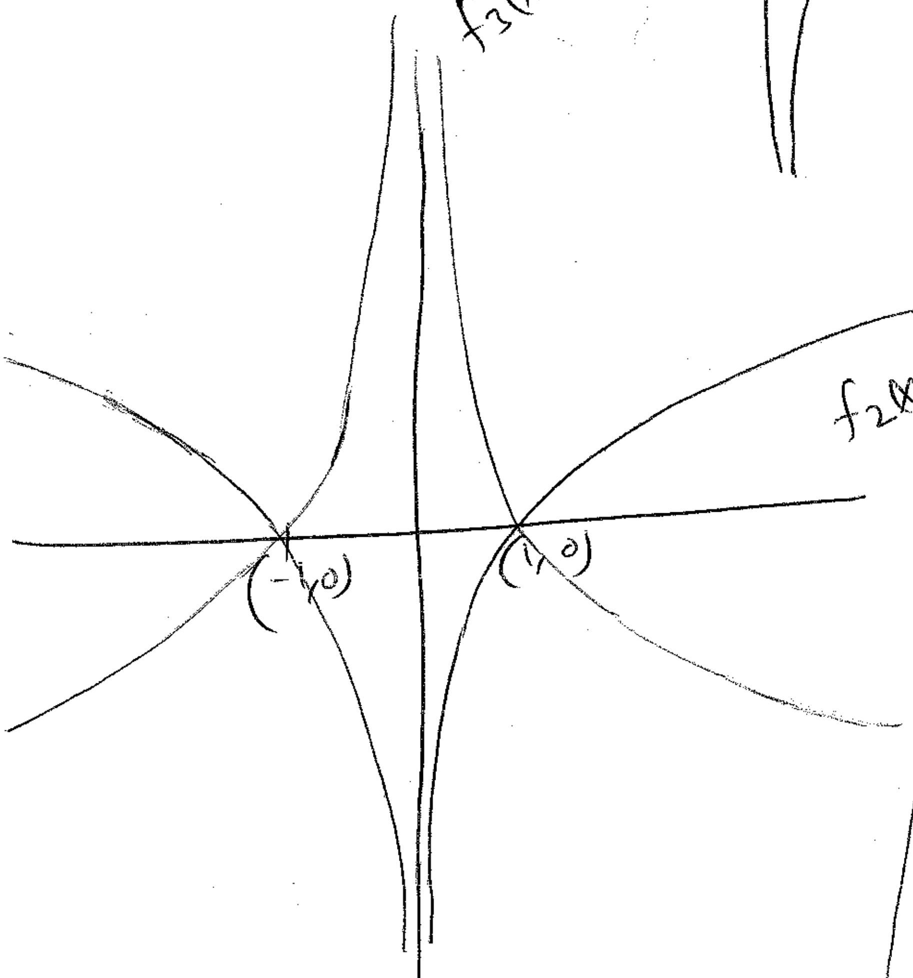
① ⓐ $y = -\ln|x+2|$. ①

$f_1(x) = \ln x$




$0 = \ln|x+2|$

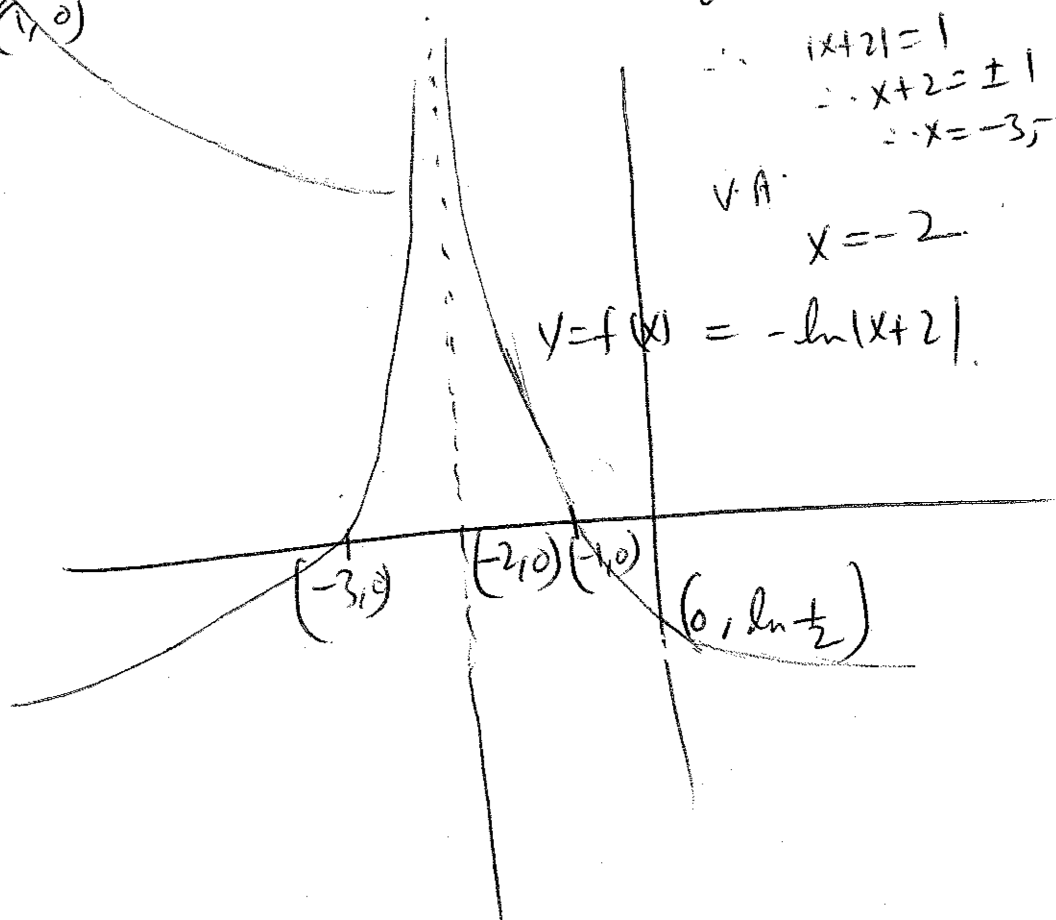
$\therefore |x+2| = 1$

$\therefore x+2 = \pm 1$

$\therefore x = -3, -1$

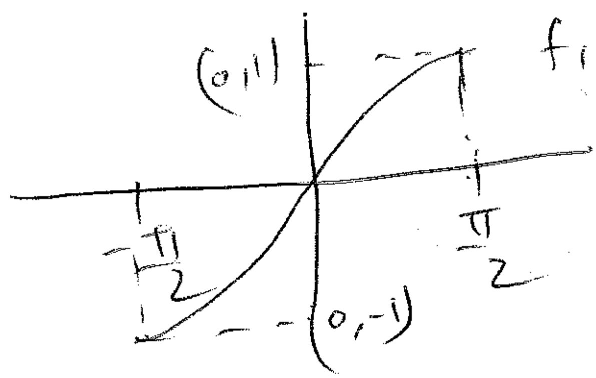
V.A.

$x = -2$



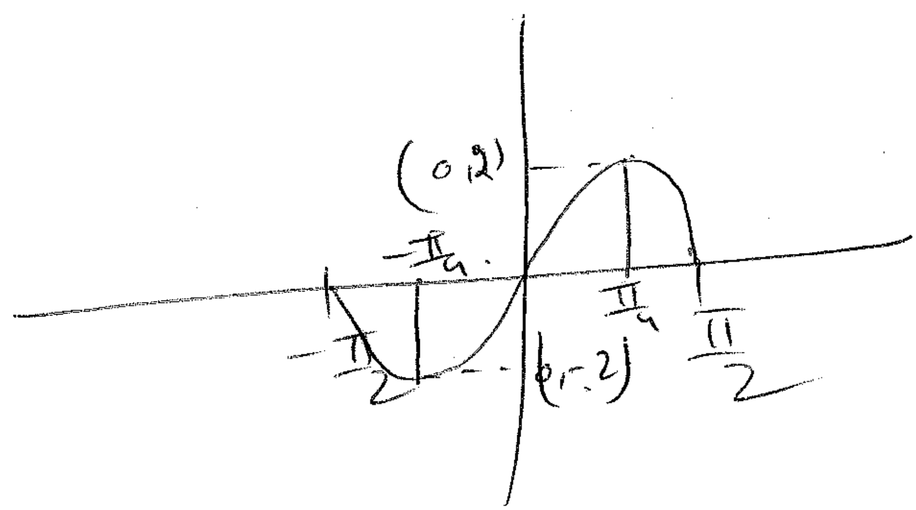
(b) $f(x) = 2\sin\left(2x+\frac{\pi}{2}\right) = 2\sin 2\left(x+\frac{\pi}{4}\right)$ (2)

$f_1(x) = \sin x$.

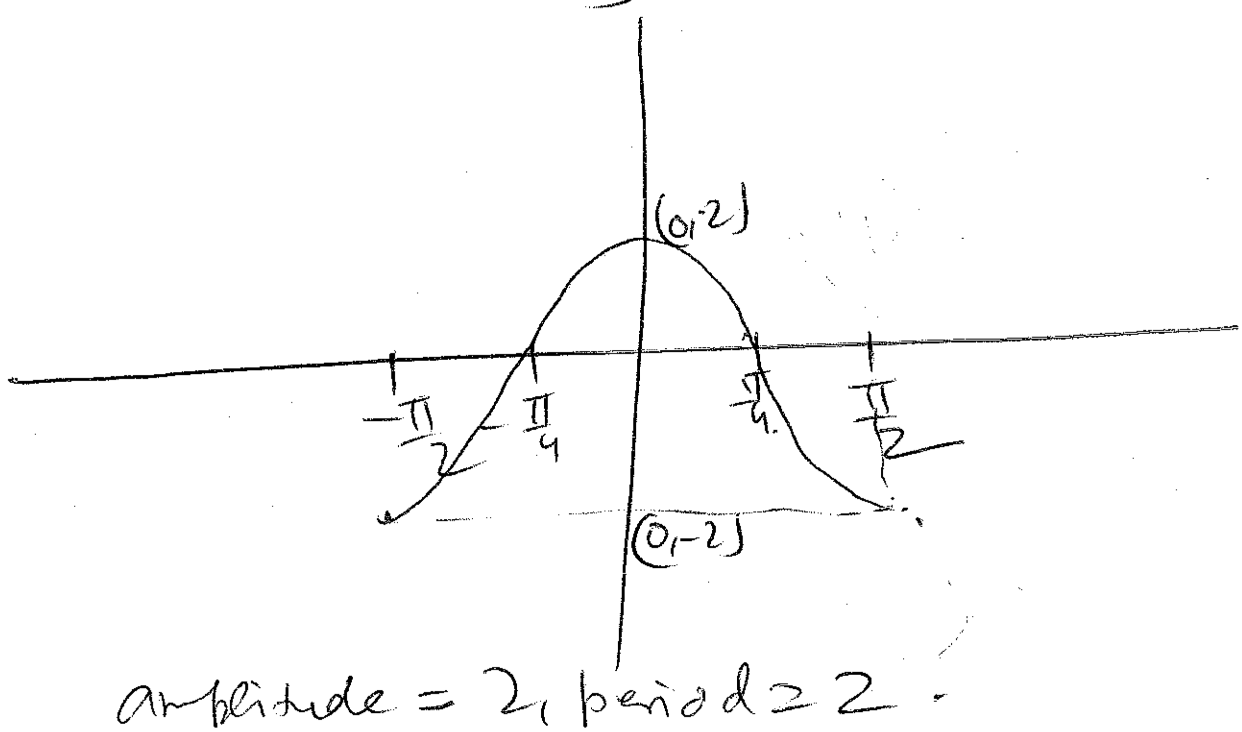



$f_1(x) = \sin x$.

$f_2(x) = 2\sin 2x$




$f(x) = 2\sin 2\left(x+\frac{\pi}{4}\right)$




amplitude $= 2$, period $= 2$.

3

2 (i) Range $(f) = \mathbb{R}$ as Range $\tan\left(\frac{\pi x}{2}\right) = \mathbb{R}$ for $-1 < x < 1$.

$\therefore$ Domain$(f^{-1})$ = Range $(f) = \mathbb{R}$.

0 Let $f^{-1}(3) = \alpha$

$$\Rightarrow f(\alpha) = 3$$

$$\Rightarrow 3 + \alpha^2 + \tan\left(\frac{\pi\alpha}{2}\right) = 3$$

$$\Rightarrow \alpha^2 + \tan\left(\frac{\pi\alpha}{2}\right) = 0$$

Since $\tan\left(\frac{\pi x}{2}\right)$ is one-to-one on $-1 < x < 1$.

we get that $\alpha = 0$ as $\tan 0 = 0$.

$$\therefore f^{-1}(3) = 0$$

0

Since $5 \in$ Domain$(f^{-1})$, we have

$$\therefore f(f^{-1}(5)) = 5.$$

[6] 3. Find a formula for the inverse of the function $f(x) = \frac{1+e^{-x}}{1-e^{-x}}$. What is the Range of $f^{-1}(x)$.

$$\text{Domain of } f = \{x : 1-e^{-x} \neq 0\}$$

$$\text{But} \quad 1-e^{-x} = 0 \iff x = 0$$

$$\therefore \text{Domain}(f) = \mathbb{R} - \{0\} = \text{Range}(f^{-1})$$

$$\text{Let} \quad y = \frac{1+e^{-x}}{1-e^{-x}} \quad \therefore y - ye^{-x} = 1 + e^{-x}$$

$$\therefore y - 1 = e^{-x}(1+y)$$

$$\therefore e^{-x} = \frac{y-1}{y+1} \quad \therefore -x = \ln\left(\frac{y-1}{y+1}\right)$$

$$\therefore x = \ln\left(\frac{y+1}{y-1}\right)$$

$$\therefore f^{-1}(x) = \ln\left(\frac{1+x}{x-1}\right)$$

[4] 4. Explain why the function $f(x)$ is discontinuous at $x = -3$. Also Sketch the graph of the function

$$f(x) = \begin{cases} \frac{x^2-x-12}{x+3} & \text{if } x \neq -3 \\ -5 & \text{if } x = -3 \end{cases}$$

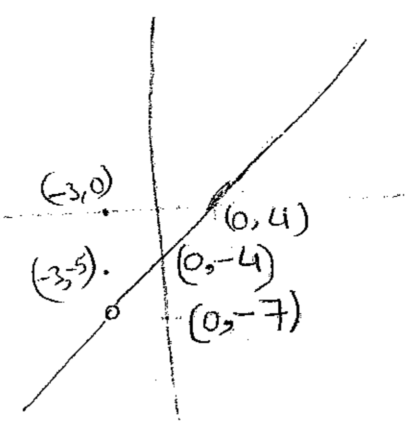
Let $x \neq -3$, then

$$f(x) \quad \frac{x^2-x-12}{x+3} = \frac{(x-4)(x+3)}{x+3} = x-4$$

Hence $\lim_{x \to -3} f(x) = -3-4 = -7$ (as '$x-4$' is continuous at $x=-3$)

But $f(-3) = -5 \neq -7$

Hence '$f$' is discontinuous at $x = -3$



(5)

[4] 5. Evaluate the $\lim_{t\to 0}\left(\frac{1}{t} - \frac{1}{t^2+t}\right)$

For $t \neq 0$

$$\frac{1}{t} - \frac{1}{t^2+t} = \frac{t^2+t-t}{t(t+1)} = \frac{1}{t+1}$$

Since $f(t) = \frac{1}{t+1}$ is continuous on $\mathbb{R} - \{-1\}$.

we get that $\lim_{t\to 0}\left(\frac{1}{t} - \frac{1}{t^2+t}\right) = f(0)$

$= 1$

[5] 6. Find all values of $x$ in the interval $[0, 2\pi]$ that satisfy the equation $2 + \cos 2x = 3\cos x$.

$$2 + \cos 2x = 3\cos x$$

$$\Rightarrow 2 + 2\cos^2 x - 1 = 3\cos x$$

$$\Rightarrow 2\cos^2 x - 3\cos x + 1 = 0$$

$$\Rightarrow \cos x = \frac{3 \pm \sqrt{9-8}}{4} = 1 \text{ or } \frac{1}{2}$$

$\therefore$ $x = 0,\ 2\pi,\ \frac{\pi}{3},\ \frac{5\pi}{3}$

[5] 7. Solve the inequality $x^3 > x$ for all real numbers $x$.

$$x^3 > x \Rightarrow x^3 - x > 0 \Rightarrow x(x^2-1) > 0$$

$$\Rightarrow x(x-1)(x+1) > 0$$

$(-)(-)(-)=(-)$ $(-)(-)(+)=(+)$ $(+)(-)(+)=(-)$ $(+)(+)(+)=(+)$

$-1$ $\quad 0$ $\quad 1$

$\therefore$ Answer: $(-1, 0) \cup (1, \infty)$