Solutions #7

4.9

11. let $f(x) = x^3 - 30$ $\therefore f'(x) = 3x^2$

Then $\sqrt[3]{30}$ is a root of $f(x)$.

let $x_0 = 3$,

$$\therefore \quad x_1 = 3 - \frac{f(3)}{f'(3)} = 3.1111111$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 3.107237339$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 3.107232506$$

$$\boxed{x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 3.107232506}$$

33. Newton's Method gives

6pt

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

2

if they just draw appropriate graphs, but can't do the calculation give 2 pts

$$= x_n - \frac{x_n^{1/3}}{\frac{1}{3}x_n^{-2/3}}$$

$$= x_n - 3x_n$$

**

$$= -2x_n$$

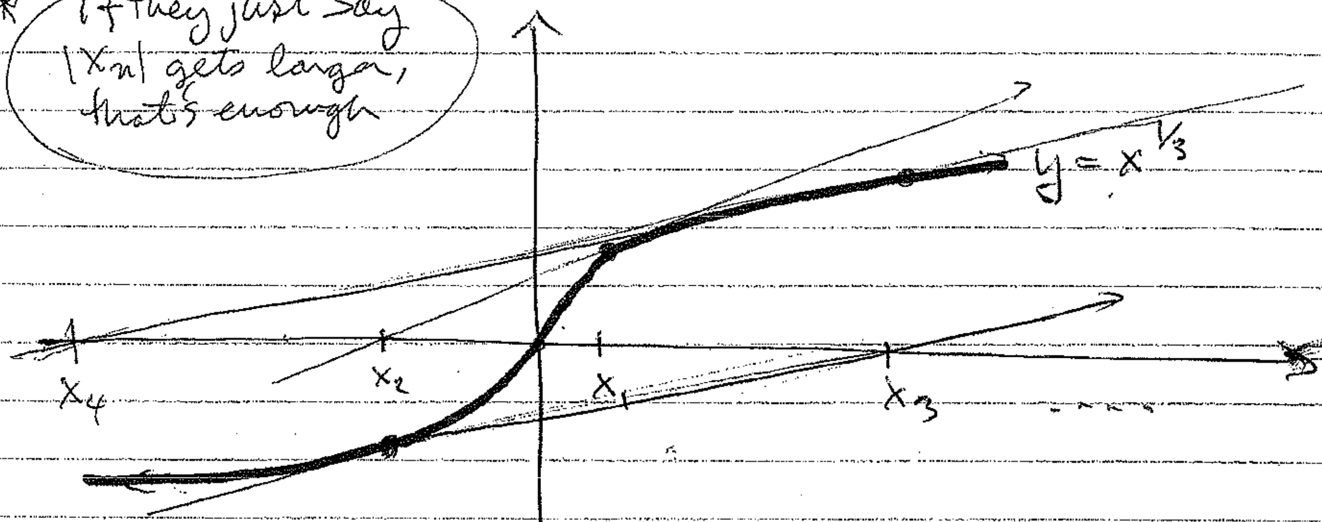
2

$\therefore \quad |x_{n+1}| = 2|x_n|$

$\therefore \quad \lim_{n\to\infty} |x_n| = \infty$ provided $x_1 \neq 0$.

(2)

** If they just say $|x_n|$ gets larger, that's enough



$|x_n|$ gets larger as $n \to \infty$.

36. Assume $x^2 + \sin x$ does have an absolute minimum at $x = c$.

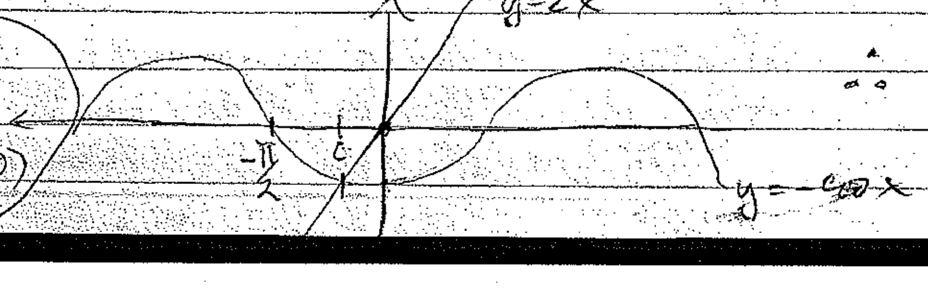
Then we must have

$$f'(c) = 0$$

$\therefore$ (2) $\quad 2c + \cos c = 0 \quad (2c = -\cos c)$

$\therefore$ c is where graphs $y = 2x$ and $y = -\cos x$ intersect

** give 2 pts as long as ~~c~~ $c \in (-\pi/2, 0)$



$\therefore$ $c \sim -\pi/4$

(2)

now use Newtons Method to approx the root of $f'(x)$ using $x_0 = -\pi/4$.

This gives

$$x_0 = -\pi/4 = -.7853982$$

4

$$x_1 = x_0 - \frac{f'(x_0)}{f''(x_0)} = -.4663529$$

$$x_2 = x_1 - \frac{f'(x_1)}{f''(x_1)} = -.4502314$$

$$x_3 = x_2 - \frac{f'(x_2)}{f''(x_2)} = -.4501836$$

$$\boxed{x_4 = x_3 - \frac{f'(x_3)}{f''(x_3)} = -.4501836}$$

(plug x_4 into your calculator and see how close $f'(x_4)$ is to zero!)

4.1 50. **5pt** critical points in $(-1,4)$

$$f'(x) = 3x^2 - 12x + 9$$

$$= 3(x+1)(x+3) = 0$$

2 $\Rightarrow x = -1, -3$ ∴ no critical points in $(-1,4)$

endpoints 1

2 $f(-1) = -1 - 6 - 9 + 2 = -14 \leftarrow$ absolute Min

$f(4) = 64 - 96 + 36 + 2 = 6 \leftarrow$ absolute Max

$\therefore$ absolute Max $= 6$
" Min $= -14$

52. 1. critical points in $(-1, 2)$
5pt

$$f'(x) = 3(x^2-1)^2(2x)$$
$$= 0$$
$$\Rightarrow x = \pm 1, 0$$ (2)

2. Endpoints

$$f(-1) = 0$$
$$f(0) = -1$$ (2)
$$f(1) = 0$$
$$f(2) = 27$$

$\therefore$ absolute Max is 27
" Min is 0 (1)

70. 1. critical points in $(0, \pi/2)$
8pt.

$$\frac{dF}{d\theta} = \frac{-\mu W}{(\mu\sin\theta + \cos\theta)^2}(\mu\cos\theta - \sin\theta) = 0$$

$$\Rightarrow (\mu\cos\theta - \sin\theta) = 0$$ (2)

$$\Rightarrow \mu = \frac{\sin\theta}{\cos\theta} = \tan\theta, \qquad \theta \in (0, \pi/2)$$

2. Endpoints.

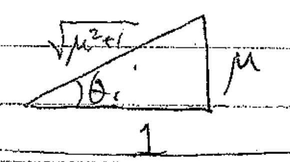
let critical point be θ_1. Then

$$F(\theta_1) = \frac{\mu W}{\mu \sin\theta_1 + \cos\theta_1}$$

$$= \frac{\left(\frac{\sin\theta_1}{\cos\theta_1}\right) W}{\left(\frac{\sin\theta_1}{\cos\theta_1}\right)\sin\theta_1 + \cos\theta_1}$$

$$= \frac{\sin\theta_1 W}{\sin^2\theta_1 + \cos^2\theta_1} = \sin\theta_1 W$$ 2

but $\tan\theta_1 = \mu \Rightarrow \sin\theta_1 = \frac{\mu}{\sqrt{\mu^2+1}} < \mu$

since 

2

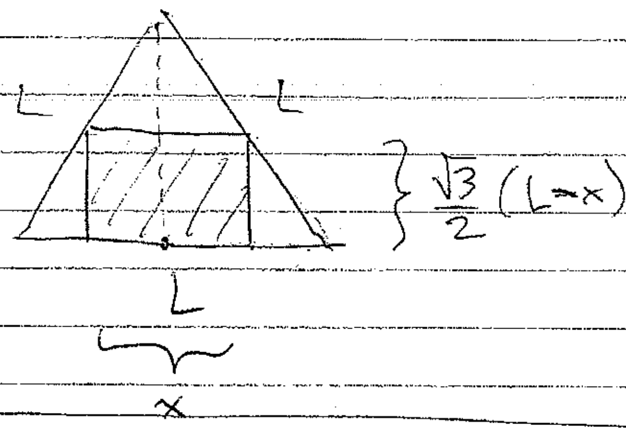
$\therefore \quad F(\theta_1) < \mu W = F(0)$ 2

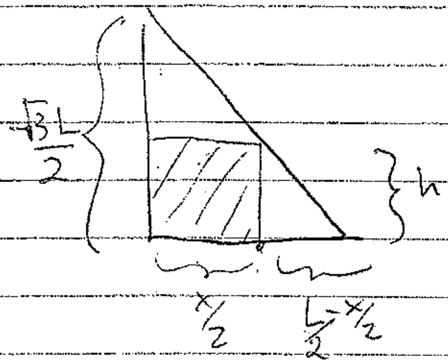
also $F(\theta_1) < W = F(\pi/2)$

$$\boxed{\therefore F(\theta_1) = \sin\theta_1 W \text{ is absolute Min of } F. \text{ and } \tan\theta_1 = \mu}$$

(P337)

21.





$$\therefore \quad \frac{\frac{\sqrt{3}\cancel{L}}{\cancel{2}}}{\frac{\cancel{L}}{\cancel{2}}} = \frac{h}{\frac{L}{2}-\frac{x}{2}}$$

$$\therefore \ h = \frac{\sqrt{3}}{2}(L-x)$$

$\therefore$ $A(x)$ = area of rectangle

$$= x\left(\frac{\sqrt{3}}{2}(L-x)\right)$$

$$= \frac{\sqrt{3}}{2}(Lx - x^2) \quad ; \quad x \in [0, L]$$

1. critical point

$$A'(x) = \frac{\sqrt{3}}{2}(L-2x) = 0$$

$$\Longrightarrow x = L/2$$

2. endpoints

$$A(0) = 0$$

$$A(L/2) = L/2 \, \sqrt{3}/2 \, (L/2) = \frac{\sqrt{3}L^2}{8}$$

$$A(L) = 0$$

$$\boxed{\therefore \text{ Max area } = A(L/2) = \frac{\sqrt{3}}{8}L^2}$$

4.2

6. $f'(x) = -2(x-1)^{-3} = \frac{-2}{(x-1)^3}$

5pt.

$$\begin{cases} \therefore f'(x) > 0 & \text{when } x < 1 \\ f'(x) < 0 & \text{when } x > 1 \\ f'(x) \text{ DNE} & \text{at } x = 1 \end{cases}$$ 2

$\therefore f'(x)$ is never zero.

$f(x)$ is not differentiable at $x=1$, $\therefore$ 3
can't apply MVT on $(0,2)$

7. want $f'(c) = \frac{f(8)-f(0)}{8-0} = \frac{6-4}{8} = \frac{1}{4}$

$c \sim .9,\ 3.1,\ 4.2,\ 6.1$

("roughly")

17. **6pt.** 1. $f(x)$ has at least one root since;

*** Dont deduct marks if they dont say (IVT), just check they get a root in some interval.

$f(x)$ is cont'nous

(2)

$f(-1) = -6 < 0 < f(1) = 8$.

$\boxed{\therefore \text{by (IVT) } f \text{ has root in } (-1, 8)}$

2. $f(x)$ has at most one root since;

** dont deduct for not saying (MVT)

if $f(x)$ had 2 roots.

(2) $\Rightarrow f'(c) = 0$ for some c by (MVT)

(or Rolles Theorem)

(2) $\Rightarrow f'(c) = 2 + 3x^2 + 20x^4 = 0$ for some c.

But $f'(c) > 2$ for all c. $\therefore$ we get contradiction $\therefore f$ has at most one root.

(assume $a<b$) ** don't deduct for not saying MVT

29. 5pt
for any a, b, by (MVT) we know

(2) there is $c \in (a,b)$ such that

$$f'(c) = \frac{f(b)-f(a)}{b-a} \quad (\text{where } f = \sin x)$$

$$\Rightarrow \cos(c) = \frac{\sin b - \sin a}{b-a}$$

(1) $$\Rightarrow |\cos c| = \frac{|\sin b - \sin a|}{|b-a|}$$

(2) $$\boxed{\Rightarrow \quad 1 \geq \frac{|\sin b - \sin a|}{|b-a|}}$$

$$\left(\text{i.e.} \quad 1 \geq \frac{|\sin a - \sin b|}{|a-b|} \right)$$