Solutions #5

1

3.2

4. $g'(x) = \frac{1}{2\sqrt{x}} e^x + \sqrt{x}\, e^x = e^x\left(\frac{1}{2\sqrt{x}} + \sqrt{x}\right)$ prod. rule

14. $\boxed{4pt}$ $y' = \dfrac{(t^4-2)(t^3+t)' - (t^3+t)(t^4-2)'}{(t^4-2)^2}$ Quot. rule   2

$= \dfrac{(t^4-2)(3t^2+1) - (t^3+t)(4t^3)}{(t^4-2)^2}$ 2

18. $\boxed{4pt}$ $z' = \frac{3}{2} w^{1/2}(w+ce^{w}) + w^{3/2}(1+ce^{w})$ prod. rule   2

$= w^{1/2}\left[\frac{3}{2}(w+ce^{w}) + w(w+ce^{w})\right]$ 2

40. b) $R'(p) = 1\cdot f(p) + p\cdot f'(p)$

$\therefore R'(20) = f(20) + 20\cdot f'(20)$

$= 10\,000 + 20\cdot(-350)$

$= 10\,000 - 700$

$= 9300$

3.3

14. $A(r) = \pi r^2$; $\frac{dr}{dt} = 60$ cm/s

5pt

$\therefore \frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt} = (2\pi r) \cdot 60$ cm^2/s

$= 120\pi r$ 2

a) when $t = 1$; $r = 60$

$\therefore \frac{dA}{dt} = 120 \cdot \pi \cdot 60$ 1

b) " $t = 3$; $r = 180$

$\therefore \frac{dA}{dt} = 120 \cdot \pi \cdot 180$ 1

c) " $t = 5$; $r = 300$

$\therefore \frac{dA}{dt} = 120 \cdot \pi \cdot 300$ 1

$\frac{dA}{dt}$ is a linear function of time

28. $\boxed{12pt}$ $f = \frac{1}{2L}\sqrt{\frac{T}{p}}$ (3)

a) i) $\frac{df}{dL} = \left(\frac{1}{2}\sqrt{\frac{T}{p}}\right)\cdot\left(-\frac{1}{L^2}\right) = -\frac{\sqrt{T}}{2\sqrt{p}\,L^2}$

(2)

(2) ii) $\frac{df}{dT} = \left(\frac{1}{2L\sqrt{p}}\right)\cdot\left(\frac{1}{2\sqrt{T}}\right) = \frac{1}{4L\sqrt{Tp}}$

(2) iii) $\frac{df}{dp} = \left(\frac{\sqrt{T}}{2L}\right)\left(-\frac{1}{2}p^{-3/2}\right) = -\frac{\sqrt{T}}{4Lp^{3/2}}$

b) i) note; $\frac{df}{dL} < 0$. (above)

(2) $\therefore$ as L decreases, f increases and pitch goes up

ii) note; $\frac{df}{dT} > 0$.

(2) $\therefore$ as T increases, f increases and pitch goes up

iii) note; $\frac{df}{dp} < 0$.

(2) $\therefore$ as p increases, f decreases and pitch goes down.

3.4.

8. 3pt

$$y' = e^u(\cos u + cu) + e^u(-\sin u + c)$$

$$\boxed{= e^u(\cos u - \sin u + cu + c)}$$

10.

$$y' = \frac{(x+\cos x)(\cos x) - (1+\sin x)(1-\sin x)}{(x+\cos x)^2}$$

$$= \frac{x\cos x + \cos^2 x - 1 + \sin^2 x}{(x+\cos x)^2}$$

$$\boxed{= \frac{x\cos x}{(x+\cos x)^2}}$$

22. 6pt

$$y' = e^x \cos x + e^x(-\sin x)$$ 2

$\therefore$ at $(0,1)$

$$y' = e^0 \cos 0 + e^0(-\sin 0) = 1$$ 2

$\therefore$ line is

$$\frac{y-1}{x-0} = 1$$

$$\boxed{y = x + 1}$$ 2

30. $y' = \dfrac{(2+\sin x)(-\sin x) - (\cos x)(\cos x)}{(2+\sin x)^2}$ (5)

$= \dfrac{-2\sin x - \sin^2 x - \cos^2 x}{(2+\sin x)^2}$

$= \dfrac{-2\sin x - 1}{(2+\sin x)^2}$

$\therefore\ y' = 0$ when $2\sin x + 1 = 0$

$\Longrightarrow \sin x = -\frac{1}{2}$

$\Longrightarrow \boxed{x = -\frac{\pi}{6} + 2\pi n \ ; \ -\frac{5\pi}{6} + 2\pi n}$; any $n \in \mathbb{Z}$; any $n \in \mathbb{Z}$

3.5

24. 3pt $y' = \left[(\log 10)\, 10^{(1-x^2)}\right] \cdot (-2x)$

28. $y' = \dfrac{(e^u + e^{-u})(2e^{2u}) - e^{2u}(e^u - e^{-u})}{(e^u + e^{-u})^2}$

$= \dfrac{2e^{3u} + 2e^{u} - e^{3u} + e^{u}}{(e^u + e^{-u})^2} = \boxed{\dfrac{e^{3u} + 3e^{u}}{(e^u + e^{-u})^2}}$

6

34. $y = x \sin\left(\frac{1}{x}\right)$

3pt

$y' = \sin\left(\frac{1}{x}\right) + x\cos\left(\frac{1}{x}\right)\cdot\left(-\frac{1}{x^2}\right)$

42. $y = 2^{3^{x^2}}$ (recall $(a^x)' = (\log a)a$

4pt

$y' = (\log 2)\cdot 2^{3^{x^2}}\cdot\left(3^{x^2}\right)'$ 2

$= (\log 2)\cdot 2^{3^{x^2}}\cdot(\log 3)3^{x^2}(x^2)'$ 2

$\boxed{= (\log 2)(\log 3)\cdot 2x\cdot 2^{3^{x^2}}\cdot 3^{x^2}}$

44. $y' = \cos x + 2\sin x\cos x$

at $(0,0)$

$y' = 1 + 0$

$\therefore$ equation is

$\frac{y-0}{x-0} = 1$; $\boxed{y = x}$

70. a) $\lim_{x\to\infty} \frac{1}{1+ae^{-kt}} = \frac{1}{1} = 1$

b) $\frac{dp}{dt} = -\frac{1}{(1+ae^{-kt})}\left(1+ae^{-kt}\right)' = \boxed{\frac{ake^{-kt}}{1+ae^{-kt}}}$

$$\boxed{= \frac{ak}{e^{kt}+a}}$$

7

3.6

12. $1 = \cos(xy^2)\cdot(y^2 + x2yy')$

4pt

$\therefore\ 2xy\cos(xy^2)\,y' = 1 - y^2\cos(xy^2)$ 2

$\therefore\ \boxed{y' = \dfrac{1-y^2\cos(xy^2)}{2xy\cos(xy^2)}}$ 2

26. $x^2 + 2xy - y^2 + x = 2$

$\therefore\ 2x + 2y + 2xy' - 2yy' + 1 = 0$

$\therefore$ at $(1,2)$

$2 + 4 + 2y' - 4y' + 1 = 0$

$7 = 2y'$

$y' = 7/2$

$\therefore$ line is

$\dfrac{y-2}{x-1} = 7/2$; $\boxed{\begin{aligned} y &= \tfrac{7}{2}x - \tfrac{7}{2} + 2 \\ &= \tfrac{7}{2}x - \tfrac{3}{2} \end{aligned}}$