Solutions #4

2.5

36. $f(x)$ is cont. on $(-\infty, \pi/4) \cup (\pi/4, \infty)$.

also
$$\begin{cases} \lim\limits_{x \to \pi/4^+} f(x) = \lim\limits_{x \to \pi/4^+} \cos x = \frac{1}{\sqrt{2}} \\ \lim\limits_{x \to \pi/4^-} f(x) = \lim\limits_{x \to \pi/4^-} \sin x = 1/\sqrt{2} \end{cases}$$

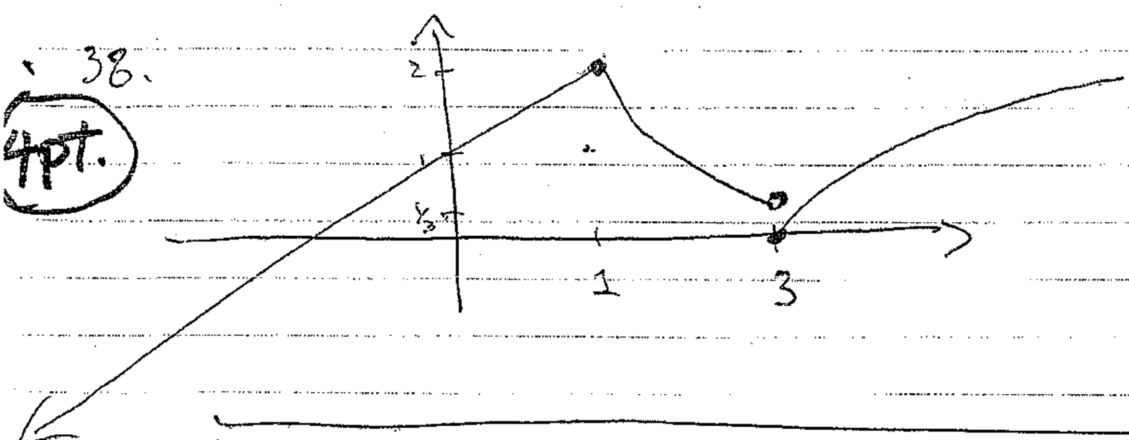
$$\therefore \lim_{x \to \pi/4} f(x) = 1/\sqrt{2} = \cos \pi/4 = f(\pi/4)$$

$\therefore f$ is cont at $x = \pi/4$

$\boxed{\therefore f(x) \text{ is cont. on } \mathbb{R}}$

38.

4pt.



f is discont. at $x=3$ — 2

f is continuous from right at $x=3$ — 2

42. $g(x)$ is cont. on $(-\infty, 4) \cup (4, \infty)$ for all c

~~dt~~ also

$$\begin{cases} \lim_{x \to 4^+} g(x) = \lim_{x \to 4^+} cx + 20 = 4c + 20 \\ \lim_{x \to 4^-} g(x) = \lim_{x \to 4^-} x^2 - c^2 = 16 - c^2 \end{cases}$$

$\therefore$ $\lim_{x \to 4} g(x)$ exists when $16 - c^2 = 4c + 20$

$$\Rightarrow c^2 + 4c + 4 = 0$$
$$\Rightarrow c = -2.$$

$\therefore$ if $c = -2$, $\lim_{x \to 4} g(x) = 4 \cdot (-2) + 20 = 16 = g(4)$

and $g(x)$ is then cont. at $x = 4$ $\therefore$ on $(-\infty, \infty)$

48. let $f(x) = \sqrt[3]{x} + x - 1$ 1

5pt

1) Then $f(x)$ is cont. on $(-\infty, \infty)$ 2

2) $f(0) = -1 < 0 < f(1) = 2$ 2

$\boxed{\therefore \text{By IVT}, \ f(c) = 0 \text{ for some } c \in (0, 1)}$ 1

50. let $f(x) = \log x - e^{-x}$ 1

6pt

1. Then $f(x)$ is cont. on $(0,\infty)$ 2

2. $f(1) = -e^{-1} \leq 0 \leq f(2) = \log 2 - \frac{1}{e^2}$ 2

1

$\boxed{\therefore\ f(c) = 0 \text{ for some } c \in (1,2) \text{ by IVT}}$

2.6

19. $\lim_{u\to\infty} \frac{4u^4+5}{(u^2-2)(2u^2-1)} = \lim_{u\to\infty} \frac{4+5/u^4}{(1-2/u^2)(2-1/u^2)}$

$$\begin{cases} \lim_{u\to\infty} 4+5/u^4 = 4 \\ \lim_{u\to\infty} (1-2/u^2)(2-1/u^2) = 2 \end{cases}$$

$\boxed{\therefore\ = 4/2 = 2 \text{ by (LQR)}}$

20. $\lim_{x\to\infty} \frac{x+2}{\sqrt{9x^2+1}} = \lim_{x\to\infty} \frac{1+2/x}{\sqrt{9+1/x^2}}$ 2

4pt

$$\begin{cases} \lim_{x\to\infty} 1+2/x = 1 \\ \lim_{x\to\infty} \sqrt{9+1/x^2} = 3 \end{cases}$$

2

$\therefore\ \boxed{= 1/3 \text{ by (LQR)}}$

2.7

(4)

18. a) $H' = 58 - 2(.83)t$

(6pt)

$\boxed{H'(1) = 58 - 2(.83)}$ (1)

b) $\boxed{H'(a) = 58 - 2(.83)a}$ (1)

c) arrow stops ascending when

$H'(t) = 0$

(2)

$\Rightarrow 58 - 2(.83)t = 0$

$\Rightarrow \boxed{t = \frac{58}{2(.83)}}$

arrow then takes same amt of time to fall back down to moon. (2)

$\therefore$ total time to go up + come down

$\boxed{\text{is} \quad t = 2\left(\frac{58}{2(.83)}\right) = \frac{58}{.83}}$

So The arrow strikes the moon when the height is 0, that is,

$58t - 0.83t^2 = 0 \Leftrightarrow t = 0$ or $t = \frac{58}{0.83} \approx 69.9s$

Since $t \neq 0$, $t = 69.9s$.

(d) Using the time from part (c), $v\left(\frac{58}{0.83}\right) = -58 \, m/s$.

Thus, the arrow will have a velocity of $-58 \, m/s$.

2.8

16. $f(x) = \dfrac{x^2+1}{x-2}$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\dfrac{(x+h)^2+1}{x+h-2} - \dfrac{x^2+1}{x-2}}{h}$$

$$= \lim_{h\to 0} \frac{(x-2)((x+h)^2+1) - (x+h-2)(x^2+1)}{(x+h-2)(x-2)\,h}$$

simplify!

$$= \lim_{h\to 0} \frac{(2x\cancel{h}+h^2)(x-2) - \cancel{h}(x^2+1)}{(x+h-2)(x-2)\cancel{h}}$$

$$= \lim_{h\to 0} \frac{(2x+h)(x-2)-(x^2+1)}{(x+h-2)(x-2)}$$

$$\boxed{= \frac{2x(x-2)-(x^2+1)}{(x-2)^2} = f'(x)}$$

by (L.Q.R)

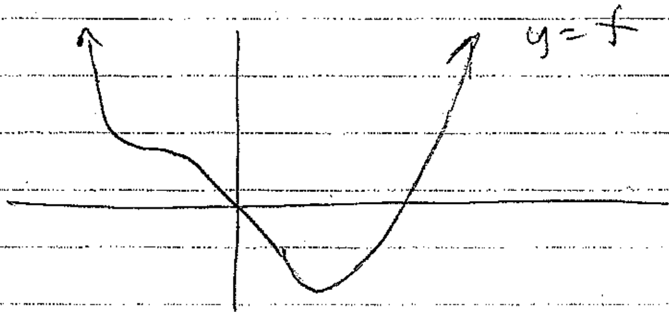
22. let $\begin{cases} f(x) = \tan x \\ a = \pi/4 \end{cases}$

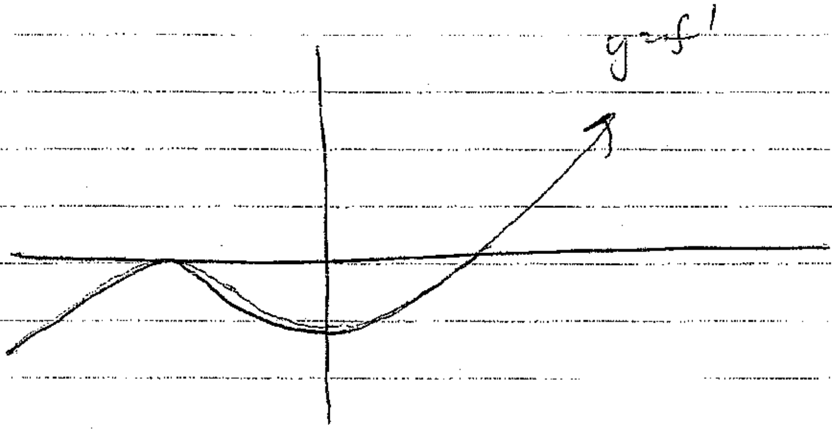
Then $\displaystyle \lim_{x\to \pi/4} \frac{\tan x - 1}{x-\pi/4} = \lim_{x\to\pi/4} \frac{f(x)-f(\pi/4)}{x-\pi/4} = f'(\pi/4)$

2.9

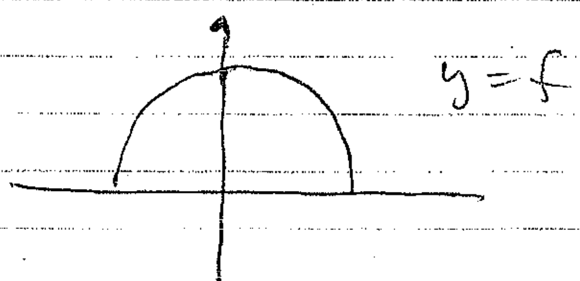
4pt 4. (a) II , (b) IV , (c) I , (d) III

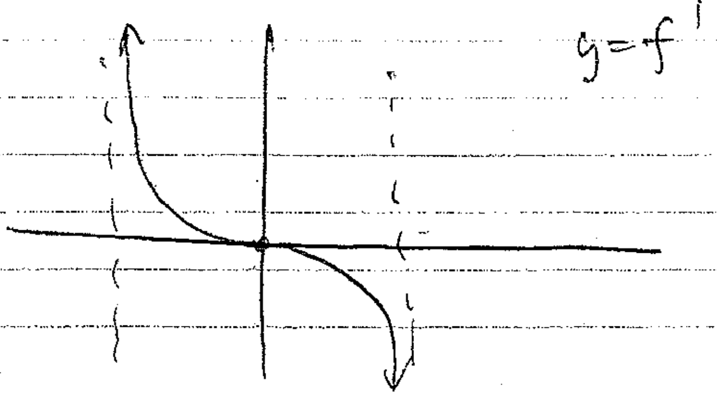
6.





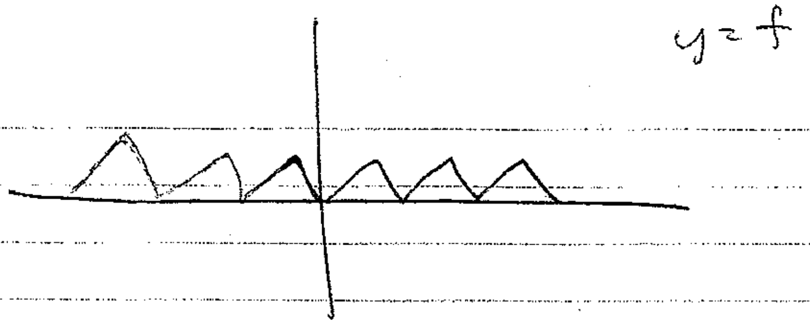
10.
4pt

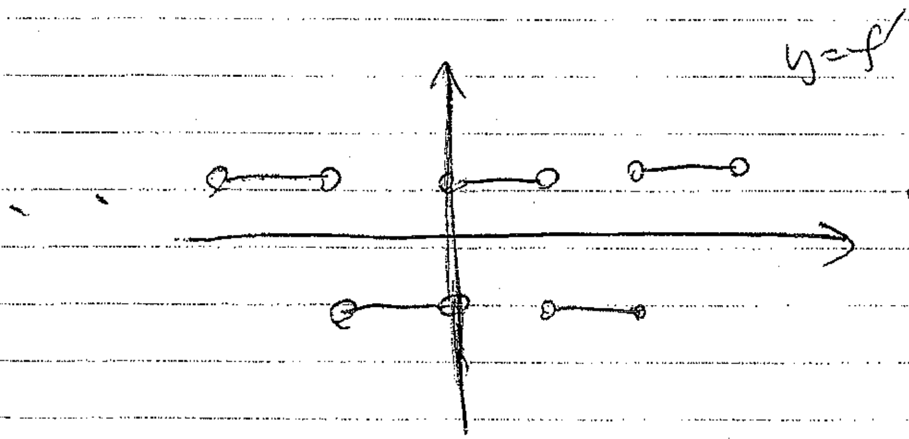




7

12.
4pt





26.
5pt.

$$\lim_{h\to 0} \frac{\left((x+h)+\sqrt{x+h}\right) - \left(x+\sqrt{x}\right)}{h}$$ 1

$$= \lim_{h\to 0} \frac{h + \sqrt{x+h} - \sqrt{x}}{h}$$

$$= \lim_{h\to 0} \frac{h}{h} + \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{(\sqrt{x+h} + \sqrt{x})}{(\sqrt{x+h} + \sqrt{x})}$$ 2

$$= \lim_{h\to 0} 1 + \frac{(\cancel{x}+h) - \cancel{x}}{h(\sqrt{x+h} + \sqrt{x})}$$

$$\therefore f'(x) \boxed{= \lim_{h\to 0} 1 + \frac{1}{\sqrt{x+h} + \sqrt{x}} = 1 + \frac{1}{2\sqrt{x}}}$$ 2

30. $\lim_{h\to 0} \dfrac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} = \lim_{h\to 0} \dfrac{x^2 - (x+h)^2}{x^2(x+h)^2 h}$

$= \lim_{h\to 0} \dfrac{-2x\cancel{h} - h^{\cancel{2}}}{x^2(x+h)^2 \cancel{h}}$

$= \lim_{h\to 0} \dfrac{-2x - h}{x^2(x+h)^2} = \dfrac{-2x}{x^4} = \dfrac{-2}{x^3}$

$\boxed{\therefore f'(x) = \dfrac{-2}{x^3}}$

38. a) $x = -2, 0, 5$

(6pt)

(2) $x=-2$; since $\lim_{x\to 2} g \neq g(-2)$

(2) $x=0$; since $g(x)$ not defined at 0

(2) $x=5$; since $\lim_{x\to 5^+} g(x) \neq \lim_{x\to 5^-} g(x)$