Solutions #3

1.6

24. $x = \dfrac{4y-1}{2x+3}$

$x(2y+3) = 4y-1$

$2xy - 4y = -1 - 3x$

$y(2x-4) = -1-3x$

$y = \dfrac{1+3x}{4-2x}$

$\boxed{\therefore f^{-1}(x) = \dfrac{1+3x}{4-2x}}$

28. $x = \dfrac{1+e^y}{1-e^y}$

$x(1-e^y) = 1+e^y$

$-xe^y - e^y = 1-x$

$e^y = \dfrac{x-1}{x+1}$

$y = \log\left(\dfrac{x-1}{x+1}\right)$

$\boxed{\therefore f^{-1}(x) = \log\left(\dfrac{x-1}{x+1}\right)}$

37. b) $\log_5 10 + \log_5 20 - 3\log_5 2$

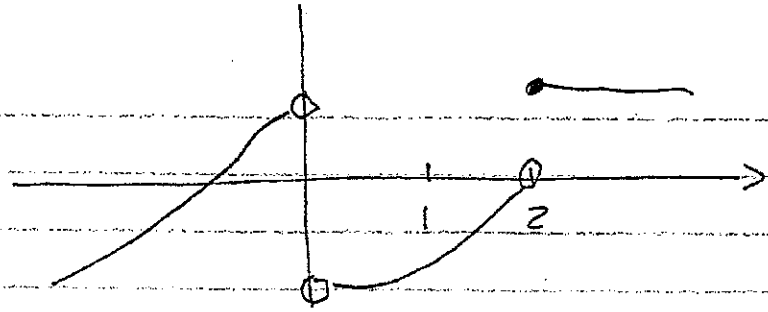
$= \log_5 10 + \log_5 20 - \log_5 2^3$

$= \log_5\left(\dfrac{10 \times 20}{2^3}\right) = \log_5(25) = 2$

2.2

8 a) $-\infty$ b) ∞ c) $-\infty$ d) ∞ e) $x=-3$, $x=2$, $x=5$

14.



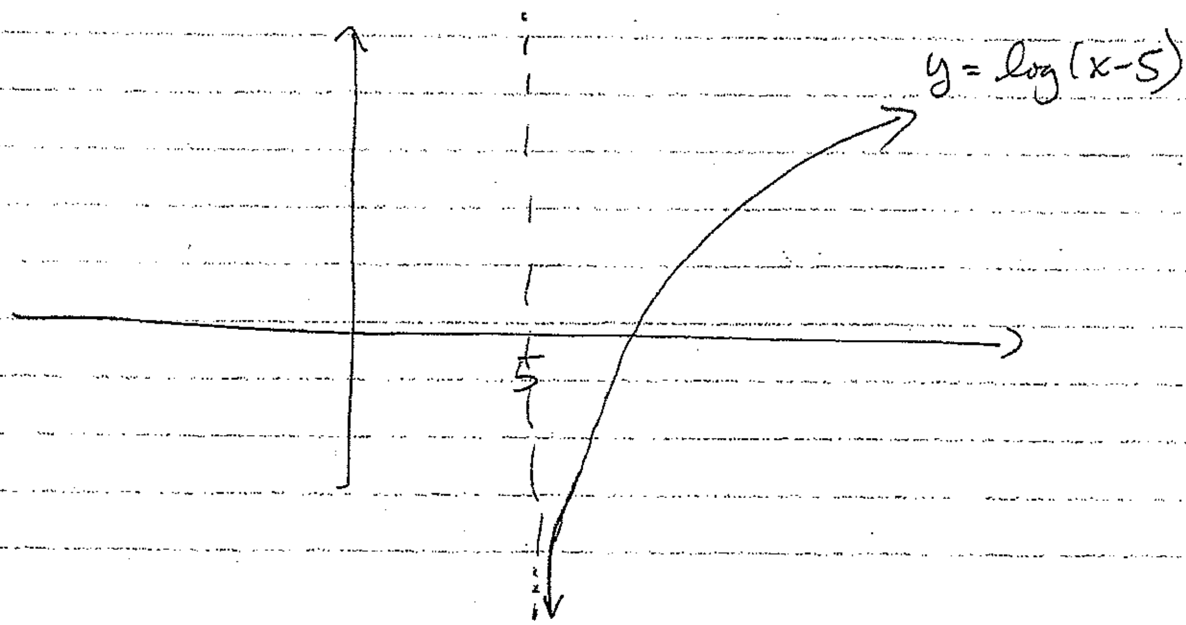
26. $\lim_{x \to 0} \frac{x-1}{x^2(x+2)}$

$$\left\{ \begin{array}{l} \lim_{x \to 0} x - 1 = -1 \\ \\ \lim_{x \to 0} x^2(x+2) = 0 \end{array} \right\}$$

also, $x^2(x+2) > 0$ as $x \to 0$

$$\boxed{\therefore \lim_{x \to 0} \frac{x-1}{x^2(x+2)} = -\infty}$$

30. $\lim_{x \to 5^+} \log(x-5) = -\infty$; from graph



2.3

16. $\lim\limits_{x \to -1} \dfrac{x^2-4x}{x^2-3x-4} = \lim\limits_{x \to -1} \dfrac{x\cancel{(x-4)}}{(x+1)\cancel{(x-4)}}$

$\left\{ \begin{array}{l} \lim\limits_{x \to -1} x = -1 \\ \lim\limits_{x \to -1} (x+1) = 0 \end{array} \right\}$ but $x+1 > 1$ for $x > -1$ and $x+1 < 1$ for $x < -1$

$\boxed{\therefore \text{ lim doesn't exist}}$

24. $\lim\limits_{x \to 2} \dfrac{x^4-16}{x-2} = \lim \dfrac{\overset{x+2}{\cancel{(x^2-4)}}(x^2+4)}{\cancel{x-2}} = \lim (x+2)(x^2+4)$

$\boxed{= 4 \cdot 8 = 32}$

26. $\lim\limits_{t \to 0} \dfrac{1}{t} - \dfrac{1}{t^2+t} = \lim\limits_{t \to 0} \dfrac{\cancel{t} + \cancel{1} - \cancel{1}}{t^2+t}$

$= \lim\limits_{t \to 0} \dfrac{1}{t+1} = 1$

37. $-1 \le \cos\left(\frac{2}{x}\right) \le 1$ for all x

$\therefore -x^4 \le x^4 \cos\left(\frac{2}{x}\right) \le x^4$

but $\lim\limits_{x \to 0} -x^4 = \lim\limits_{x \to 0} x^4 = 0$

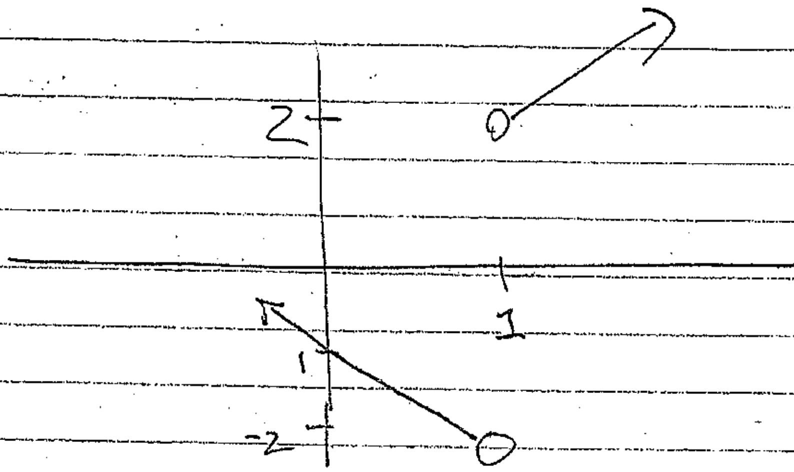
$$\boxed{\therefore \lim_{x \to 0} x^4 \cos\left(\frac{2}{x}\right) = 0 \quad \text{by } \underline{SQ}}$$

47. a) $\lim_{x \to 1^+} \frac{x^2-1}{|x-1|} = \lim_{x \to 1^+} \frac{x^2-1}{(x-1)} = \lim_{x \to 1^+} (x+1) = 2$

$\lim_{x \to 1^-} \frac{x^2-1}{|x-1|} = \lim_{x \to 1^-} \frac{x^2-1}{-(x-1)} = \lim_{x \to 1^-} -(x+1) = -2$

b) No

c)



55. Use Squeeze Theorem!

$0 \le f(x) \le x^2$ for $\underline{\text{all}}$ $x \in \mathbb{R}$

but $\lim_{x \to 0} 0 = \lim_{x \to 0} x^2 = 0$

$$\boxed{\therefore \lim_{x \to 0} f(x) = 0 \quad \text{by } \underline{SQ}}$$

1.6

70 $\tan(\sin^{-1} x) = ??$

Let $\sin^{-1} x = \theta$. Then $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$.

Now $\tan(\sin^{-1} x) = \tan\theta$ and $\sin\theta = x$

$$\Rightarrow \cos\theta = \sqrt{1-\sin^2\theta}$$

$$\left(\text{as } -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}\right)$$

$$= \sqrt{1-x2}.$$

$$\therefore \tan(\sin^{-1} x) = \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{x}{\sqrt{1-x2}}.$$

APPENDIX D

52 $$\frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta} = \frac{1+\sin\theta+1-\sin\theta}{(1-\sin^2\theta)}$$

$$= \frac{2}{\cos^2\theta} = 2\sec^2\theta$$

70 $2\cos x + \sin 2x = 2\cos x + 2\sin x\cos x$

$= 2\cos x(1+\sin x) = 0$

$\Rightarrow \cos x = 0$ or $\sin x = -1$

$\therefore x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\qquad$ $\therefore x = \frac{3\pi}{2}$

$\therefore x = \frac{\pi}{2}, \frac{3\pi}{2}$.