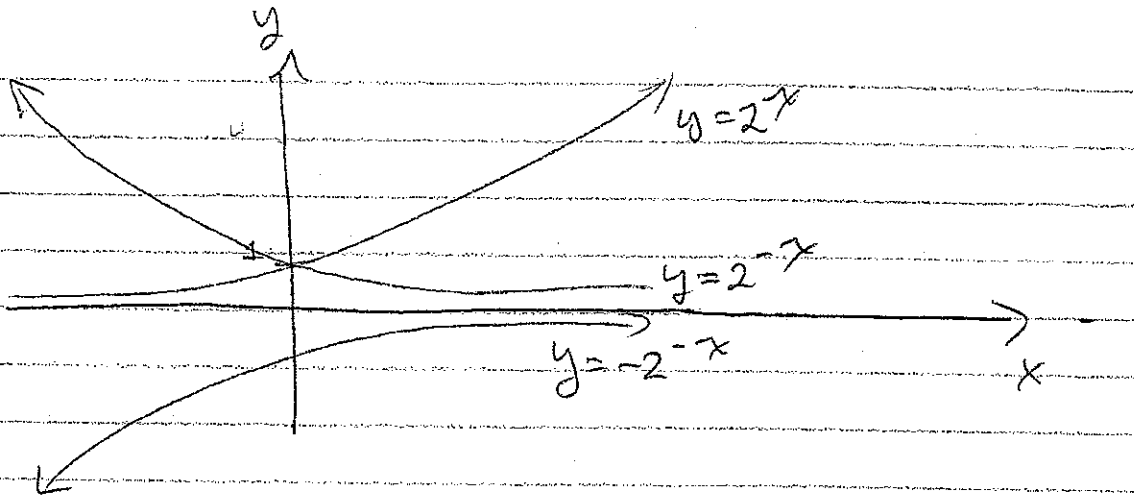
SOLUTIONS #2

1.5

9.



14. reflect about line $y=4$; same as

1) translate down by 4; $y=e^x-4$

a) 2) reflect about $y=0$ (x axis); $y=-(e^x-4)$

3) translate up by 4; $y=-(e^x-4)+4$

$=-e^x+8$

$\therefore$ new equation is $y=-e^x+8$

b) reflect about line $x=2$; same as

1) translate left by 2; $y=e^{x+2}$

1.6

3. no 5. yes 7. no

4. yes 6. no 8. yes

9. yes 10. no 11. no

12. yes 13. no 14. I hope so. But probably not.

25. $f(x) = e^{x^3}$

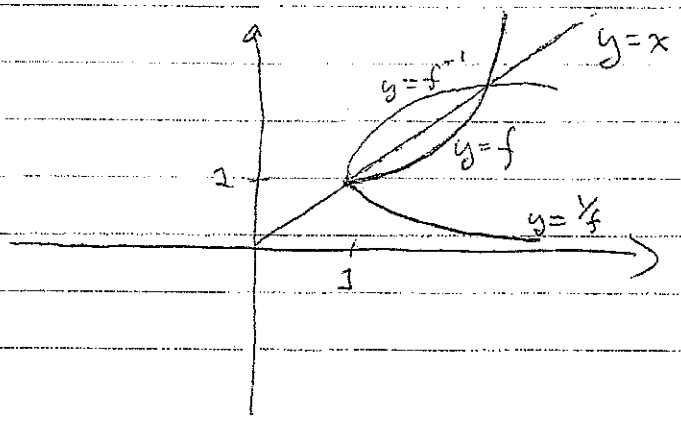
let $x = e^{y^3}$

$\therefore \log x = y^3$

$\therefore (\log x)^{1/3} = y$

$\therefore f^{-1}(x) = (\log x)^{1/3}$

32.



49. a) $2\log x = 1$

$\log x = \frac{1}{2}$

$x = e^{\frac{1}{2}}$

b) $e^{-x} = 5$

$-x = \log 5$

$x = -\log 5$

50. a) $e^{2x+3} = 7$

$2x + 3 = \log 7$

$2x = \log 7 - 3$

$x = \frac{\log 7}{2} - \frac{3}{2}$

b) $\log(5 - 2x) = -3$

$5 - 2x = e^{-3}$

$-2x = e^{-3} - 5$

$x = -\frac{1}{2}e^{-3} + \frac{5}{2}$

54. a) $2 < \log x < 9$

$\therefore e^2 < x < e^9$

b) $e^{2-3x} > 4$

$\therefore 2 - 3x > \log 4$

$\therefore -3x > \log 4 - 2$

$\therefore x < -\frac{\log 4}{3} + \frac{2}{3}$

56. a) $\log(2+\log x)$

$\therefore 2+\log x > 0$

$\therefore \log x > -2$

$\therefore x > e^{-2}$

$\therefore \text{Dom } f = (e^{-2}, \infty)$

b) let $x = \log(2+\log y)$

$\therefore e^x = 2+\log y$

$e^x - 2 = \log y$

$e^{e^x-2} = y$

$\therefore f^{-1} = e^{e^x-2}$

and $\text{Dom}(f^{-1}) = (-\infty, \infty)$

a) graph of f has equ^{n}

$y = f(x)$

reflection about $y = x$ has equat^{n}

$x = f(y)$

graph of translate of f has equ^{n}

$y = f(x+c)$

reflect this about $y = x$ has equ^{n}

$x = f(y+c)$

But $x = f(y+c)$ is just $x = f(y)$ shifted down by c

∴ answer to question in a) is: its reflection is shifted down by c

$g(x) = f(x+c)$

∴ $g^{-1}(x) = f^{-1}(x) - c$ "reflection of $y = f$ shifted down by c"

b) $h(x) = f(cx)$

Set $x = h(y)$

$\therefore x = f(cy)$

$\therefore f^{-1}(x) = cy$

$\therefore y = \frac{1}{c} f^{-1}(x)$

$\boxed{\therefore h^{-1}(x) = \frac{1}{c} f^{-1}(x)}$

note: answer to a) could have been derived similarly as

$g(x) = f(x + c)$

Set $x = g(y)$

$\therefore x = f(y + c)$

$\therefore f^{-1}(x) = y + c$

$\therefore y = f^{-1}(x) - c$

$\boxed{\therefore g^{-1}(x) = f^{-1}(x) - c}$