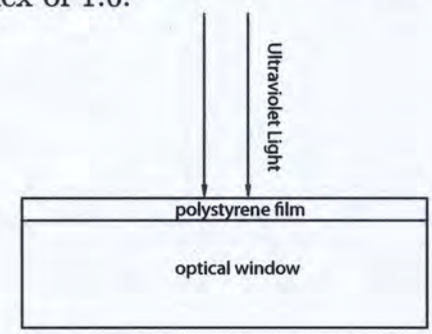
Quick Solutions

Name: ______________________ Activity ID: ______________________

Problem 1. A synthetic crystal material Fabulite (Strontium titanate $SrTiO_3$) with refractive index of 2.394 has many applications. For example, it can used as a substrate for making optical windows. Optical windows are used because they allow (UV) ultraviolet light to pass through them.

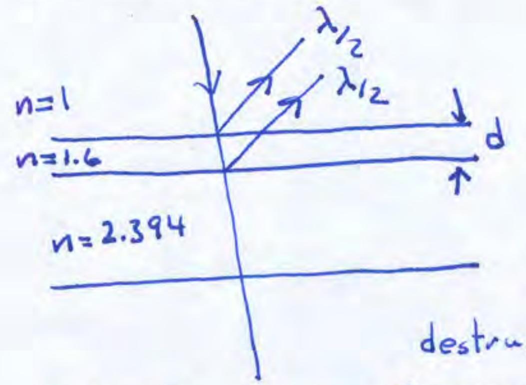
In order to obtain the maximum transmission of ultraviolet light through an optical window made of Fabulite, the window is coated with a thin film of polystyrene which has a refractive index of 1.6.



a) Find the minimum thickness of the polystyrene film required to allow the maximum transmission of ultraviolet light at $\lambda = 350$ nm.

b) Find the minimum thickness of the polystyrene film required to allow the maximum transmission of ultraviolet light if the Fabulite optical window is replaced with a glass window of refractive index of 1.5.

a)



$$\Delta d = 2d$$

$$\lambda = \frac{\lambda_{air}}{n_{film}}$$

$$\begin{cases} \Delta d = m\lambda & (1) \\ \Delta d = (m+\tfrac{1}{2})\lambda & (2) \end{cases}$$

$m = 0, \pm 1, \pm 2, \pm 3, \ldots$

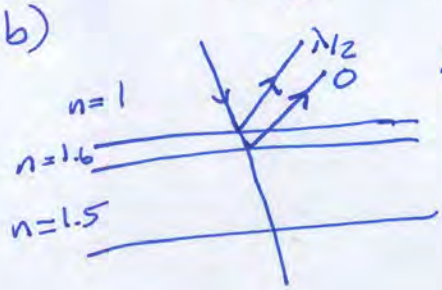
For maximum transmission, you want destructive interference of reflection.

$\Delta\phi = \frac{\lambda}{2} - \frac{\lambda}{2} = 0$. Because $\Delta\phi = 0$ (1) is constructive and (2) is destructive.

$$2d = (m+\tfrac{1}{2})\frac{\lambda_{air}}{n_{film}} \Rightarrow d = \frac{\lambda_{air}}{2n_{film}}(m+\tfrac{1}{2})$$

$$d = \frac{\lambda_{air}}{4n_{film}} = \frac{350\text{ nm}}{4(1.6)} = \underline{54.7\text{ nm}}$$

b)



$\Delta\phi = \frac{\lambda}{2} - 0 = \frac{\lambda}{2}$ this means (1) destructive and (2) is constructive.

$\Delta d = m\lambda$ is destructive.

$$d = \frac{\lambda_{air}}{2(n_{film})} = \frac{350\text{ nm}}{2(1.6)} = \underline{109.4\text{ nm}}$$

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Problem 2. A circuit used in a camera flash system has the following circuit diagram shown below.

$\mathcal{E} = 160\,V$, $R_1 = 8\,\Omega$, $R_2 = 3\,\Omega$, $C = 0.0180\,F$.

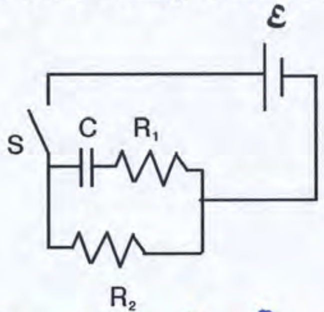
a) Find all currents immediately after the switch S is closed ($t = 0^+$).

b) Find the maximum energy stored in the capacitor.

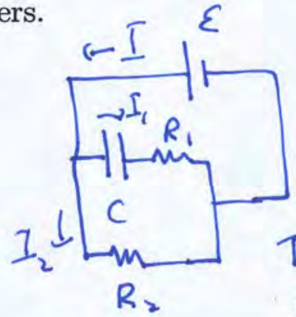
c) Find all the currents as a function of time t.

d) A very long time after closing the switch S, it is opened once again. Find all currents as a function of time.

- Please clearly indicate your answers.



a)



At $t=0$, $Q_1 = 0$.

$I_1 = \frac{dQ_1}{dt}$

$I = I_1 + I_2$

Take a loop including C

$\mathcal{E} - \frac{Q_1}{C} - I_1 R_1$ (1)

Take a loop with R_2

$\mathcal{E} - I_2 R_2 = 0$ (2)

So $I_2 = \mathcal{E}/R_2$, $I_1 = \mathcal{E}/R_1$ at $t=0$

at $t=0$
$$\begin{cases} I = I_1 + I_2 = \frac{\mathcal{E}}{R_1} + \frac{\mathcal{E}}{R_2} = \underline{73.3A} \\ I_1 = \frac{\mathcal{E}}{R_1} = \frac{160V}{8\Omega} = \underline{20A},\ I_2 = \frac{160V}{3\Omega} = \underline{53.3A} \end{cases}$$

b) Using (1) $\mathcal{E} - \frac{Q_1}{C} - I_1 R_1 = 0$

Q_1 is a max when $I_1 = 0$.

So $\underline{Q_1 = C\mathcal{E}}$ or $\underline{V_c = \mathcal{E}}$.

$U = \frac{1}{2} C V^2 = \frac{1}{2} C \mathcal{E}^2$

$U = \frac{1}{2}(0.018F)(160V)^2$

$U = \underline{230J}$

c) Note from equation (2) in part a) $I_2 = \mathcal{E}/R_2$ always.

Equation (1) $\Rightarrow I_1 = \frac{\mathcal{E}}{R_1} e^{-t/\tau}$ $\tau = R_1 C$

$$\begin{cases} I_1 = \frac{\mathcal{E}}{R_1} e^{-t/R_1 C} \\ I_2 = \mathcal{E}/R_2 \\ I = \mathcal{E}/R_2 + \frac{\mathcal{E}}{R_1} e^{-t/R_1 C} \end{cases}$$

d) When switch is opened

$V_c = \mathcal{E}$ at the start from b).

$V_c = I(R_1 + R_2)$ so

$$\underline{I = \left(\frac{\mathcal{E}}{R_1 + R_2}\right) e^{-t/\tau},\ \tau = C(R_1 + R_2)}$$

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Problem 3. Given the following circuit with $\mathcal{E} = 12\,V$, $R_1 = 30\,\Omega$, $R_2 = 50\,\Omega$, $R_3 = 10\,\Omega$, $L = 30\,mH$, $C_1 = 200\,\mu F$, $C_2 = 300\,\mu F$,

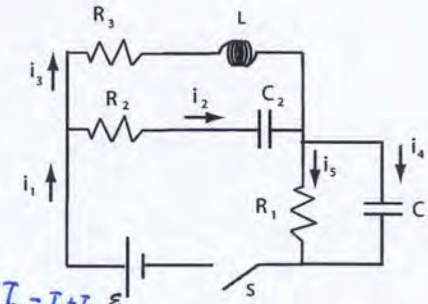
(a) Find all the currents i_1, i_2, i_3, i_4, and i_5 at $t = 0^+$, just after the switch S is closed.

(b) Find all currents after the switch S has been closed for a long time.

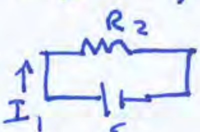
(c) After a very long time the switch S is opened. Find all the currents just after the switch S is opened.

(d) Find all the currents at $t = \infty$ after the switch S is opened.

Please clearly indicate your answers.

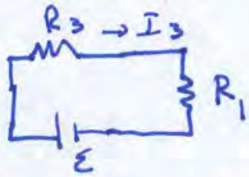


a) At $t=0$, when S just closed.
$I_3 = 0$ because L allows no charge at start. $Q_2 = 0$ so $V_{C_2} = 0$. The same is true for C_1 so $Q_1 = 0, V_{C_1} = 0$.
$V_{C_1} = V_{R_1}$, $0 = I_5 R_1 \Rightarrow I_5 = 0$
So $\underline{I_3 = I_5 = 0}$. $I_1 = I_3 + I_2$, $I_2 + I_3 = I_5 + I_4$

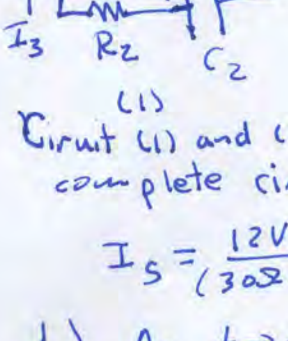
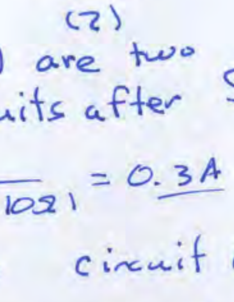
The equivalent circuit is  $I_1 = \mathcal{E}/R_2 = \frac{12V}{50\Omega} = 0.24A$

$I_1 = I_2 = I_4 = \mathcal{E}/R_2$ and $I_3 = I_5 = 0$

b) After a long time. $V_L = 0$, $I_2 = 0$ because V_{C_2} is max and $I_4 = 0$ because V_{C_1} is max.

Equivalent circuit  $I_1 = I_3 = I_5 = \frac{\mathcal{E}}{R_1 + R_3}$

Thus, $I_1 = I_3 = I_5 = \frac{\mathcal{E}}{R_1 + R_3}$ and $I_2 = I_4 = 0$ $\quad I_1 = I_3 = I_5 = \frac{12V}{40\Omega} = \underline{0.3A}$

c)  

For circuit (1) L maintains the current it had before the change, $I_3 = \frac{\mathcal{E}}{R_1 + R_3} = \underline{0.3A}$

Circuit (1) and (2) are two complete circuits after S is opened. $I_5 = \frac{V_{C_1}}{R_1}$ ~ value before opening switch.

$I_5 = \frac{12V}{(30\Omega + 10\Omega)} = \underline{0.3A}$ $\quad$ so $I_5 = \frac{V_{C_1}}{R_1} = \frac{I_5^{before} R_1}{R_1} = \frac{\mathcal{E}}{R_1 + R_3}$

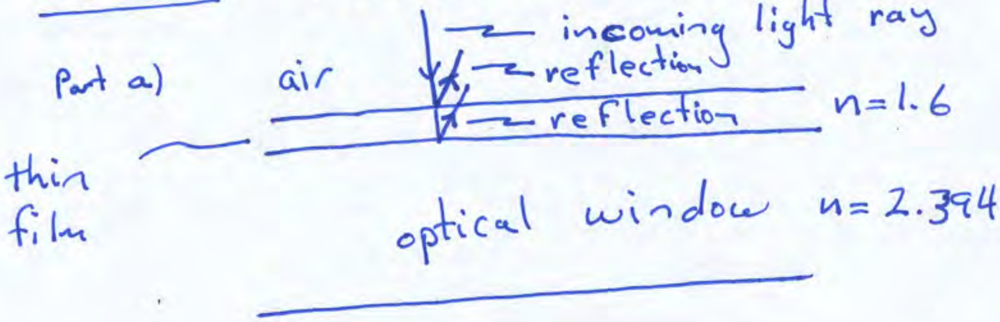
d) As $t \to \infty$ circuit (1) from c) is damped so $I_3 \to 0$.
At $t \to \infty$ circuit (2) from c) goes like $e^{-t/\tau}$ so $I_5 \to 0$.

Thus ~~both~~ all are $\underline{\text{zero}}$

More Detailed Solutions

Problem 1 This is a thin film interference problem.

Part a)



The first step is to find the phase shifts of any reflected waves.

This is determined by comparing the refractive index of each material.

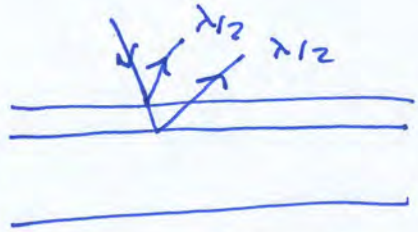
The incoming ray is traveling in air with $n=1$ and hits material with $n=1.6$. This means there is a phase shift of $\frac{\lambda}{2}$ for the first reflection.

The remaining incoming ray continues in the ~~thin~~ film and is reflected at the optical window. So it is traveling in a material with $n=1.6$ and is reflected at material with $n=2.394$.

Therefore, another phase of $\frac{\lambda}{2}$ happens because the wave is moving in material 1.6 to 2.394 so low to high.

Problem 1 (cont'd)

Thus, the two reflected waves have no net phase change.



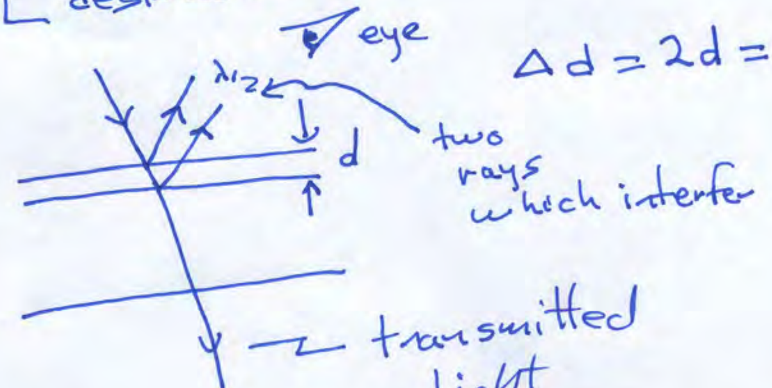
The interference formulas are $\Delta d = m\lambda$ constructive
$\Delta d = m\frac{\lambda}{2}$ destructive

if both waves are <u>in phase</u>.

Note: Since the optical window is thick no further interference takes place.

For maximum transmission one wants minimum reflected light or in other words destructive interference.

wave length in thin film



$$\Delta d = 2d = \frac{m}{2}\lambda$$

$m = 1, 3, 5, \ldots$

$$2d = \frac{m\lambda_{air}}{2n}$$

$$d = \frac{m\lambda_{air}}{4n}$$

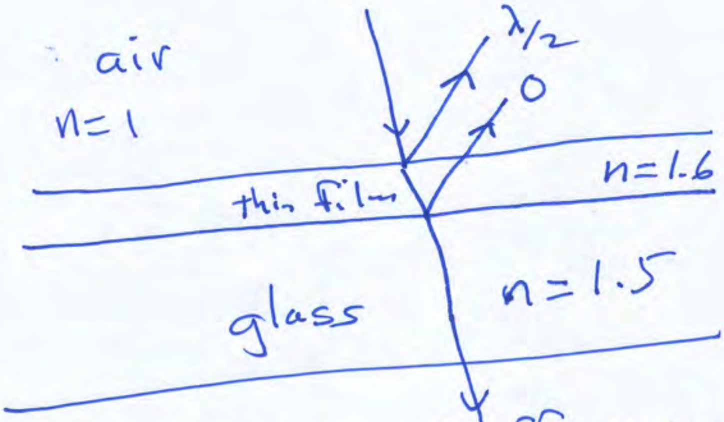
d smallest for $m = 1$

$$d = \frac{\lambda_{air}}{4n} = \frac{350\text{ nm}}{4(1.6)} = \underline{54.7\text{ nm}}$$

Problem 1 (cont'd)

b) The only change in part b) of problem is the phase differences.

Namely,



So there is a net phase difference initially because reflections.

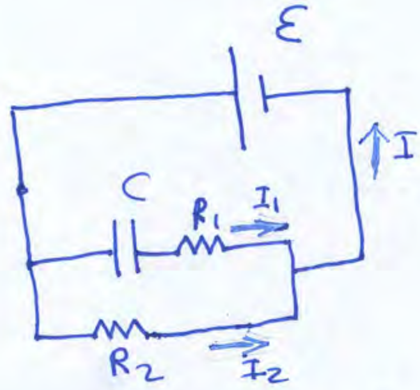
Now,
$\Delta d = \frac{m\lambda}{2}$ (constructive)
$\Delta d = m\lambda$ (destructive)

Again, we want destructive interference for maximum transmitted light.

So $2d = \frac{m\lambda_{air}}{n} \longrightarrow d = \frac{m\lambda_{air}}{2n}$

now minimum film thickness is $d = \frac{(350\,nm)}{2(1.6)} = \underline{109.4\,nm}$

<u>Problem 2</u>



a) There is no charge initially on the capacitor since

$V_C = \frac{Q_1}{C} \Rightarrow V_C = 0.$

$Q_1 = 0$

Since equation (1) gives $\mathcal{E} - I_1 R_1 = 0$

$I_1 = \mathcal{E}/R_1$ at $t = 0^+$

Taking loops in the circuit

$$+\mathcal{E} - \frac{Q_1}{C} - I_1 R_1 = 0 \quad (1)$$

$$+\mathcal{E} - I_2 R_2 = 0 \quad (2)$$

$$I = I_1 + I_2 \quad (3)$$

Note: that (1) & (2) are not dependent on each other so (2) gives $I_2 = \mathcal{E}/R_2$ for all time.

$I = I_1 + I_2 = \frac{\mathcal{E}}{R_1} + \mathcal{E}/R_2$

$I_1 = \frac{160V}{8\Omega} = \underline{20A}$ $\quad I_2 = \frac{160V}{3\Omega} = \underline{53A}$

$I = \underline{73\ A}$

b) The maximum energy stored in the capacitor is when the charge is a maximum, recall $U = \frac{1}{2}\frac{Q^2}{C}$

Q_1 is a maximum when $I_1 = 0$ (remember $I_1 = \frac{dQ_1}{dt}$)

when $I_1 = 0$ equation (1) yields

$\mathcal{E} - \frac{Q_1}{C} = 0 \Rightarrow Q_1 = \mathcal{E}C = (160V)(0.018F)$

$Q_1 = \underline{2.88\ C}$

$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{(2.88C)^2}{(0.018F)}$ or $U = \frac{1}{2}CV^2 = \frac{1}{2}(0.018F)(160V)^2$

$U = \underline{230\ J}$

Problem 2 (cont'd)

c)

Again equation (1) and (2) are independent.

So $I_2 = \mathcal{E}/R_2$ for all time.

$$I_2 = \frac{160\text{ V}}{3\,\Omega} = \underline{53\text{ A}}$$

I_1 does change as a function of time because

equation (1) is $\quad \mathcal{E} - \frac{Q_1}{C} - I_1 R_1 = 0 \qquad I_1 = \frac{dQ_1}{dt}$

So $\quad I_1 = I_{max} e^{-t/\tau} \qquad \tau = R_1 C$

$I_{max} = \mathcal{E}/R_1$ from part a) of the

problem. $\quad \underline{I_1 = \frac{\mathcal{E}}{R_1} e^{-t/R_1C}} \quad , \underline{I_2 = \mathcal{E}/R_2}$

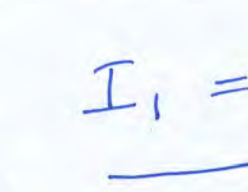
and $\quad \underline{I = I_1 + I_2 = \frac{\mathcal{E}}{R_2} + \frac{\mathcal{E}}{R_1} e^{-t/R_1C}}$

d) After a very long time $I_1 = 0$ and $I_2 = \mathcal{E}/R_2$

So capacitor C has $V_C = \mathcal{E}$, from equation (1) & (2).

Now, open switch and the circuit becomes

with $V_C = \mathcal{E}$ initial



So $\quad I_{max} = \frac{\mathcal{E}}{R_1 + R_2}$ because

loop gives $\quad V_C = IR_1 + IR_2$

$I(t) = I_{max} e^{-t/\tau} \qquad \tau = (R_1 + R_2) C$

So $\quad \underline{I = \frac{\mathcal{E}}{R_1 + R_2} e^{-t/C(R_1+R_2)}}$

Problem 3

a) At instant the switch is closed

$\underline{I_3 = 0}$ because I_3 is going through an inductor. $V_{C_2} = 0$ because $Q_2 = 0$ and

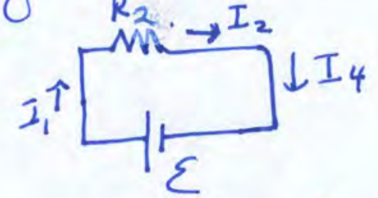
$V_{C_1} = 0$ because $Q_1 = 0$.

$V_{C_1} = V_{R_1}$ so $V_{C_1} = 0$ implies $V_{R_1} = I_5 R_1 = 0$,

because R_1 is parallel to C_1. This means $\underline{I_5 = 0}$

Thus the equivalent circuit is

$$\boxed{\begin{array}{l} I_1 = I_2 = I_4 = \mathcal{E}/R_2 \\ I_3 = I_5 = 0 \end{array}} = \frac{12V}{50\Omega} = 0.24A$$

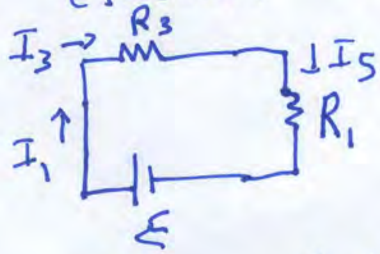


b) After a long time $I_2 = 0$ because the capcitor C_2 becomes full charged. In addition

$I_4 = 0$ because the capacitor C_1 becomes fully charged.

After a long time $V_L \to 0$ so, the equivalent circuit is

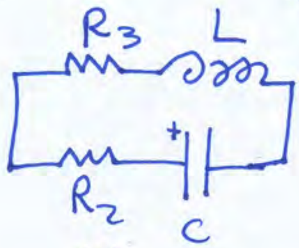
$$\boxed{\begin{array}{l} I_1 = I_3 = I_5 = \dfrac{\mathcal{E}}{R_1 + R_3} \\ I_4 = I_2 = 0 \end{array}} = \frac{12V}{30\Omega + 10\Omega} = 0.3\text{ A}$$



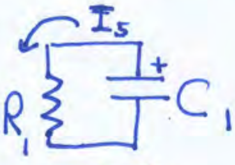
c) After the switch is opened the circuit reduces to two different independent circuits.

Problem 3 (cont'd)

c) The two circuits are



(1)



(2)

because the switch S removes the current path.

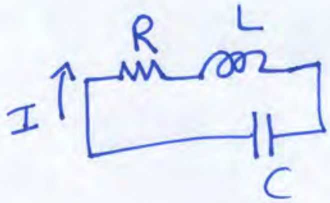
For circuit (2) $I_5(t) = I_{max} e^{-t/\tau}$ as a function of time but at the start $I_5(0) = \frac{V_{C_1}}{R_1}$.

V_{C_1} is equal to voltage $I_5 R_1$ right before the switch is opened.

$$I_5(0) = \frac{I_5^{(\infty)} R_1}{R_1} = I_5(\infty)$$ answer from part b)

$$I_5(0) = \frac{\mathcal{E}}{R_1 + R_3} = 0.3A$$

For circuit (1) first reduce circuit



where $R = R_2 + R_3$

where I is current right before switch is opened.

$$I = I_3^{before} = \frac{\mathcal{E}}{R_1 + R_3} = 0.3A$$

(from part b)

d) Finally, all currents will be zero as $t \to \infty$.
The reason is $I_5(t) = I_{max} e^{-t/\tau}$ and circuit (1) is also a damped oscillator.