

# Assignment 3

## Applied Linear Algebra

### Math 232 - D100 (Fall 2012)

**Quiz date:** Friday, September 28

Complete this assignment by Wednesday in your *homework journal*. This will give you plenty of time to make sure you understand the material before the quiz at the end of Wednesday's class. Quiz questions will be taken from items 1 (Questions from Textbook) or 2 (Additional Questions) below.

Some suggestions for using your homework journal are:

- Do rough work on scratch paper.
- If you find one solution, try to find another (a simpler solution may reveal itself).
- When you find a solution, try to see it as a whole without all the little details.
- Do questions in order and clearly label question and section numbers.
- Grade your own assignment when solutions are posted. Catch your mistakes now when the stakes are low rather than making them on exams.

To obtain maximum marks on the quiz, your answer should be in a form that another student could understand without undue effort: a poorly expressed but correct result is not sufficient.

#### 1. Questions from textbook:

<i>section</i>	<i>question</i>	<i>done</i>	<i>checked</i>	<i>corrected</i>	<i>study MT</i>	<i>study final</i>	<i>type <sup>1</sup></i>	<i>comment</i>
2.3	4	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	RE	
	11	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	RE	
	15	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	RE	
3.1	5	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	CD	
	7	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	CD	
	11	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	CD	
	13	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	CD	
	18	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	RE	
	23	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	RE	
	D2	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	CE	
	D4	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	RE	
	D9	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	CE	
3.2	7	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	RE	
	14	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	RE	
	17	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	RE	
	20	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	RE	
	21	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	RE	
	24	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	CE	
	32	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	CE	
	35	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	RE	
	D1	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	CE	
	D4	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	CE	
	D5	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	CE	

<sup>1</sup>See the legend on last page of this assignment for what these acronyms mean.

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## 2. Additional questions:

These questions are made up by your instructor and may require a blend of ideas that we have encountered so far in the course. They are similar to exam style questions in that it is not entirely clear what part of the text is directly related to solving the problem. You will have to decide what tools/techniques are required.

**A1.** A  $2 \times 2$  matrix is said to be **of full rank** if its reduced row echelon form is  $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ . For which real numbers  $a$  and  $b$  is  $\begin{pmatrix} a & b \\ b & a \end{pmatrix}$  of full rank? [Hint: when you do row operations with unknowns, be careful that you don't unwittingly multiply a row by 0 or try to divide anything by 0.]

**A2.** A square matrix  $A$  is called **idempotent** if  $A^2 = A$ .

- (a) Find three idempotent  $2 \times 2$  matrices.
- (b) Prove that the only invertible idempotent  $n \times n$  matrix is the identity matrix.

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### 3. Extra-Practice Questions:

Try these questions for some more practice. The more practice you get the better you will understand the material and the better you will do on quizzes and exams.

- Section 2.3: 2, 10, 13
  - Section 3.1: 1, 3, 17, 19, 21, 27, 30, 33, D7, D8
  - Section 3.2: 1, 3, 5, 15, 25, 31, 33, 34, 36, D2, D3, D7, D8
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#### **Legend (for "type" of question):**

**RE** = Routine Exercise: This is something you should be able to do in your sleep ;-). Your goal is to be able to answer these questions quickly and accurately every time. These form the foundations of your skill set.

**TC** = Time Challenge: Speed and accuracy are important factors in solving this type of routine exercise. Try to do these exercises within the time limit, usually 5 minutes. If you need more time than that, its o.k., but keep practicing! Solving these routine exercises provides a foundation for solving more involved problems, and is essential in performing well on quizzes and exams.

**WP** = Word Problem: Translating words into expressions (also known as modeling): Master this skill now, we will be using this all term.

**CD** = Concepts and Definitions: These questions relate to your understanding of the "new language" we are introducing. They should help you remember the important definitions and theorems.

**CE** = Concepts and Explorations: This indicates a question which is testing your understanding of the fundamentals. It is not a routine exercise since the solution process may not be obvious at first glance. It may take a little bit of thought to figure out what to do, don't be afraid to play around with some ideas. You'll learn more by making mistakes and taking routes which lead to dead ends. You must be able to do these types of questions to succeed in learning this material.

**HL** = Higher Level Understanding: This indicates a question which is testing understanding at a higher level. These questions will require more thought than a RE or CE so don't be discouraged if you can't see how to do this immediately. Perseverance and playing around with ideas is the key to these questions. Understanding this material at this level is an expected outcome of this course.

**CM** = Computer of Computational Device: This indicates a question in which a computer or calculator is needed.

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**Selected Hints & Answers:**

2.3: 2 (a)  $x_1 + x_3 = 200$ ,  $x_1 - x_2 = -25$ ,  $x_2 + x_4 - x_6 = 175$ ,  $x_3 - x_4 - x_5 = -150$ ,  $x_5 + x_6 = 200$ , (b)  $x_1 = -s + t + 150$ ,  $x_2 = -s + t + 175$ ,  $x_3 = s - t + 50$ ,  $x_4 = s$ ,  $x_5 = -t + 200$ ,  $x_6 = t$  for  $-\infty < s, t < \infty$ , (c)  $x_1 = 100$ ,  $x_2 = 125$ ,  $x_3 = 100$ ,  $x_4 = 50$ ,  $x_5 = 200$ ,  $x_6 = 0$ . The positive flows  $x_1, x_2, x_3, x_4, x_5$  move in the direction of the arrows on the diagram.  $x_6 = 0$  represents no flow.

2.3: 4 (a)  $x_1 + x_3 = 800$ ,  $x_1 - x_2 + x_4 = 200$ ,  $x_2 - x_5 = 500$ ,  $x_5 - x_7 = -50$ ,  $x_4 + x_6 - x_7 = 600$ ,  $x_3 + x_6 = 750$ , (b)  $x_1 = s + 50$ ,  $x_2 = t + 450$ ,  $x_3 = -s + 750$ ,  $x_4 = -s + t + 600$ ,  $x_5 = t - 50$ ,  $x_6 = s$ ,  $x_7 = t$  for  $-\infty < s, t < \infty$ , (c) it is not possible

2.3: 10 the balanced equation is  $\text{C}_6\text{H}_{12}\text{O}_6 \rightarrow 2\text{CO}_2 + 2\text{C}_2\text{H}_5\text{OH}$

3.1: 18 (a)  $\begin{pmatrix} 6 & -6 & 70 \end{pmatrix}$ , (b)  $\begin{pmatrix} 63 & 41 & 122 \end{pmatrix}$ , (c)  $\begin{pmatrix} -6 \\ 17 \\ 41 \end{pmatrix}$

3.1: 30 (a) If  $\mathbf{c}_j(B) = \mathbf{c}_k(C)$ , then  $\mathbf{c}_j(AB) = A\mathbf{c}_j(B) = A\mathbf{c}_k(C) = \mathbf{c}_k(AC)$ , (b) If  $\mathbf{r}_j(B) = \mathbf{r}_k(C)$ , then  $\mathbf{r}_j(BA) = \mathbf{r}_j(B)A = \mathbf{r}_k(C)A = \mathbf{r}_k(CA)$

3.1: D2 One possible choice is  $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

3.1: D4 Only one

3.1: D7 Yes

3.1: D8 Yes

3.1: D9 (a) F, (b) T, (c) T, (d) F, (e) T, (f) F

3.2: 14.  $\frac{1}{11} \begin{pmatrix} -88 & 37 \\ 66 & -29 \end{pmatrix}$

3.2: 20(a).  $\frac{1}{5} \begin{pmatrix} -2 & 5 \\ -1 & 3 \end{pmatrix}$ , (b).  $\frac{1}{13} \begin{pmatrix} -9 & 1 \\ 2 & -6 \end{pmatrix}$

3.2: 24. There are many possibilities, such as  $\begin{pmatrix} 0 & 1 & 2 \\ -1 & 0 & 3 \\ -2 & -3 & 0 \end{pmatrix}$

3.2: 32. (a) Just calculate  $(I - A)^2$ , replacing  $A^2$  with  $A$  (because of idempotency) when you see it, (b) Just calculate  $(2A - I)^2$  and check that you get  $I$

3.2: 34. Multiply  $A$  and what they claim is  $A^{-1}$  together (in both orders). Use the fact that  $\mathbf{u}^T \mathbf{v} = \mathbf{u} \cdot \mathbf{v} = \mathbf{v}^T \mathbf{u}$  to simplify.

3.2: 36. Calculate the trace of both sides of the equation; show they don't match.

3.2: D1. (a) Many possibilities: a simple pair is  $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$  and  $B = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$ , (b)  $A^2 - AB - BA + B^2$ , (c)  $AB = BA$

3.2: D2. All matrices of the form  $\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$  with  $a, b, c \in \{-1, 1\}$  work

3.2: D3. (a) rearrange the polynomial in two ways, one of which is  $A(-A - 2I) = I$ , (b) generalize the trick used in part (a)

3.2: D4. rearrange the equation  $A^3 = I$  as in the hint for D3

3.2: D5. FTTFF

3.2: D7. Six are invertible, ten are not

3.2: D8. Yes.