Solutions to Assignment 2

Applied Linear Algebra
Math 232 (Fall 2012)

Section 1.2

1. (a) The norm of $\mathbf{v}$ is $\|\mathbf{v}\| = \sqrt{4^2 + (-3)^2} = \sqrt{25} = 5$. The vector $\frac{\mathbf{v}}{\|\mathbf{v}\|} = (\frac{4}{5}, -\frac{3}{5})$ is a unit vector having the same direction as $\mathbf{v}$, and $-\frac{\mathbf{v}}{\|\mathbf{v}\|} = (-\frac{4}{5}, \frac{3}{5})$ is a unit vector in the opposite direction.

(c) $\|\mathbf{v}\| = \sqrt{1^2 + 0^2 + 2^2 + 1^2 + 3^3} = \sqrt{15} \qquad \pm\frac{\mathbf{v}}{\|\mathbf{v}\|} = \pm\frac{1}{\sqrt{15}}(1, 0, 2, 1, 3)$

5. (a) $3\mathbf{u} - 5\mathbf{v} + \mathbf{w} = (-27, -6, 38, -19) \qquad \|3\mathbf{u} - 5\mathbf{v} + \mathbf{w}\| = \sqrt{729 + 36 + 1444 + 361} = \sqrt{2570}$

(b) $\|3\mathbf{u}\| - 5\|\mathbf{v}\| + \|\mathbf{u}\| = 3\|\mathbf{u}\| - 5\|\mathbf{v}\| + \|\mathbf{u}\| = 4\|\mathbf{u}\| - 5\|\mathbf{v}\| = 4\sqrt{46} - 5\sqrt{84}$

(c) $\|\mathbf{u}\| = \sqrt{46} \qquad \|-\|\mathbf{u}\|\mathbf{v}\| = \|-\sqrt{46}\mathbf{v}\| = \sqrt{46}\|\mathbf{v}\| = \sqrt{46}\sqrt{84} = \sqrt{3864} = 2\sqrt{966}$

8. $\|k\mathbf{v}\| = |k|\,\|\mathbf{v}\| = |k|\sqrt{16} = 4|k|$; thus $\|k\mathbf{v}\| = 4$ if and only if $k = \pm 1$.

9. (a) $\mathbf{u}\cdot\mathbf{v} = (3)(2) + (1)(2) + (4)(-4) = -8 \qquad \mathbf{u}\cdot\mathbf{u} = \|\mathbf{u}\|^2 = 26 \qquad \mathbf{v}\cdot\mathbf{v} = \|\mathbf{v}\|^2 = 24$

(b) $\mathbf{u}\cdot\mathbf{v} = (1)(2) + (1)(-2) + (4)(3) + (6)(-2) = 0 \qquad \mathbf{u}\cdot\mathbf{u} = \|\mathbf{u}\|^2 = 1 + 1 + 16 + 36 = 54$
$\mathbf{v}\cdot\mathbf{v} = \|\mathbf{v}\|^2 = 4 + 4 + 9 + 4 = 21$

11. (a) $\|\mathbf{u} - \mathbf{v}\| = \sqrt{(3-1)^2 + (3-0)^2 + (3-4)^2} = \sqrt{4 + 9 + 1} = \sqrt{14}$

13. (a) $\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{(3)(1) + (3)(0) + (3)(4)}{\sqrt{9+9+9}\sqrt{1+0+16}} = \frac{15}{\sqrt{27}\sqrt{17}} \qquad \theta$ is acute

(b) $\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{(0)(-3) + (-2)(2) + (-1)(4) + (1)(4)}{\sqrt{0+4+1+1}\sqrt{9+4+16+16}} = \frac{-4}{\sqrt{6}\sqrt{45}} \qquad \theta$ is obtuse

(c) $\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{(3)(-4) + (-3)(1) + (-2)(-1) + (0)(5) + (-3)(0) + (13)(-11) + (5)(4)}{\sqrt{9+9+4+0+9+169+25}\sqrt{16+1+1+25+0+121+16}}$
$= \frac{-136}{\sqrt{225}\sqrt{180}} \qquad \theta$ is obtuse

15. If θ is the angle between two vectors $\mathbf{a}$ and $\mathbf{b}$, then $\mathbf{a}\cdot\mathbf{b} = \|\mathbf{a}\|\|\mathbf{b}\|\cos\theta$. In this problem we have $\|\mathbf{a}\| = 9$, $\|\mathbf{b}\| = 5$, and $\theta = 30°$; thus $\mathbf{a}\cdot\mathbf{b} = (9)(5)\cos 30° = 45\frac{\sqrt{3}}{2}$.

24. We must verify that $\mathbf{v_i}\cdot\mathbf{v_j} = \mathbf{0}$ for $\mathbf{i} \neq \mathbf{j}$, and $\mathbf{v_i}\cdot\mathbf{v_i} = \|\mathbf{v_i}\|^2 = 1$ for $\mathbf{i} = 1,\ 2,\ 3$. Here are the required calculations:

$$\mathbf{v}_1\cdot\mathbf{v}_2 = 0 - \tfrac{1}{3} + \tfrac{1}{3} = 0 \qquad \mathbf{v}_1\cdot\mathbf{v}_3 = -\tfrac{1}{2} + \tfrac{1}{6} + \tfrac{1}{3} = 0$$
$$\mathbf{v}_2\cdot\mathbf{v}_3 = 0 - \tfrac{1}{3} + \tfrac{1}{3} = 0 \qquad \mathbf{v}_1\cdot\mathbf{v}_1 = \tfrac{1}{2} + \tfrac{1}{6} + \tfrac{1}{3} = 1$$
$$\mathbf{v}_2\cdot\mathbf{v}_2 = 0 + \tfrac{4}{6} + \tfrac{1}{3} = 1 \qquad \mathbf{v}_3\cdot\mathbf{v}_3 = \tfrac{1}{2} + \tfrac{1}{6} + \tfrac{1}{3} = 1$$

25. We begin with the observation that $\mathbf{v} = (b, -a)$ and $-\mathbf{v} = (-b, a)$ are both orthogonal to the vector $\mathbf{u} = (a, b)$. This follows from $\mathbf{v}\cdot\mathbf{u} = ba - ab = 0$ and $(-\mathbf{v})\cdot\mathbf{u} = -ba + ab = 0$. To get unit vectors with this property we only need to normalize. The vectors $\pm\frac{\mathbf{v}}{\|\mathbf{v}\|} = \pm(\frac{b}{\sqrt{a^2+b^2}}, -\frac{a}{\sqrt{a^2+b^2}})$ are unit vectors which are orthogonal to $\mathbf{u} = (a, b)$.

D6. (a) $\|\mathbf{x}-\mathbf{x}_0\| = 1$ (b) $\|\mathbf{x}-\mathbf{x}_0\| < 1$ (c) $\|\mathbf{x}-\mathbf{x}_0\| > 1$

Section 1.3

5. (a) Vector equation: $(x,y) = (0,0) + t(3,5) = t(3,5)$
Parametric equations: $x = 3t,\ y = 5t$

(b) Vector equation: $(x,y,z) = (1,1,1) + t((0,0,0) - (1,1,1)) = (1,1,1) - t(1,1,1)$
Parametric equations: $x = 1-t,\ y = 1-t,\ z = 1-t$

(c) Vector equation: $(x,y,z) = (1,-1,1) + t((2,1,1) - (1,-1,1)) = (1,-1,1) + t(1,2,0)$
Parametric equations: $x = 1+t,\ y = -1+2t,\ z = 1$
Note. Once again these equations represent just one of the many possibilities.

7. (a) Vector equation: $(x,y) = (1,1) + t(1,2)$
Parametric equations: $x = 1+t,\ y = 1+2t$
Points other than P_0: $Q(2,3)$ corresponds to $t=1$; $R(3,5)$ corresponds to $t=2$

(b) Vector equation: $(x,y,z) = (2,0,3) + t(1,-1,1)$
Parametric equations: $x = 2+t,\ y = -t,\ z = 3+t$
Points other than P_0: $Q(3,-1,4)$ corresponds to $t=1$; $R(1,1,2)$ corresponds to $t=-1$

(c) Vector equation: $(x,y,z) = (0,0,0) + t(3,2,-3) = t(3,2,-3)$
Parametric equations: $x = 3t,\ y = 2t,\ z = -3t$
Points other than P_0: $Q(-3,-2,3)$ corresponds to $t=-1$; $R(6,4,-6)$ corresponds to $t=2$

9. A point-normal form is $3(x+1) + 2(y+1) + (z+1) = 0$. The corresponding general equation is $3x + 2y + z = -6$.

15. First note that the points $P(1,2,4)$, $Q(1,-1,6)$, $R(1,4,8)$, all lie on the vertical plane $x = 1$. This is the general equation of the plane. To find a vector equation, we observe that the points which are on this plane are exactly those points which are of the form $(x,y,z) = (1,t_1,t_2)$ where $-\infty < t_1, t_2 < \infty$. This corresponds to the parametric equations $x = 1$, $y = t_1$, $z = t_2$, or to the vector equation

$$(x,y,z) = (1,0,0) + t_1(0,1,0) + t_2(0,0,1)$$

25. The plane $x+y+z=0$ has normal vector $\mathbf{n} = (1,1,1)$; thus any line perpendicular to the plane must be parallel to $\mathbf{n}$. Parametric equations for the line that passes through the point $P(2,0,1)$ and is parallel to $\mathbf{n} = (1,1,1)$ are

$$x = 2+t, \quad y = t, \quad z = 1+t$$

D5. (a) True. The line is parallel to $\mathbf{n} = (a,b,c)$, and this is a normal vector for the plane.

(b) False. For example, the lines with vector equations $(x,y,z) = (t,0,0)$ and $(x,y,z) = (0,s,1)$ are not parallel and do not intersect each other.

(c) True. If $\mathbf{u}+\mathbf{v}+\mathbf{w} = \mathbf{0}$, then $\mathbf{w} = -(\mathbf{u}+\mathbf{v})$; thus $\mathbf{w}$ lies in the plane determined by $\mathbf{u}$ and $\mathbf{v}$.

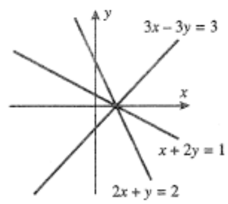
(d) False. If $\mathbf{v} = \mathbf{0}$, then the equation represents a point (the origin); otherwise it will represent a line through the origin.

Section 2.1

3. (a) is linear. (b) is linear if $k \neq 0$. (c) is linear only if $k = 1$.

5. (a), (d), and (e) are solutions; these sets of values satisfy all three equations. (b) and (c) are not solutions.

7. The three lines intersect at the point $(1, 0)$ (see figure). The values $x = 1$, $y = 0$ satisfy all three equations and this is the unique solution of the system.



The augmented matrix of the system is

$$\left[\begin{array}{cc|c} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 3 & -3 & 3 \end{array}\right].$$

Add -2 times row 1 to row 2 and add -3 times row 1 to row 3:

$$\left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & -3 & 0 \\ 0 & -9 & 0 \end{array}\right]$$

Multiply row 2 by $-\frac{1}{3}$ and add 9 times the new row 2 to row 3:

$$\left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

From the last row we see that the system is redundant (reduces to only two equations). From the second row we see that $y = 0$ and, from back substitution, it follows that $x = 1 - 2y = 1$.

15. If $k \neq 6$, then the equations $x - y = 3$, $2x - 2y = k$ represent nonintersecting parallel lines and so the system of equations has no solution. If $k = 6$, the two lines coincide and so there are infinitely many solutions: $x = 3 + t$, $y = t$, where $-\infty < t < \infty$.

17. The augmented matrix of the system is $\left[\begin{array}{cc|c} 3 & -2 & -1 \\ 4 & 5 & 3 \\ 7 & 3 & 2 \end{array}\right]$.

21. A system of equations corresponding to the given augmented matrix is:

$$\begin{aligned} 2x_1 \qquad\quad &= 0 \\ 3x_1 - 4x_2 &= 0 \\ x_2 &= 1 \end{aligned}$$

27.
$$\begin{aligned} 2x + 3y + z &= 7 \\ 2x + y + 3z &= 9 \\ 4x + 2y + 5z &= 16 \end{aligned}$$

D5. The parabola $y = ax^2 + bx + c$ will pass through the points $(1,1)$, $(2,4)$, and $(-1,1)$ if and only if

$$\begin{aligned} a + b + c &= 1 \\ 4a + 2b + c &= 4 \\ a - b + c &= 1 \end{aligned}$$

Since there is a unique parabola passing through any three non-collinear points, one would expect this system to have exactly one solution.

Section 2.2

1. The matrices (a), (c), and (d) are in reduced row echelon form. The matrix (b) does not satisfy property 4 of the definition, and the matrix (e) does not satisfy property 2.

3. The matrices (a) and (b) are in row echelon form. The matrix (c) does not satisfy property 1 or property 3 of the definition.

8. The possible 3 by 3 reduced row echelon forms are

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and

$$\begin{bmatrix} 0 & 1 & * \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & * & * \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & * & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & * \\ 0 & 1 & * \\ 0 & 0 & 0 \end{bmatrix}$$

with any real numbers substituted for the *'s.

23. The augmented matrix of the system is

$$\begin{bmatrix} 1 & 1 & 2 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{bmatrix}$$

Add row 1 to row 2. Add -3 times row 1 to row 3.

$$\begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & -10 & -2 & -14 \end{bmatrix}$$

Multiply row 2 by -1. Add 10 times the new row 2 to row 3.

$$\begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & -52 & -114 \end{bmatrix}$$

Multiply row 3 by $-\frac{1}{52}$.

$$\begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

Add 5 times row 3 to row 2. Add -2 times row 3 to row 1.

$$\begin{bmatrix} 1 & 1 & 0 & 4 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

Add -1 times row 2 to row 1.

$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

Thus the solution is $x_1 = 3$, $x_2 = 1$, $x_3 = 2$.

24. The augmented matrix of the system is

$$\begin{bmatrix} 2 & 2 & 2 & 0 \\ -2 & 5 & 2 & 1 \\ 8 & 1 & 4 & -1 \end{bmatrix}$$

Multiply row 1 by $\frac{1}{2}$. Add 2 times the new row 1 to row 2. Add -8 times the new row 1 to row 3.

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 7 & 4 & 1 \\ 0 & -7 & -4 & -1 \end{bmatrix}$$

Multiply row 2 by $\frac{1}{7}$. Add 7 times the new row 2 to row 3.

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & \frac{4}{7} & \frac{1}{7} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Add -1 times row 2 to row 1.

$$\begin{bmatrix} 1 & 0 & \frac{3}{7} & -\frac{1}{7} \\ 0 & 1 & \frac{4}{7} & \frac{1}{7} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Finally, assigning an arbitrary value to the free variable x_3, the solution set is represented by the parametric equations

$$x_1 = -\tfrac{1}{7} - \tfrac{3}{7}t, \quad x_2 = \tfrac{1}{7} - \tfrac{4}{7}t, \quad x_3 = t$$

28. The augmented matrix of the system is

$$\begin{bmatrix} 3 & 2 & -1 & -15 \\ 5 & 3 & 2 & 0 \\ 3 & 1 & 3 & 11 \\ 6 & -4 & 2 & 30 \end{bmatrix}$$

and the reduced row echelon form of this matrix is

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Thus the system is inconsistent.

34. **(a)** There are more unknowns than equations in this homogeneous system; thus there are infinitely many nontrivial solutions.

(b) The second equation is a multiple of the first. Thus the system reduces to only one equation in two unknowns and there are infinitely many solutions.

46. The augmented matrix of the system is

$$\begin{bmatrix} 1 & 1 & 7 & -7 \\ 2 & 3 & 17 & -16 \\ 1 & 2 & a^2+1 & 3a \end{bmatrix}$$

This reduces to

$$\begin{bmatrix} 1 & 1 & 7 & -7 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & a^2-9 & 3a+9 \end{bmatrix}$$

The last row corresponds to $(a^2-9)z = 3a+9$. If $a=-3$ this becomes $0=0$, and the system will have infinitely many solutions. If $a=3$, then the last row corresponds to $0=18$; the system is inconsistent. If $a \neq \pm 3$, then $z = \frac{3a+9}{a^2-9} = \frac{3}{a-3}$ and, from back substitution, y and x are uniquely determined as well; the system has exactly one solution.

50. This system is linear in the variables $X = x^2$, $Y = y^2$, and $Z = z^2$, with augmented matrix

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & -1 & 2 & 2 \\ 2 & 1 & -1 & 3 \end{bmatrix}$$

The reduced row-echelon form for this matrix is

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

It follows from this that $X=1$, $Y=3$, and $Z=2$; thus $x=\pm 1$, $y = \pm\sqrt{3}$, and $z = \pm\sqrt{2}$.

D5. **(a)** At most three (the number of rows in the matrix).

(b) At most five (if B is the zero matrix). If B is not the zero matrix, then there are at most 4 free variables ($5-r$ where r is the number of non-zero rows in a row echelon form).

(c) At most three (the number of rows in the matrix).

D7. **(a)** False. For example, $x+y+z=0$ and $x+y+z=1$ are inconsistent.

(b) False. If there is more than one solution then there are infinitely many solutions.

(c) False. If the system is consistent then, since there is at least one free variable, there will be infinitely many solutions.

(d) True. A homogeneous system always has (at least) the trivial solution.

D8. (a) True. For example $\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$ can be reduced to either $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ or $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

(b) False. The reduced row echelon form of a matrix is unique.

(c) False. The appearance of a row of zeros means that there was some redundancy in the system. But the remaining equations may be inconsistent, have exactly one solution, or have infinitely many solutions. All of these are possible.

(d) False. There may be redundancy in the system. For example, the system consisting of the equations $x+y=1$, $2x+2y=2$, and $3x+3y=3$ has infinitely many solutions.

Homework 2
Solutions to Instructor's Questions

A1. Refer to Figure 2.1.2 of Anton & Busby.

(a) First of all, we should start with a consistent linear system. Let $\mathbf{p} = (x_0, y_0, z_0)$ be a solution to the system and let $\mathbf{n}_1$, $\mathbf{n}_2$, and $\mathbf{n}_3$ be normal vectors for the three planes. Then the solution set is the set of all solutions $\mathbf{r} = (x, y, z)$ satisfying

$$\mathbf{n}_1 \cdot (\mathbf{r} - \mathbf{p}) = \mathbf{0} \quad (1)$$
$$\mathbf{n}_2 \cdot (\mathbf{r} - \mathbf{p}) = \mathbf{0} \quad (2)$$
$$\mathbf{n}_3 \cdot (\mathbf{r} - \mathbf{p}) = \mathbf{0} \quad (3)$$

To avoid the situation with three coincident planes, we should have at least one pair of non-parallel normal vectors, say, $\mathbf{n}_1$ and $\mathbf{n}_2$.

In order for the solution set to be a line rather than a point, the direction vector, $\mathbf{v}$, for the line should be orthogonal to each of the normal vectors for the three planes. This implies that the vectors $\mathbf{n}_1$, $\mathbf{n}_2$, and $\mathbf{n}_3$ should be coplanar (they lie in the plane orthogonal to $\mathbf{v}$), and so we should take $\mathbf{n}_3$ to be a linear combination of $\mathbf{n}_1$ and $\mathbf{n}_2$,

$$\mathbf{n}_3 = a_1\mathbf{n}_1 + a_2\mathbf{n}_2$$

(If either $a_1 = 0$ or $a_2 = 0$, two of the planes are coincident.)

We can generate one example setting $\mathbf{p} = (0, 0, 0)$, $\mathbf{n}_1 = (1, 0, 0)$, $\mathbf{n}_2 = (0, 1, 0)$, and $\mathbf{n}_3 = \mathbf{n}_1 + \mathbf{n}_2 = (1, 1, 0)$:

$$\begin{aligned} x &= 0 \\ y &= 0 \\ x + y &= 0 \end{aligned}$$

These planes intersect in the line $(x, y, z) = (0, 0, t)$.

(b) Continuing the discussion from part (a), if we want the solution set to be a single point then we should choose the third plane to have a normal vector $\mathbf{n}_3$ that does not lie in the same plane as $\mathbf{n}_1$ and $\mathbf{n}_2$. This will ensure that the the third plane will intersect the line of intersection of the first and second plane in a single point. Thus we choose $\mathbf{n}_3$ such that $\mathbf{n}_3$ cannot be expressed as a linear combination of $\mathbf{n}_1$ and $\mathbf{n}_2$.

We can generate one example setting $\mathbf{p} = (0,0,0)$, $\mathbf{n}_1 = (1,0,0)$, $\mathbf{n}_2 = (0,1,0)$, and $\mathbf{n}_3 = (0,0,1)$:

$$\begin{aligned} x &= 0 \ (yz\text{-plane}) \\ y &= 0 \ (xz\text{-plane}) \\ z &= 0 \ (xy\text{-plane}) \end{aligned}$$

(c) Perhaps the most straightforward way of constructing three planes with no common intersection is to construct at least one pair of distinct parallel planes.

Here is one example with three distinct parallel planes:

$$\begin{aligned} x + 2y + 3z &= -1 \\ x + 2y + 3z &= 0 \\ x + 2y + 3z &= 1 \end{aligned}$$

Note that these planes are parallel since $\mathbf{n} = (1,2,3)$ is a normal vector to all of them.

Can you find an example of three planes with no common intersection where no two of the planes are parallel?

A2. (a) First note that an inhomogeneous system may be inconsistent, in which case there are 0 solutions. Now consider the case where the system is consistent. Since there are more variables than equations the total number of variables exceeds the number of pivot variables (there is at most one pivot variable for each equation). This implies that in this case there is at least one free variable, giving rise to infinitely many solutions. There are no other possibilities. Therefore an inhomogenous linear system with more variables than equations can have either 0 solutions or infinitely many solutions.

(b) Again, an inhomogenous system may be inconsistent, in which case there are 0 solutions. Now consider the case where the system is consistent. If there are no free variables then the system has a unique solution. On the other hand, it is also possible that the system may have free variables, since row reducing the augmented matrix can lead to zero rows. In the latter case there are infinitely many solutions. There are no other possibilities. Thus an inhomogenous linear system with more equations than variables can have 0, 1, or infinitely many solutions.