Assignment 2

Applied Linear Algebra
Math 232 - D100 (Fall 2012)

Quiz date: Friday, Sept 21

Complete this assignment by Wednesday in your *homework journal*. This will give you plenty of time to make sure you understand the material before the quiz at the end of Wednesday's class. Quiz questions will be taken from items 1 (Questions from Textbook) or 2 (Additional Questions) below.
Some suggestions for using your homework journal are:

- Do rough work on scratch paper.
- If you find one solution, try to find another (a simpler solution may reveal itself).
- When you find a solution, try to see it as a whole without all the little details.
- Do questions in order and clearly label question and section numbers.
- Grade your own assignment when solutions are posted. Catch your mistakes now when the stakes are low rather than making them on exams.

To obtain maximum marks on the quiz, your answer should be in a form that another student could understand without undue effort: a poorly expressed but correct result is not sufficient.

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1. **Questions from textbook:**

section	question	done	checked	corrected	study MT	study final	type [1]	comment
1.2	1(a)(c)	☐	☐	☐	☐	☐	CD	norm, normalization, scaling
	5(b)(c)	☐	☐	☐	☐	☐	CD	norm, scaling
	8	☐	☐	☐	☐	☐	RE	scaling
	9	☐	☐	☐	☐	☐	CD	calculating inner product
	11(a)	☐	☐	☐	☐	☐	CD	calculating distance
	13(a)(c)	☐	☐	☐	☐	☐	CD	calculating angles
	15	☐	☐	☐	☐	☐	RE	inner products and angles
	24	☐	☐	☐	☐	☐	CD	checking orthonormality
	25	☐	☐	☐	☐	☐	CE	turning definitions into equations
	D6	☐	☐	☐	☐	☐	HL	distance and geometry
1.3	5(a)(c)	☐	☐	☐	☐	☐	CD	vector and parametric equations
	7	☐	☐	☐	☐	☐	CD	vector and parametric equations
	9	☐	☐	☐	☐	☐	CD	point-normal equation
	15	☐	☐	☐	☐	☐	RE	equations for a plane
	25	☐	☐	☐	☐	☐	CE	combining concepts
	D5	☐	☐	☐	☐	☐	CE	test your grasp of the concepts
2.1	3	☐	☐	☐	☐	☐	CD	definition of linear equation
	5(b)(d)	☐	☐	☐	☐	☐	CD	definition of solution
	7	☐	☐	☐	☐	☐	CD	geometry of linear systems
	15	☐	☐	☐	☐	☐	CE	solution sets
	17	☐	☐	☐	☐	☐	CD	augmented matrix form
	21	☐	☐	☐	☐	☐	CD	augmented matrix form
	27	☐	☐	☐	☐	☐	RE	translating word problems
	D5	☐	☐	☐	☐	☐	CE	fitting curves with linear algebra
2.2	1	☐	☐	☐	☐	☐	CD	echelon forms
	3	☐	☐	☐	☐	☐	CD	echelon forms
	8	☐	☐	☐	☐	☐	CE	echelon forms
	23	☐	☐	☐	☐	☐	RE	solving a linear system
	24	☐	☐	☐	☐	☐	RE	solving a linear system
	28	☐	☐	☐	☐	☐	RE	solving a linear system
	34	☐	☐	☐	☐	☐	RE	solving a linear system
	46	☐	☐	☐	☐	☐	RE	number of solutions
	50	☐	☐	☐	☐	☐	CE	reducing to a linear system
	D5	☐	☐	☐	☐	☐	CE	pivot variables and free variables
	D7	☐	☐	☐	☐	☐	CE	number of solutions
	D8	☐	☐	☐	☐	☐	CE	echelon forms

[1] See the legend on last page of this assignment for what these acronyms mean.

2. **Additional questions:**
These questions are made up by your instructor and may require a blend of ideas that we have encountered so far in the course. They are similar to exam style questions in that it is not entirely clear what part of the text is directly related to solving the problem. You will have to decide what tools/techniques are required.

A1. The solutions (x, y, z) of a single linear equation

$$ax + by + cz = d$$

form a plane in $\mathbb{R}^3$ (when a, b, and c are not all zero). Construct sets of three linear equations in x, y, and z whose graphs
(a) intersect in a single line;
(b) intersect in a single point;
(c) have no points in common.

A2. (a) If we have an inhomogeneous system of linear equations with more variables than equations, what are the possible sizes of the solution sets?
(b) If we have an inhomogeneous system of linear equations with more equations than variables, what are the possible sizes of the solution sets?

For each part of A2, you should give an exhaustive list of possible sizes. Justify each value on your list, and if one of the possible solution set sizes for a linear system is not on your list, explain why it is not.

3. **Extra-Practice Questions:**
   Try these questions for some more practice. The more practice you get the better you will understand the material and the better you will do on quizzes and exams.

   - Section 1.2: 16, 18, 19, 21, 27, 28, 29, D2, D3, D4, D7, D9, D10
   - Section 1.3: 1, 3, 11, 13, 21, 23, 29, D2(b), D2(a), 31, 33, D4
   - Section 2.1: 9, 11, 13, D8, D9
   - Section 2.2: 25, 26, 33, 45, 47, D1, D6, D9

Legend (for "type" of question):
RE = Routine Exercise: This is something you should be able to do in your sleep ;-). Your goal is to be able to answer these questions quickly and accurately every time. These form the foundations of your skill set.
TC = Time Challenge: Speed and accuracy are important factors in solving this type of routine exercise. Try to do these exercises within the time limit, usually 5 minutes. If you need more time than that, its o.k., but keep practicing! Solving these routine exercises provides a foundation for solving more involved problems, and is essential in performing well on quizzes and exams.
WP = Word Problem: Translating words into expressions (also known as modeling): Master this skill now, we will be using this all term.
CD = Concepts and Definitions: These questions relate to your understanding of the "new language" we are introducing. They should help you remember the important definitions and theorems.
CE = Concepts and Explorations: This indicates a question which is testing your understanding of the fundamentals. It is not a routine exercise since the solution process may not be obvious at first glance. It may take a little bit of thought to figure out what to do, don't be afraid to play around with some ideas. You'll learn more by making mistakes and taking routes which lead to dead ends. You must be able to do these types of questions to succeed in learning this material.
HL = Higher Level Understanding: This indicates a question which is testing understanding at a higher level. These questions will require more thought than a RE or CE so don't be discouraged if you can't see how to do this immediately. Perseverance and playing around with ideas is the key to these questions. Understanding this material at this level is an expected outcome of this course.
CM = Computer of Computational Device: This indicates a question in which a computer or calculator is needed.

Selected Hints & Answers:
1.2: 8. $k = \pm 1$
1.2: 24. You need to check three dot products and three lengths.
1.2: D6. For (a), recall the geometric definition of a circle. For (b) and (c), start with your answer for (a), but now use inequalities.
1.3: D5. (a) T, (b) F, (c) T, (d) F
2.1: D5. The system is

$$\begin{aligned} a + b + c &= 1 \\ 4a + 2b + c &= 4 \\ a - b + c &= 1 \end{aligned}$$

We should expect one solution: three non-collinear points define a unique parabola.

2.2: 8. The 3×3 reduced row echelon matrices are $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$; $\begin{pmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 0 \end{pmatrix}$ for any $a, b \in \mathbb{R}$; $\begin{pmatrix} 1 & c & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ for any $c \in \mathbb{R}$; $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$; $\begin{pmatrix} 1 & d & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ for any $d, e \in \mathbb{R}$; $\begin{pmatrix} 0 & 1 & f \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ for any $f \in \mathbb{R}$; $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$; and $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$.
2.2: 24 $x_1 = -\frac{1}{7} - \frac{3}{7}t$, $x_2 = \frac{1}{7} - \frac{4}{7}t$, $x_3 = t$ for $-\infty < t < \infty$.
2.2: 26 inconsistent
2.2: 28 inconsistent
2.2: 34 (a) has nontrivial solutions, (b) has nontrivial solutions
2.2: 46 no solution if $a = 3$, infinitely many solutions if $a = -3$, one solution for any other value of a
2.2: 50 there are eight solutions embracing all possible combinations of $x = \pm 1$, $y = \pm\sqrt{3}$, and $z = \pm\sqrt{2}$
2.2: D1 there is exactly one solution
2.2: D5 (a) 3, (b) 5, (c) 3
2.2: D6 (a) 3, (b) 3, (c) 3
2.2: D7 (a) F, (b) F, (c) F, (d) T
2.2: D8 (a) T, (b) F, (c) F, (d) F
2.2: D9 there are eighteen solutions embracing all possible combinations with $\alpha \in \{0, \pi, 2\pi\}$, $\beta \in \{\pi/2, 3\pi/2\}$, and $\gamma \in \{0, \pi, 2\pi\}$. No contradiction: Theorem 2.2.1 only speaks of linear equations.