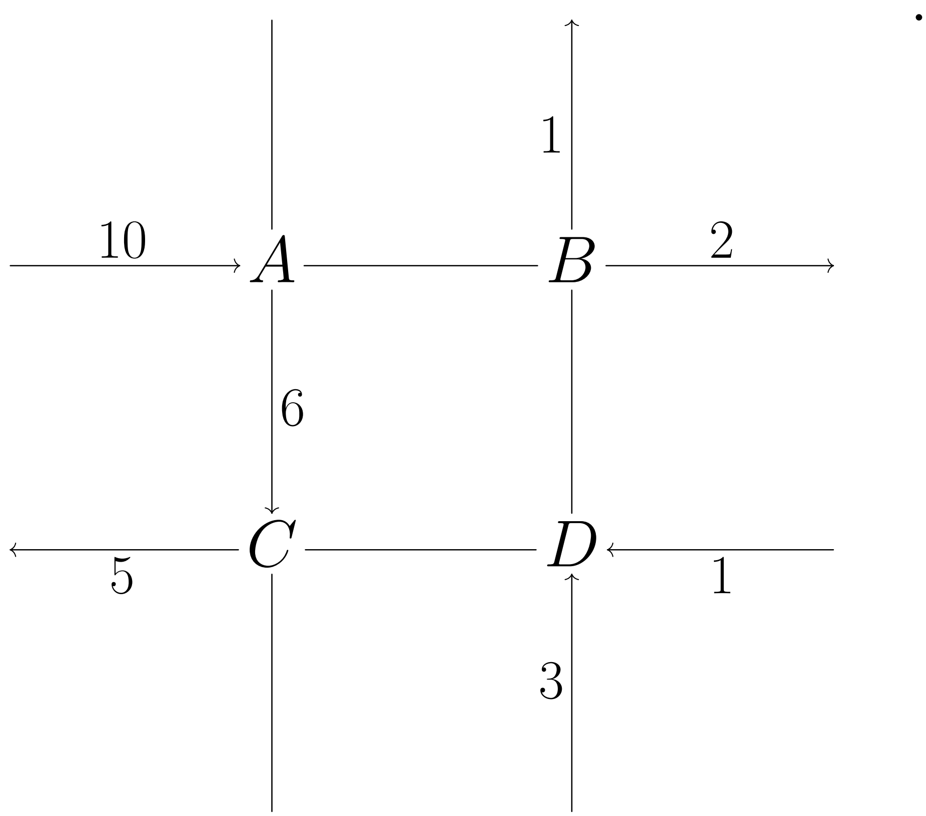
Lecture 6: Applications

1 Flows in Networks

A power delivery network has nodes and branches. For each branch a number indicates the current (flow of electricity). What are feasible currents?



Assign (arbitrarily) directions to the unknown nodes, and give each a variable for its flow.

If, when we solve, our variable turns out to be negative, it means the flow is going opposite to the direction we guessed.

To keep there from being accumulation or loss at the nodes, we need the flow into each node to equal the flow out. This gives one equation for each node.

We can put this in matrix form and solve to get all feasible flows in the network.

$$\left(\begin{array}{c|c} \quad & \quad \end{array}\right)$$

$$\left(\begin{array}{rrrrr|r} 1 & -1 & 0 & 0 & 0 & -4 \\ 0 & 1 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 & -1 & -1 \end{array}\right)$$

$$\left(\begin{array}{rrrrr|r} 1 & -1 & 0 & 0 & 0 & -4 \\ 0 & 1 & 0 & 0 & -1 & -2 \\ 0 & 0 & 1 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 & -1 & -1 \end{array}\right)$$

$$\left(\begin{array}{rrrrr|r} 1 & 0 & 0 & 0 & -1 & -6 \\ 0 & 1 & 0 & 0 & -1 & -2 \\ 0 & 0 & 1 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 & -1 & -1 \end{array}\right)$$

So the general solution is

E.g., one particular solution with $x_5 = 7$

New problem: suppose these were one-way streets, and suppose that the directions we chose at the beginning were chosen because those were the actual directions of the streets.

Then each x_j must be **nonnegative**.

Suppose we react copper metal with nitric acid

$$Cu + \quad HNO_3 \longrightarrow \quad CuN_2O_6 + \quad NO + \quad H_2O$$

To balance this equation, introduce one variable for each reactant or product.

Then write one equation for each type of atom that indicates that there are equal numbers of that kind of atom on each side of the chemical equation.

We can then put our equations in matrix form

$$\left(\begin{array}{rrrrr|r} 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -2 & 0 \\ 0 & 1 & -2 & -1 & 0 & 0 \\ 0 & 3 & -6 & -1 & -1 & 0 \end{array}\right)$$

Note that this is a homogeneous system!

Also note that all the coefficients in the first two columns are positive, while all those in third, fourth, and fifth column are negative.

$$\left(\begin{array}{rrrrr|r} 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -2 & 0 \\ 0 & 0 & -2 & -1 & 2 & 0 \\ 0 & 3 & -6 & -1 & -1 & 0 \end{array}\right)$$

$$\left(\begin{array}{rrrrr|r} 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -2 & 0 \\ 0 & 0 & -2 & -1 & 2 & 0 \\ 0 & 0 & -6 & -1 & 5 & 0 \end{array}\right)$$

$$\left(\begin{array}{rrrrr|r} 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -2 & 0 \\ 0 & 0 & -2 & -1 & 2 & 0 \\ 0 & 0 & 0 & 2 & -1 & 0 \end{array}\right)$$

$$\left(\begin{array}{rrrrr|r} 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -2 & 0 \\ 0 & 0 & 1 & 1/2 & -1 & 0 \\ 0 & 0 & 0 & 2 & -1 & 0 \end{array}\right)$$

$$\left(\begin{array}{rrrrr|r} 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -2 & 0 \\ 0 & 0 & 1 & 1/2 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1/2 & 0 \end{array}\right)$$

$$\left(\begin{array}{rrrrr|r} 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -2 & 0 \\ 0 & 0 & 1 & 0 & -3/4 & 0 \\ 0 & 0 & 0 & 1 & -1/2 & 0 \end{array}\right)$$

$$\left(\begin{array}{rrrrr|r} 1 & 0 & 0 & 0 & -3/4 & 0 \\ 0 & 1 & 0 & 0 & -2 & 0 \\ 0 & 0 & 1 & 0 & -3/4 & 0 \\ 0 & 0 & 0 & 1 & -1/2 & 0 \end{array}\right)$$

Which gives a solution of

And we let x_5 be the smallest positive number that makes all the other variables whole numbers.

Check the balance:

$$\mathrm{Cu} + \quad \mathrm{HNO_3} \longrightarrow \quad \mathrm{CuN_2O_6} + \quad \mathrm{NO} + \quad \mathrm{H_2O}$$

3 Fitting Polynomials to Sets of Points

In generality, if someone gives you a collection of points (x_1, y_1), (x_2, y_2), ..., (x_n, y_n), you can find a polynomial of degree $n-1$ or less passing through all of them.

Write the polynomial with unknown coefficients as

$$y = a_0 + a_1 x + \ldots + a_{n-1} x^n$$

You want to find a_0, a_1, ..., a_{n-1}: these are unknowns and your equations are

$$\begin{aligned} a_0 + a_1 x_1 + \ldots + a_{n-1} x_1^{n-1} &= y_1 \\ a_0 + a_1 x_2 + \ldots + a_{n-1} x_2^{n-1} &= y_2 \\ &\ldots \\ a_0 + a_1 x_n + \ldots + a_{n-1} x_n^{n-1} &= y_n \end{aligned}$$

or in augmented matrix form

$$\left(\begin{array}{cccc|c} 1 & x_1 & \cdots & x_1^{n-1} & y_1 \\ 1 & x_2 & \cdots & x_2^{n-1} & y_2 \\ \vdots & \vdots & & \vdots & \vdots \\ 1 & x_n & \cdots & x_n^{n-1} & y_n \end{array}\right)$$

You have already worked an example in Problem D5 of Section 2.1.