

MATH 1107 (Winter 2013) Test 4 (Make-up)

Instruction: This test is out of 35 points. The duration of this test is 50 minutes. Non-programmable calculators are permitted. No other aids allowed. Answer all questions.

1. Let $A \in \mathbb{R}^{3 \times 3}$ be the matrix $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & -2 \\ 0 & 1 & -1 \end{bmatrix}$ and $B = A + I_3$ where I_3 denotes the 3×3 identity matrix. Give the answer to each of the following. Justification not required and not marked.

(a) (2 points) The rank of A .

Answer. 3.

(b) (2 points) The number of distinct eigenvalues of A .

Answer. 3.

(c) (2 points) $\det(A^3)$.

Answer. 64.

(d) (2 points) The adjugate matrix of B .

Answer. $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 6 \\ 0 & -3 & 3 \end{bmatrix}$

(e) (2 points) The inverse of B .

Answer. $\begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$

2. Let $A = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 1 & -2 \\ -1 & 1 & 1 & 2 \end{bmatrix}$ be a matrix over the real numbers.

(a) (10 points) Give a basis for the column space of A . Show your work.

Solution. Row-reducing A results in the row-equivalent matrix in reduced-row echelon form: $\begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$. Columns 1, 2, and 3 are pivot columns. The corresponding columns from A give a basis for the column space of A .

(b) (5 points) Give a basis for the nullspace of A .

Solution. If one looks at the row-reduced matrix in part (a), one sees that all the solutions to $Ax = 0$ are given by $t \begin{bmatrix} 2 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ for all $t \in \mathbb{R}$. Hence, the single vector $\begin{bmatrix} 2 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ form a basis for the nullspace of A .

3. (10 points) Let U denote the vector space of polynomials in x over the real numbers with degree at most 2. Let $T : U \rightarrow \mathbb{R}^2$ be a linear transformation given by

$$T(ax^2 + bx + c) = \begin{bmatrix} a + c \\ b - c \end{bmatrix} \text{ for all } a, b \in \mathbb{R}.$$

Find a basis for the kernel of T . Justify your answer.

Solution. The kernel of T consists of all the polynomials $ax^2 + bx + c$ such that $a + c = 0$ and $b - c = 0$. Setting $c = s$, we have the polynomials $-sx^2 + sx + s$, or equivalently, $s(-x^2 + x + 1)$. Hence, a basis for the kernel of T is given by the single vector $-x^2 + x + 1$.