ECONOMICS 325 - Sample Mid Term – Instructor: Whistler

Question 1 [**5 points**] State whether the following claim is True or False. To get points you must clearly explain your answer.

If a sample of data is a random sample then the sample mean will be identical to the population mean.

Question 2 [**5 points**] For a sample of data, in what situation will the variance, standard deviation and coefficient of variation (measured as a proportion) all be identical and non-zero ?

Question 3 [**5 points**] Distinguish among joint probability, marginal probability and conditional probability. Give an example to illustrate your answer.

Question 4 The events E_1, E_2, E_3 are mutually exclusive and collectively exhaustive and each is equally likely to occur. Let A be some other event with the following probabilities:

$$P(A|E_1) = 0.2 \qquad P(A|E_2) = 0.3 \qquad P(A|E_3) = 0.4$$

(a) [**5 points**] Find P(A). (Show the calculations required - a final numerical answer is not needed).

(b) [**5 points**] Draw a Venn diagram to illustrate your answer to (a). Give an accurate picture and assign clear labels.

(c) [**5 points**] Find $P(E_1|A)$. As a general rule -- show all answers using correct notation.

Question 5 An instructor assigns grades of 40, 50, 60, 70 or 80 **only**. Let X be a discrete random variable that represents student grades. The probability function is:

x	40	50	60	70	80
P(x)	0.05	0.20	0.25	.30	0.20

(a) [**5 points**] Find $P(X = 55)$ and $P(X \leq 55)$.

(b) [**5 points**] Suppose two students are chosen at random. What is the probability that at least one of these students obtains a grade of 80 ?

(c) [**5 points**] Find E(X). (Show the numerical calculations - a final numerical answer is not required).

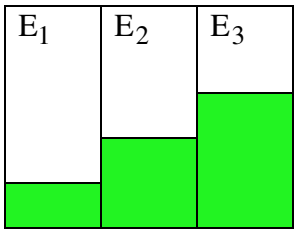
(d) [**5 points**] Find the variance of X.

SELECTED ANSWER GUIDE

Question 4 (a) Since the events are equally likely $P(E_1) = P(E_2) = P(E_3) = 1/3$

$P(A) = P(A|E_1)P(E_1) + P(A|E_2)P(E_2) + P(A|E_3)P(E_3) = (0.2 + 0.3 + 0.4) / 3$

(b)



The shaded area represents event A.

(c) $P(E_1|A) = \dfrac{P(A|E_1)P(E_1)}{P(A)} = \dfrac{0.2/3}{P(A)}$

Question 5 (a) $P(X = 55) = 0$ (55 is impossible) and $P(X \le 55) = 0.05 + 0.20 = 0.25$

(b) Denote events A - student 1 gets 80 and B - student 2 gets 80. Assume the events are independent. Then

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.2 + 0.2 - (.2)(.2)$

Alternative method: $1 - P(\overline{A})P(\overline{B}) = 1 - (0.8)^2$

(c) $E(X) = 40\ (.05) + 50\ (.20) + 60\ (.25) + 70\ (.30) + 80\ (.20)$

(d) $E(X^2) = 40^2\ (.05) + 50^2\ (.20) + 60^2\ (.25) + 70^2\ (.30) + 80^2\ (.20)$

$Var(X) = E(X^2) - [E(X)]^2$