Question #1

Loop: $[L_a s + R_a] I_a(s) + E_b(s) = E_a(s)$ where $E_b(s) = k_m s \theta_m(s)$, $I_a(s) = \frac{1}{k_m} T_m(s)$

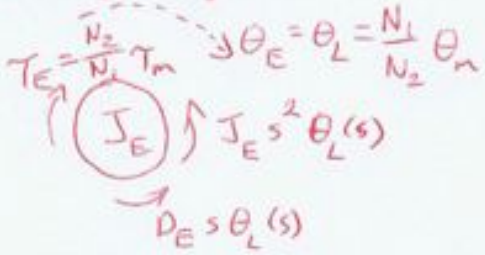
$\therefore [L_a s + R_a] \frac{1}{k_m} T_m(s) + k_m s \theta_m(s) = E_a(s)$

Reflect motor to load

$J_E = \left(\frac{N_2}{N_1}\right)^2 (J_a + J_1) + J_2 + J_L$

$D_E = \left(\frac{N_2}{N_1}\right)^2 (D_a + D_1) + D_2 + D_L$

Free-body diagram



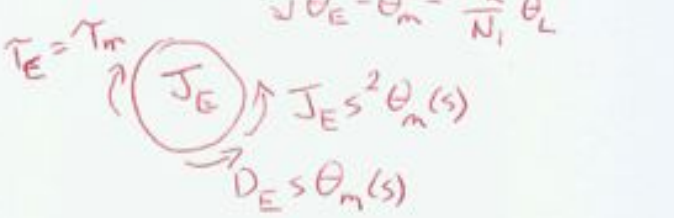
$[J_E s^2 + D_E s] \theta_L(s) = \left(\frac{N_2}{N_1}\right) T_m(s)$

Reflect load to motor

$J_E = J_a + J_1 + \left(\frac{N_1}{N_2}\right)^2 [J_2 + J_L]$

$D_E = D_a + D_1 + \left(\frac{N_1}{N_2}\right)^2 [D_2 + D_L]$

Free-body diagram



$[J_E s^2 + D_E s] \theta_m(s) = T_E(s) = T_m(s)$

$\therefore [L_a s + R_a] \frac{1}{k_m} [J_E s^2 + D_E s] \frac{N_2}{N_1} \theta_L(s) + k_m s \frac{N_2}{N_1} \theta_L(s) = E_a(s)$

$\therefore [L_a s + R_a] \frac{1}{k_m} \frac{N_1}{N_2} [J_E s^2 + D_E s] \theta_L(s) + k_m s \frac{N_2}{N_1} \theta_L(s) = E_a(s)$

$$\frac{E_a(s)}{\theta_L(s)} = \frac{1}{\frac{1}{k_m}\frac{N_1}{N_2}[J_E s^2 + D_E s][L_a s + R_a] + k_m s \frac{N_2}{N_1}}$$

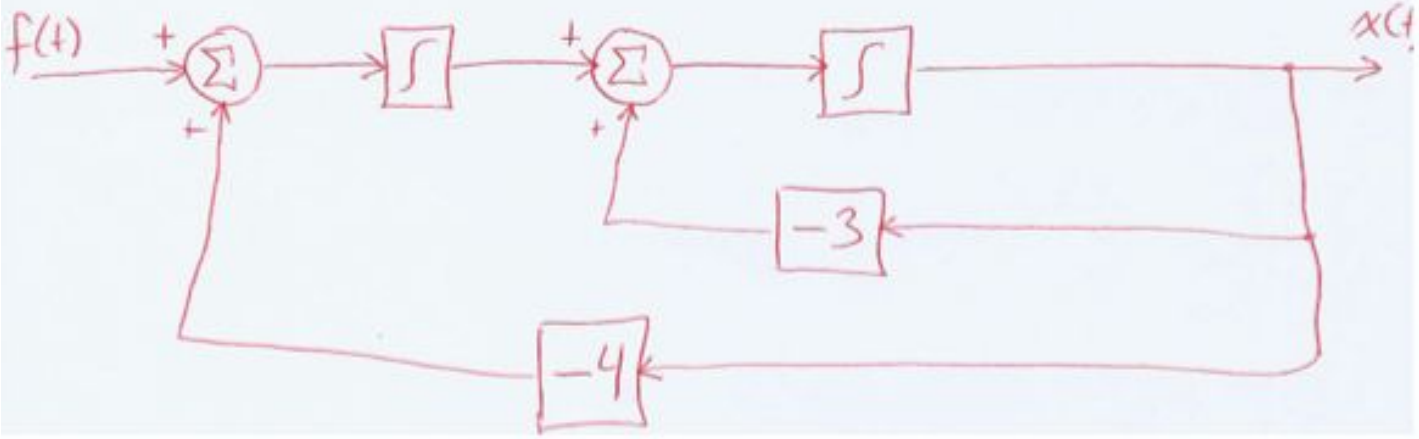
or $\frac{1}{\frac{1}{k_m}\frac{N_2}{N_1}[\cdots][\cdots] + [\cdots]}$

Question #2
(a)

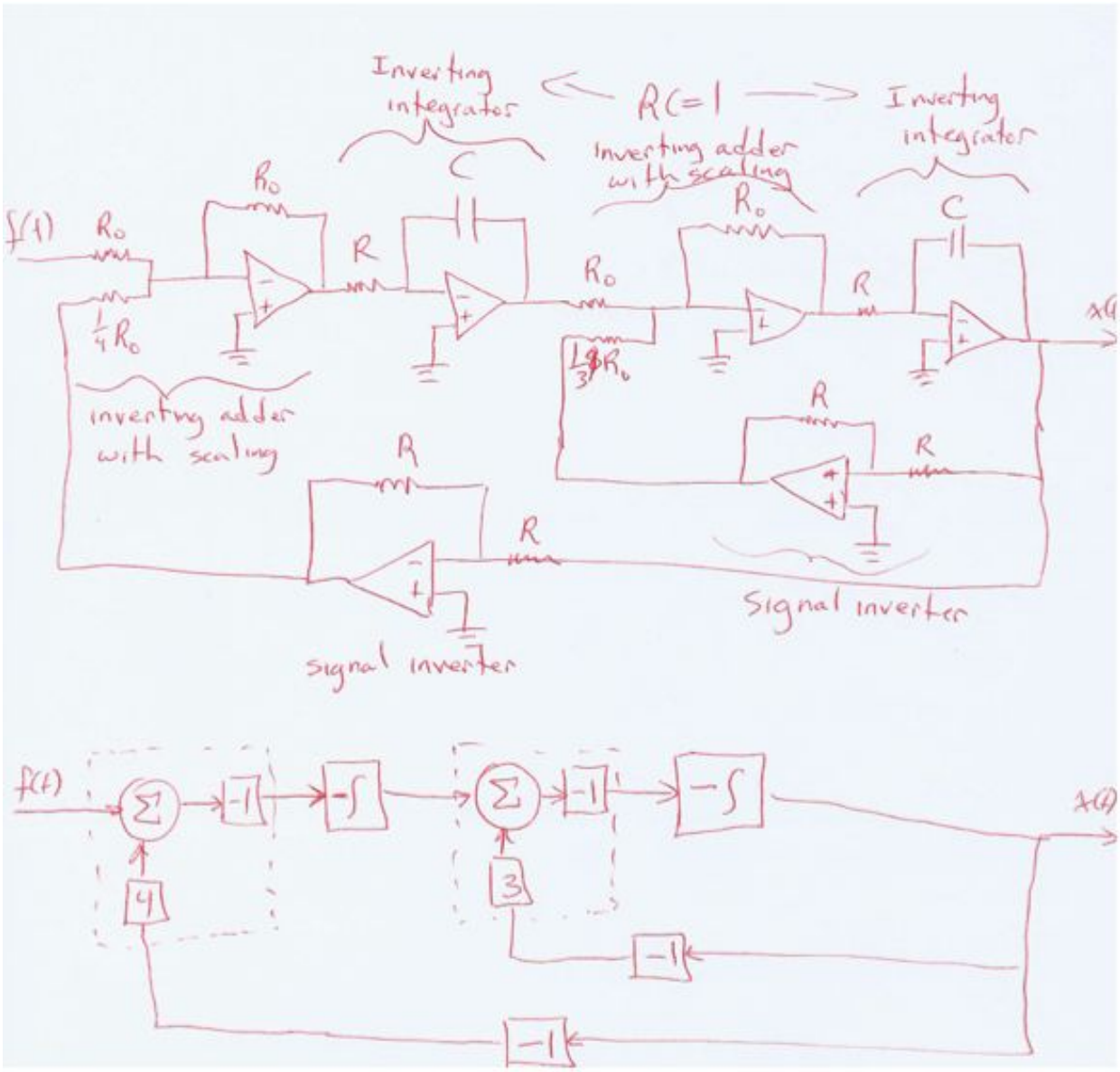
$$\ddot{x}(t) = -3\dot{x}(t) + f(t) - 4x(t)$$

$$\dot{x}(t) = -3x(t) + \int_{-\infty} f(\lambda) - 4x(\lambda)\, d\lambda$$

$$x(t) = \int_{-\infty}^{t} \left[-3x(\lambda) + \int_{-\infty}^{\lambda} \left[f(\tau) - 4x(\tau) \right] d\tau \right] d\lambda$$



Question #2
(b)

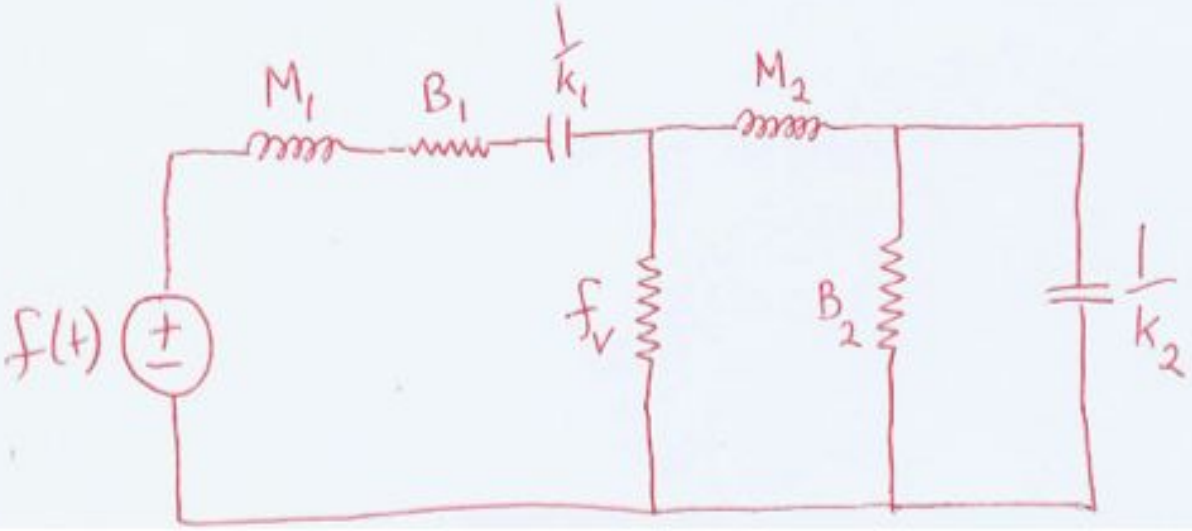

Inverting integrator
R(=1
Inverting integrator
Inverting adder with scaling
f(t)
R0
C
R
1/4 R0
inverting adder with scaling
1/3 R0
signal inverter
Signal inverter
f(t)
Σ
-1
-5
4
3
x(t)

Question #3

$$\left[M_1 s^2 + (B_1 + f_v)s + k_1\right] X_1(s) - f_v s X_2(s) = F(s)$$

$$-f_v s X_1(s) + \left[M_2 s^2 + (B_2 + f_v)s\right] X_2(s) - B_2 s X_3(s) = 0$$

$$-B_2 s X_2(s) + \left[B_2 s + k_2\right] X_3(s) = 0$$



Question #4

a)

$$\ddot{y}(t) + 5\dot{y}(t) + 6y(t) = 5\dot{x}(t) + 6x(t)$$

b)

$$X(s) = \frac{1}{s}$$

$$y(\infty) = \lim_{s \to 0} s \frac{5s+6}{s^2+5s+6} \frac{1}{s} = \frac{6}{6} = 1$$

c)

$$X(s) = 1$$

$$Y(s) = \frac{5s+6}{(s+2)(s+3)}(1) = \frac{C_1}{s+2} + \frac{C_2}{s+3}$$

$$C_1 = \frac{5s+6}{s+3}\Big|_{s=-2} = \frac{-10+6}{-2+3} = \frac{-4}{1} = -4$$

$$C_2 = \frac{5s+6}{s+2}\Big|_{s=-3} = \frac{-15+6}{-3+2} = \frac{-9}{-1} = 9$$

$$y(t) = -4e^{-2t}u(t) + 9e^{-3t}u(t)$$