SREE 3001

Sustainable and Renewable Energy Sources

Modules 1 to 4

Preparation for Final Exam

Carleton University

Department of Mechanical and Aerospace Engineering

Final Exam

- the final exam will take place on Monday, December 10th in Tory Building 340 at 9 am
- 3 hour examination
- there will be no examination booklet (write your answers in the exam paper – same format as midterm)
- a formula sheet will be provided with your exam (a copy of the formula sheet will be uploaded on WebCT)
- solutions to tutorial questions, lecture notes or laptops are not permitted

Getting Ready for Final Exam

- lecture notes and examples from class (tests will cover all 4 modules)
  - module 1 – energy use fundamentals / conventional power generation
    - energy use and environmental impact
    - power generation by fossil fuel and nuclear
  - module 2 – renewable power generation part 1
    - hydroelectric power
    - wind power
  - module 3 – renewable power generation part 2
    - solar electricity
    - solar thermal
  - module 4 – renewable power generation part 3
    - biomass
    - geothermal energy

FINAL EXAM PREPARATION

Getting Ready for Final Exam

- tutorial questions to look over
  - electricity consumption for office
  - power plant schematic (and others looked at in class)
  - nuclear power plant operation and costs
  - load duration curve
  - heat recovery steam generator and pinch point
  - pumped hydro storage
  - wind farm
  - sunchart
  - declination angle
  - solar collectors (efficiency equation)
  - ground source heat pumps

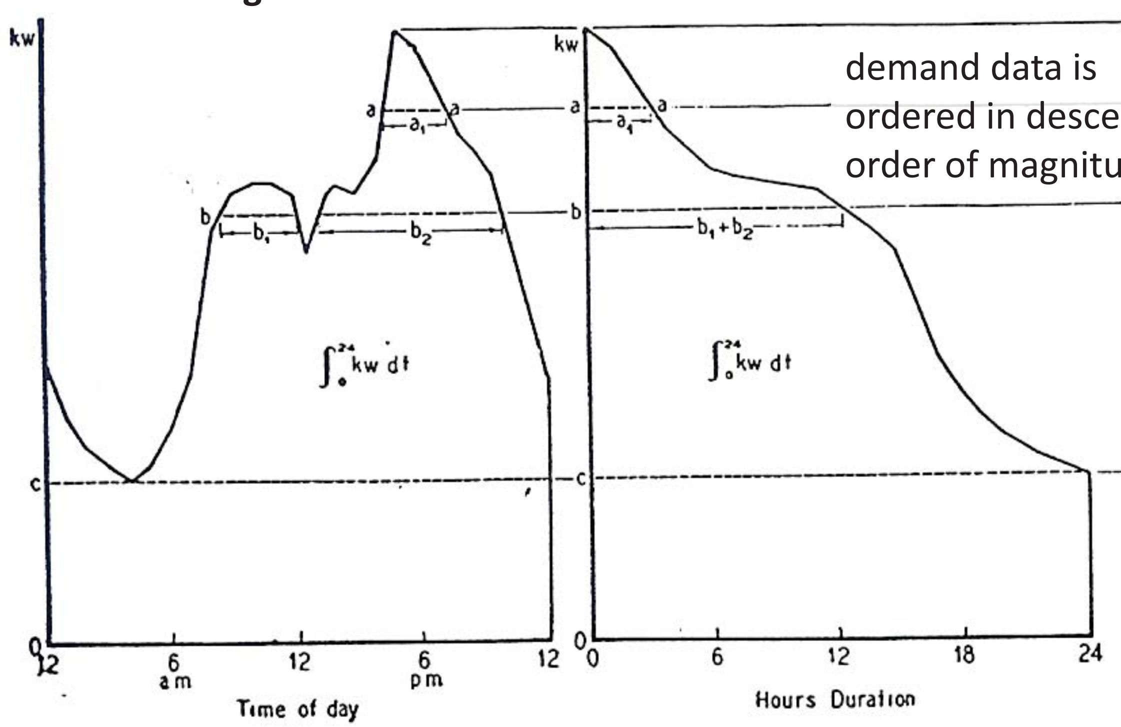
Prof. C. A. Cruickshank, Carleton University

Final Exam Format

- similar to midterm
  - short answer questions (definitions, fill in the blank (e.g., indicate components on a diagram), possible true or false questions (+ discuss your answer))
  - multi-part questions (similar to tutorial questions)
- note: there will be no questions related to any video documentaries from class

Load Duration Curves

Chronological Load Curve **Load Duration Curve**



demand data is ordered in descending order of magnitude

Load Factor

- there are also a number of other indices used to describe the characteristics of a utility load
- **Load Factor (LF)** : the average ratio of average load to maximum demand (max load); this value is typically less than 1 and is the measure of the effective use of the power station

$$\text{Load Factor} = \frac{L_{avg}}{L_{max}} = \frac{E\,/\,h}{L_{max}}$$

utilities want a high load factor (a flat load curve = 1, minimum cost per unit generated)

where:

L_{avg} = average load for period

L_{max} = peak load for period

E = total energy under the load cure, e.g., the integral

h = total number of hours in period

POWER GENERATION BY FOSSIL FUEL AND NUCLEAR

Capacity Factor and Utilization Factor

- **Capacity Factor (CF)** : the ratio of actual energy produced to the maximum possible energy that could have been produced on a given period; this indicates the reserve capacity of a plant

$$\text{Capacity Factor} = \frac{L_{avg}}{Cap}$$

don't want power plant to be oversized but want some extra capacity (e.g., CF ~0.5)

where:

Cap = rated capacity of plant

- **Utilization Factor (UF)** : the ratio of the maximum demand (max load) to the maximum possible energy that could be produced (capacity of plant) on a given period

$$\text{Utilization Factor} = \frac{L_{max}}{Cap}$$

(e.g., LF ~ 0.85)

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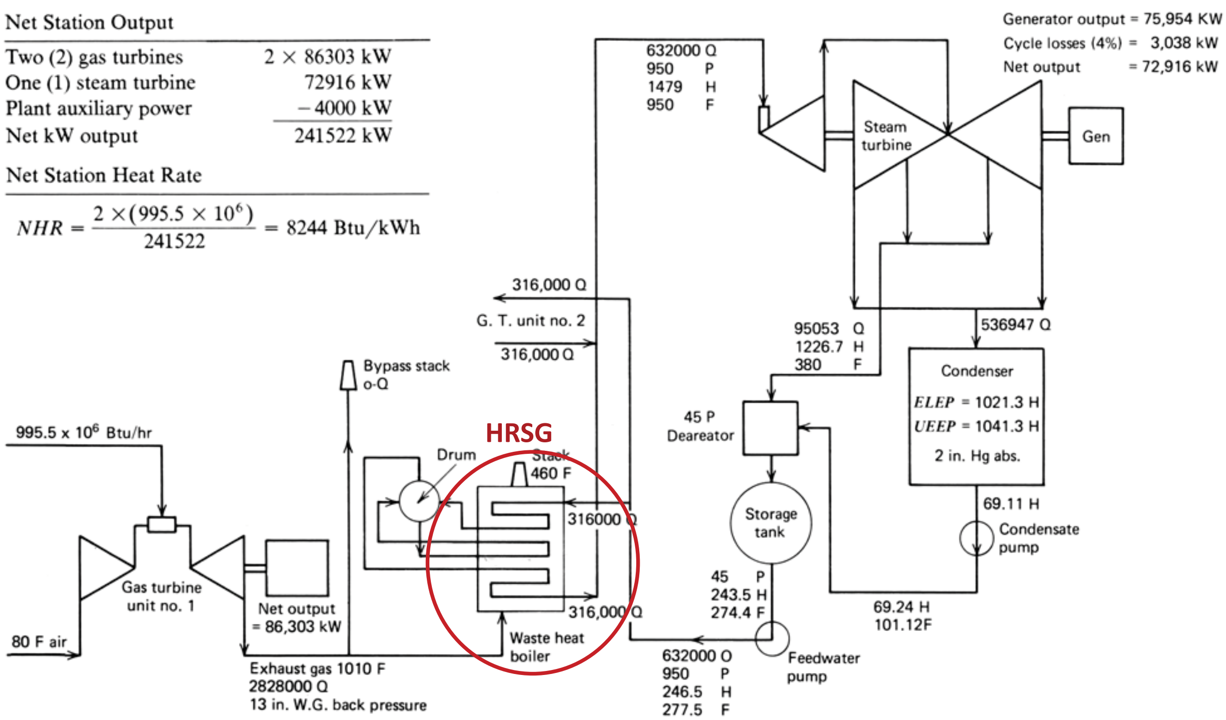
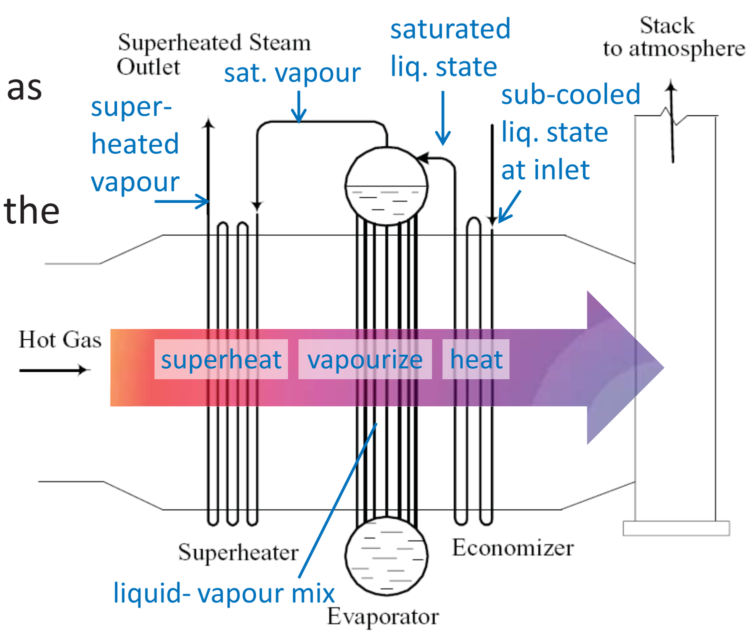
Combined Cycle Heat Balance



Heat Recovery Steam Generator (HRSG)

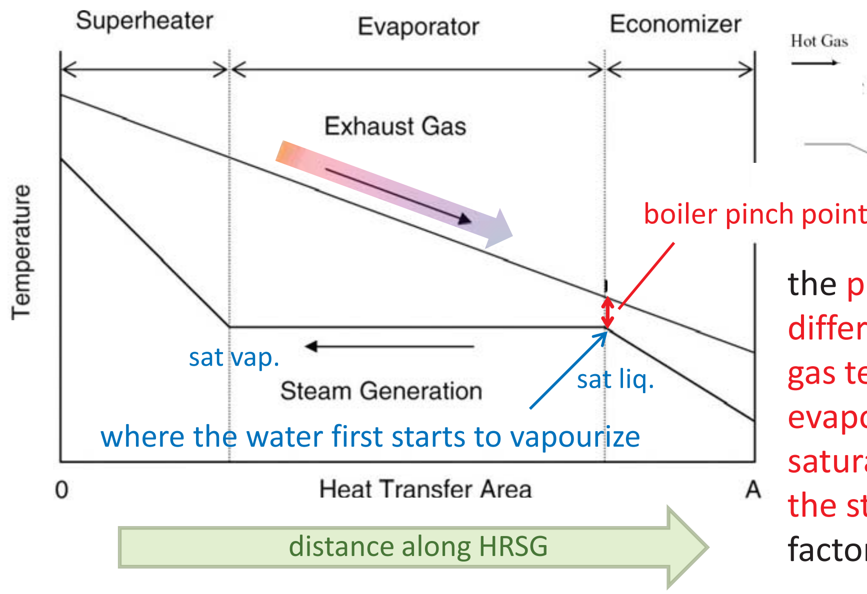
- HRSGs (sometimes called waste heat boilers) often use the exhaust gas from a gas turbine to generate steam

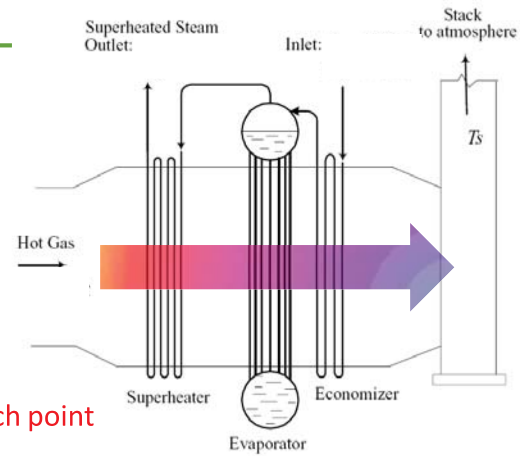
- hot exhaust enters at high temperature and is cooled as it traverses the HRSG
- the energy extracted from the hot gas is used to heat, vaporize (evaporate) and superheat the water
- a HRSG includes sections identified as an economizer, an evaporator and a superheated



flow arrangement shown is counter flow

Temperature Profile in Heat Recovery Boiler





the pinch point is defined as the difference between the exhaust gas temperature leaving the evaporator section and the saturated liquid temperature of the steam, and is the limiting factor in its overall performance

ideally , the lower the pinch point, the more heat recovered, but this calls for more surface area and consequently increases the backpressure and cost

Source: B.K. Hodge, Alternative Energy Systems and Applications

Wind Speed vs. Height from Earth's Surface

- wind speed is related to height by:

$$\frac{V_2}{V_1} = \left(\frac{H_2}{H_1}\right)^{\alpha}$$

exponent α depends on the type and roughness of the terrain

$$v(h_2) = v(h_1) \cdot \frac{\ln\left(\frac{h_2 - d}{z_0}\right)}{\ln\left(\frac{h_1 - d}{z_0}\right)}$$

recall from last class

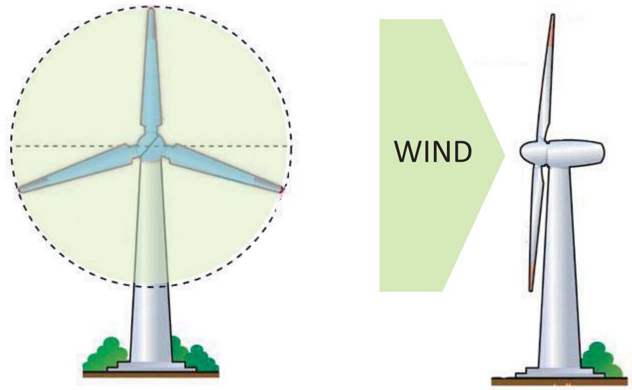
Ground class	Roughness length z_0 in m
1 – Sea	0.0002
2 – Smooth	0.005
3 – Open	0.03
4 – Open to rough	0.1
5 – Rough	0.25
6 – Very rough	0.5
7 – Closed	1
8 – Inner city	2

$v(h_2)$ = wind at desired height
$v(h_1)$ = wind at known height
z_o = surface roughness
d=boundary layer displacement in ground classes 6-8

Power and Energy in Wind

- recall from last class, the theoretical Kinetic Power in a moving air stream is given by

$$\boxed{KP = \tfrac{1}{2}\rho A V^3} = \tfrac{1}{2} m V^2$$



where:

KP = power in wind (W)

ρ = air density (kg/m^3)

(1.225 kg/m^3 at 15°C, 1.0132

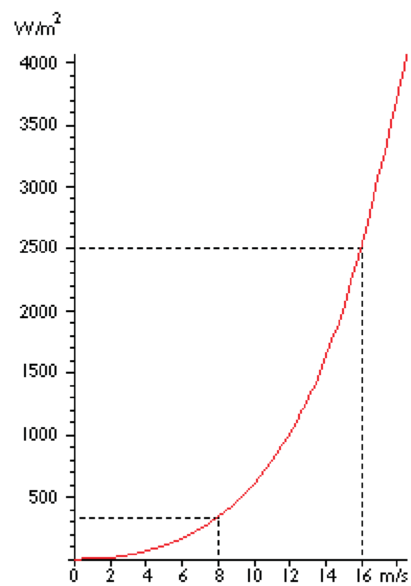
A= cross-sect. flow area (m^2)

V = wind speed (m/s)

Note: Power is proportional to the CUBE of wind speed

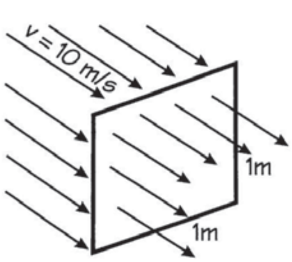
Power and Energy in Wind

- the average Kinetic Power, however, is not linearly proportional to the average wind speed!



$$\boxed{KP_{ave} = 6/\pi\,(\tfrac{1}{2}\rho A V_{ave}^{3})}$$

- the term 6/π accounts for the distribution of wind speed, and hence kinetic power with wind with time

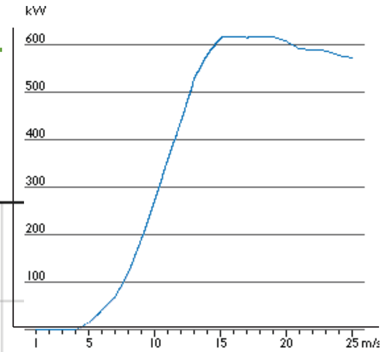


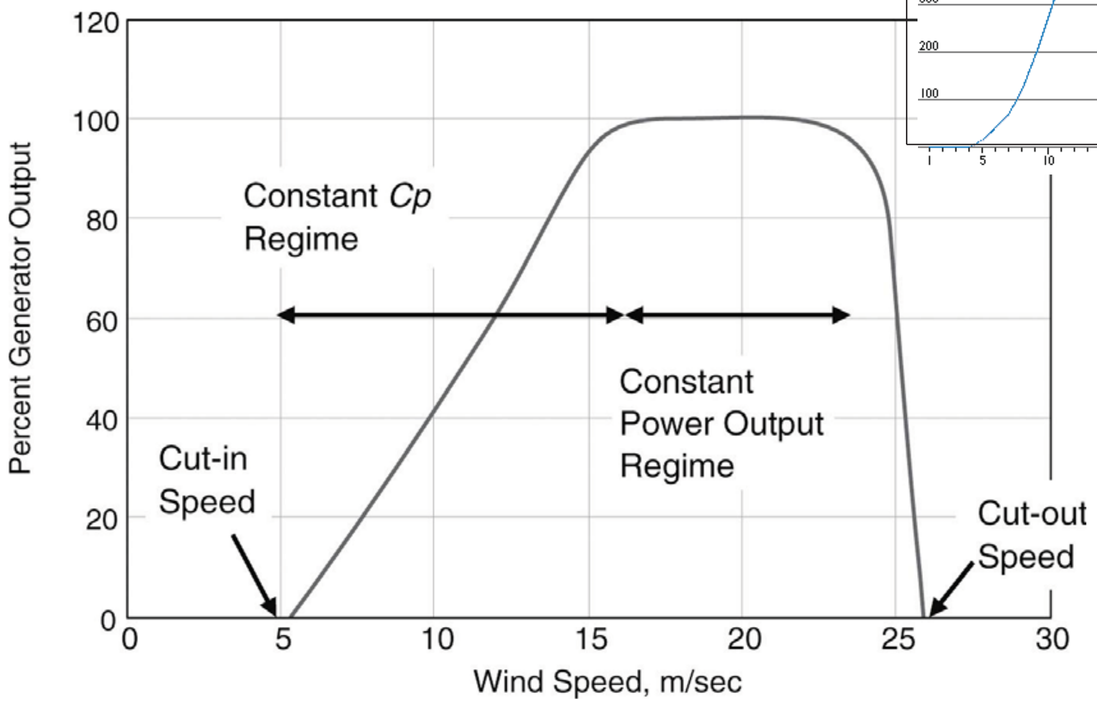
e,g., a 10 m/s wind (avg) blows through a 1m^2 window

$KP_{ave} = 6/\pi$ (½ x 1.225 kg/m^3 x 1 m^2 x 10^3)

= 1170 W = 1.17 kW

Typical Regimes of Turbine Speed Control

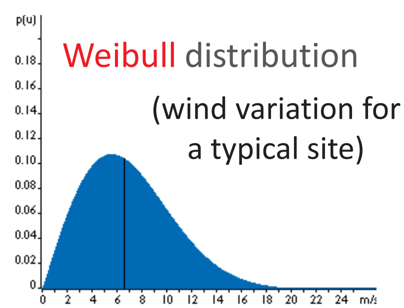


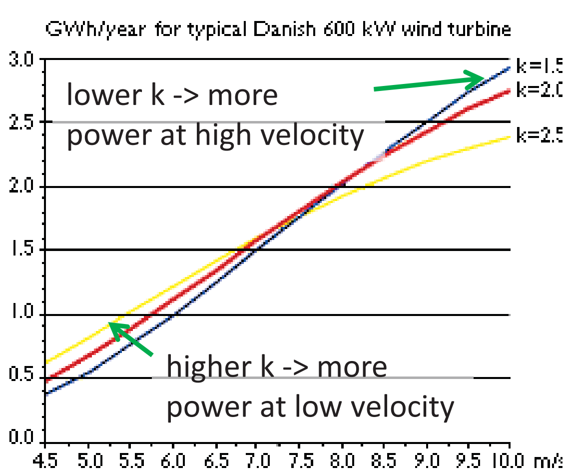


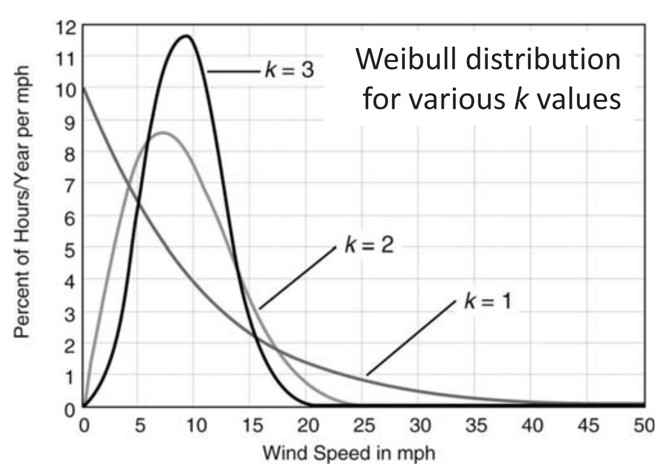
Source: 'Alternative Energy Systems and Applications', B.K. Hodge

Weibull Distribution

- the shape of the curve is determined by a so-called shape parameter, *k*, i.e., *k*= 2
- shape parameter is site dependent
- the larger the shape factor, the closer the distribution comes to being Gaussian



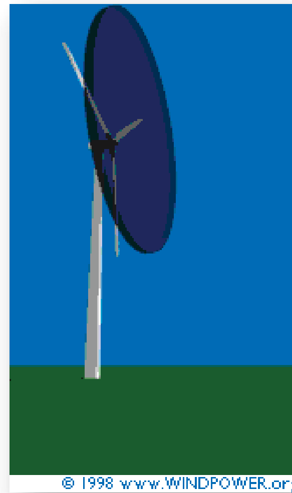




Power and Energy in Wind

- recall from last class that the power coefficient (*Cp*) is defined at the power extracted (P_T) divided by the available power (theoretical power) of the wind stream

$$C_P = \frac{2P_T}{\rho_a A_T V^3}$$



wind slows down as it flows through turbine

- the maximum value of the power coefficient (as determined by Bernoulli's Law) is known as the BETZ LIMIT and is 59.2% of the inflowing kinetic power ↓

the maximum amount of power that can be extracted from the available wind → most important single metric used in characterizing a wind turbine

Power and Energy in Wind

- recall, the average Kinetic Power is expressed as:

$$KP_{ave} = 6/\pi\ (½\ \rho\ AV_{ave}^3)$$

where the term 6/π accounts for the distribution of wind speed

- on average, good wind turbines extract about half of the theoretical maximum or 30% of the power in the airstream that passes through the rotor, thus

$$KP_{ave} = 0.5 \times 0.59 \times 6/\pi\ (½\ \rho\ AV_{ave}^3)$$

$$KP_{ave} \approx 0.3 \times 6/\pi\ (½\ \rho\ AV_{ave}^3)$$

$$KP_{ave} \approx 0.35\ \ AV_{ave}^3$$

Power and Energy in Wind

Power Example: Simplified Calculation

Consider a turbine with a 20 m diameter rotor installed on a 40 m high tower at a site where the average wind speed is 7 m/s at the standard height of 10 m above ground.

The average wind speed at the center (hub) of the rotor is

$$\frac{V_2}{V_1} = \left(\frac{H_2}{H_1}\right)^{\alpha}$$

$$V_{hub} = V_{10}\left(\frac{h_{hub}}{10}\right)^{0.16} = 7 \times (4)^{0.16} = 8.74 \text{ m/s}$$

and the average power output would be approximately

$$KP_{ave} \approx 0.35\ AV_{ave}^{3}$$

Calculate V_{hub} (m/s) and KP_{ave} (kW) and annual energy output (kWh/y); assume $\alpha = 0.16$

$$P_{ave} = 0.35 \times \frac{\pi}{4}(20)^2 \times (8.74)^3 = 74{,}000 \text{ watts} = 74\text{kW}$$

Power and Energy in Wind

Power Example: Simplified Calculation

Energy is power multiplied by time. Thus:

$$E(\text{kWh}) = \frac{P(\text{W})}{1000} \times t(\text{h})$$

So, the annual energy production of a wind turbine can be estimated from:

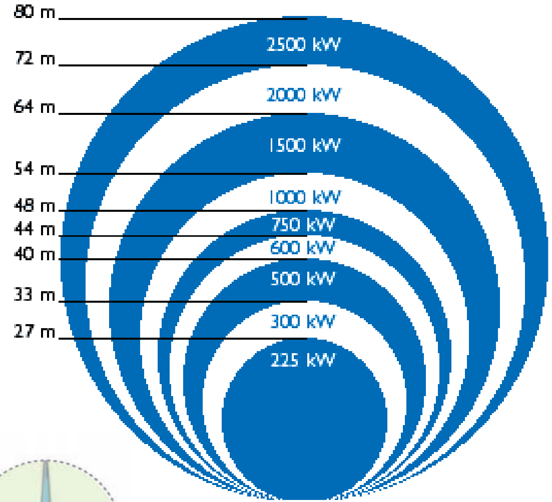
$$AEO = \frac{P_{ave} \times 8760}{1000} = \frac{0.35\ AV_{ave}^{3} \times 8760 \text{ h}}{1000} = 74\text{kW} \times 8760 \text{ h/y}$$

AEO = Annual energy output (kWh/y)
8760 = Number of hours in a year
A = Area swept by wind turbine rotor (m^2)
V_{ave} = Average wind speed (m/s)

answer: the annual energy output would be 650 MWh/y (sufficient for 80 homes)

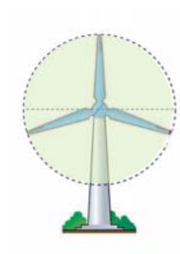
Power and Energy in Wind

- recall, the efficiency of a turbine will vary depending on the rotor cross-sectional flow area
- output of the turbine will effectively increase as the square of the rotor diameter, i.e.,



$$\frac{P_1 = \frac{1}{2}\rho A V^3 = \cancel{\frac{1}{2}}(\cancel{\pi} D^2/\cancel{4})\cancel{V^3}}{P_2 = \frac{1}{2}\rho A V^3 = \cancel{\frac{1}{2}}(\cancel{\pi} D^2/\cancel{4})\cancel{V^3}}$$

$$\boxed{(P_2/P_1) = (D_2/D_1)^2}$$



actual turbine output will vary somewhat due to specific design features and turbine performance

Example

$D_1 = 25$ m, $P_1 = 225$ kW
$D_2 = 50$ m, $P_2 = ?$

$$P_2 = (50\text{ m}/25\text{ m})^2 \times P_1 = 900\text{ kW}$$

Solar Time

- 'Solar Time' is calculated from:

$$\text{Solar Time} - \text{LST} = 4(\text{LSM} - \text{LON}) + \text{ET}$$

where:

- LST is the local standard time
- LSM is the local standard meridian
- LON is the local longitude
- ET is the equation of time given by:

$$ET = 229.2*(0.000075 + 0.001868\cos(B) - 0.032077\sin(B) - 0.014615\cos(2B) - 0.04089\sin(2B))$$

where $B = (n-1)*(360/365)$, n = day of the year

<u>all equations use degrees not radians!</u>

EXAMPLE

At Madison, WI, what is the solar time corresponding to 10:30 AM central time on February 3?

Solar Time – LST = 4(LSM - LON) + ET

In this case:

- LST is the local standard time (10:30)
- LSM is the local standard meridian (90^o W) *
- LON is the local longitude (89.4^o W) *
- ET is the equation of time given by:

* this information would be provided

ET= 229.2*(0.000075 + 0.001868 cos(B) – 0.032077 sin(B) - 0.014615 cos(2B) – 0.04089 sin(2B))

where B = (n-1)*(360/365), n = 34, therefore B = 32.55

SOLAR ENERGY RESOURCE AND AVAILABILITY

EXAMPLE

Using the equation of time,

ET= 229.2*(0.000075 + 0.001868 cos(32.55) – 0.032077 sin(32.55) - 0.014615 cos(2*32.55) – 0.04089 sin(2*32.55))

thus ET = -13.5 minutes

Solar Time – LST = 4(LSM - LON) + ET

recall,

LST is the local standard time (10:30)

LSM is the local standard meridian (90^o W)

LON is the local longitude (89.4^o W)

Solar Time – 10:30 = 4(90 – 89.4) + (-13.5) = -11 minutes

Solar Time = 10:19

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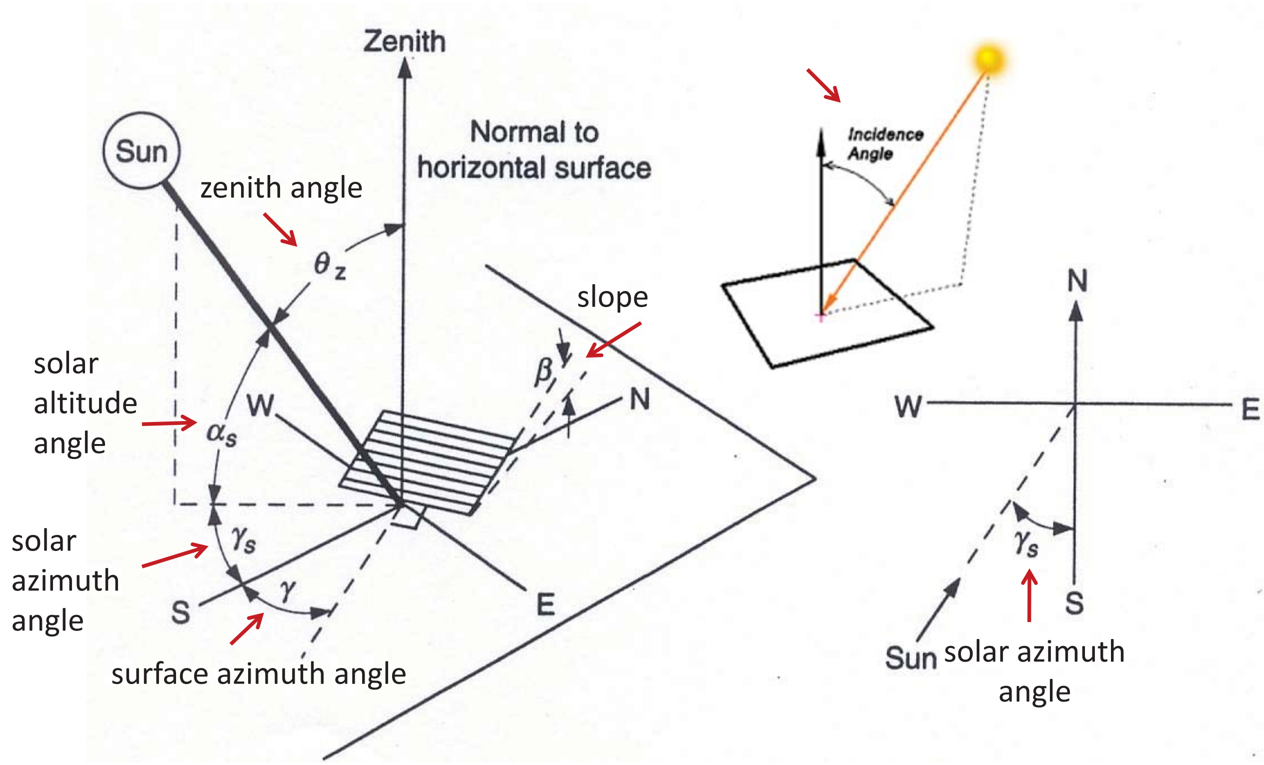
Orientation and Tilt Angle: Geometric Relationships



Source: "Solar Engineering of Thermal Processes", Duffie & Beckman

Geometric Relationships

ϕ **Latitude**, the angular location north or south of the equator, north positive; $-90^o \le \phi \le 90^o$.

δ **Declination**, the angular position of the sun at solar noon (i.e., when the sun is on the local meridian) with respect to the plane of the equator, north positive; $-23.45^o \le \delta \le 23.45^o$.

β **Slope**, the angle between the plane of the surface in question and the horizontal; $0 \le \beta \le 180^o$. ($\beta > 90^o$ means that the surface has a downward facing component.)

γ **Surface azimuth angle**, the deviation of the projection on a horizontal plane of the normal to the surface from the local meridian, with zero due south, east negative, and west positive; $-180^o \le \gamma \le 180^o$

ω **Hour angle**, the angular displacement of the sun east or west of the local meridian due to rotation of the earth on its axis at 15^o per hour, morning negative, afternoon positive.

θ **Angle of incidence**, the angle between the beam radiation on a surface and the normal to that surface.

Source: "Solar Engineering of Thermal Processes", Duffie & Beckman

Geometric Relationships

Additional angles are defined that describe the position of the sun in the sky:

θ_z **Zenith angle**, the angle between the vertical and the line to the sun, i.e., the angle of incidence of beam radiation on a horizontal surface.

α_s **Solar altitude angle**, the angle between the horizontal and the line to the sun, i.e., the complement of the zenith angle.

γ_s **Solar azimuth angle**, the angular displacement from south of the projection of beam radiation on the horizontal plane, shown in Figure 1.6.1. Displacements east of south are negative and west of south are positive.

The declination δ can be found from the equation of Cooper (1969):

$$\delta = 23.45 \sin\left(360 \frac{284 + n}{365}\right) \qquad (1.6.1)$$

Source: "Solar Engineering of Thermal Processes", Duffie & Beckman

Orientation and Tilt Angle: Declination Angle and Day of the Year

Table 1.6.1 Recommended Average Days for Months and Values of n by Months[a]

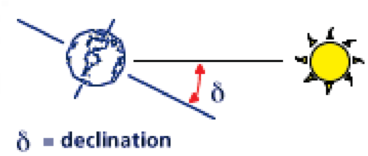
		For the Average Day of the Month		
Month	n for ith Day of Month	Date	n, Day of Year	δ, Declination
January	i	17	17	–20.9
February	$31 + i$	16	47	–13.0
March	$59 + i$	16	75	–2.4
April	$90 + i$	15	105	9.4
May	$120 + i$	15	135	18.8
June	$151 + i$	11	162	23.1
July	$181 + i$	17	198	21.2
August	$212 + i$	16	228	13.5
September	$243 + i$	15	258	2.2
October	$273 + i$	15	288	–9.6
November	$304 + i$	14	318	–18.9
December	$334 + i$	10	344	–23.0

[a] From Klein (1977)

the declination angle:

$$\delta = 23.45 \sin\left(360 \frac{284 + n}{365}\right)$$

angles are to be speci-fied in degrees not radians!!



Source: "Solar Engineering of Thermal Processes", Duffie & Beckman

Orientation and Tilt Angle: Angle of Incidence and Beam Radiation

θ - angle of incidence of beam radiation on surface
ϕ - latitude (position on the earth)
δ - declination (time of year)
ω - hour angle (time of day)
β - slope of surface (w.r.t. the horizontal)
γ - surface azimuth angle (orientation, w.r.t. the meridian)

$$\theta = f(\phi, \delta, \omega, \beta, \gamma)$$

$$\begin{aligned}\cos\theta = \; & \sin\delta \sin\phi \cos\beta \\ & - \sin\delta \cos\phi \sin\beta \cos\gamma \\ & + \cos\delta \cos\omega \cos\phi \cos\beta \\ & + \cos\delta \cos\omega \sin\phi \sin\beta \cos\gamma \\ & + \cos\delta \sin\beta \sin\gamma \sin\omega\end{aligned}$$

the intensity of the beam radiation on a tilted surface or horizontal surface, G_s, is equivalent to the direct normal beam radiation, G_{DN}, multiplied by the cosine of the angle of incidence of beam radiation on the surface:

$$G_s = G_{DN} \cos\theta$$

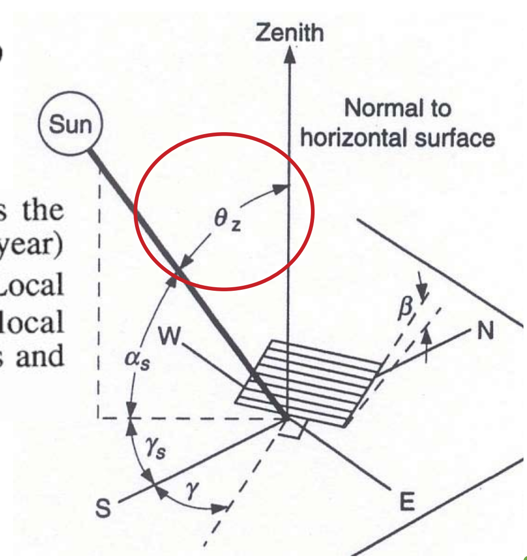
Source: "Solar Engineering of Thermal Processes", Duffie & Beckman

Orientation and Tilt Angle

θ_Z is the zenith angle, the angle between the line to the sun and the normal to a horizontal surface. For a horizontal surface β is zero. All but two of the terms in the above expression disappear.

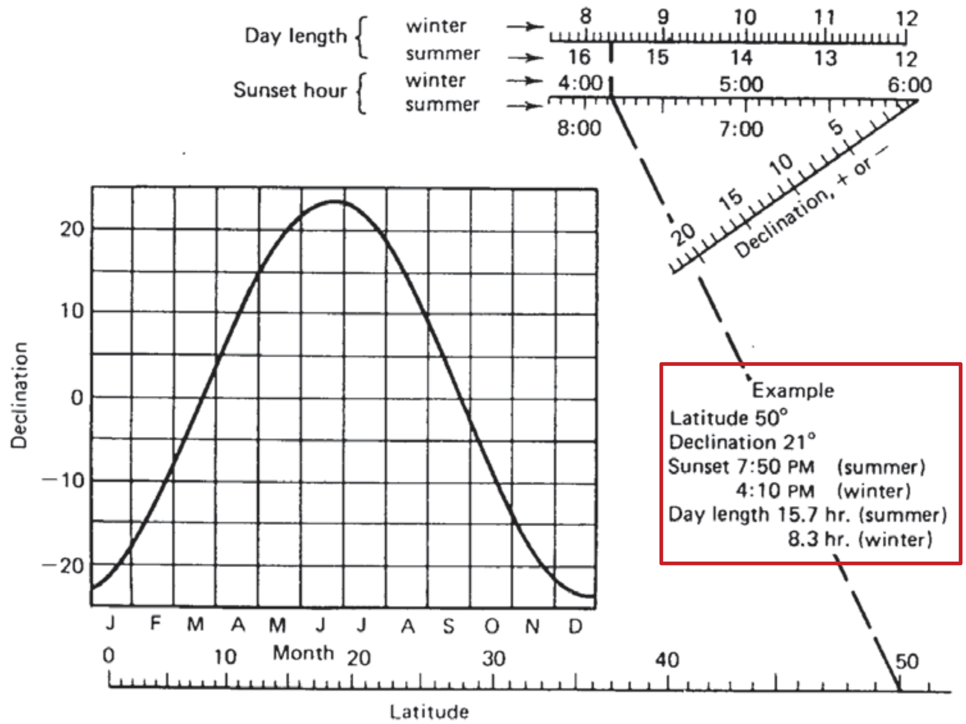
$$\cos\theta_Z = \sin\delta \sin\phi + \cos\delta \cos\omega \cos\phi$$

A zenith angle of -90° is the sunrise angle; +90° is the sunset angle. Knowing declination (i.e., the time of year) and latitude we can then calculate sunrise time (ω_s). Local sunrise time is solar noon (expressed in terms of local time) less the sunset hour angle, converted into hours and minutes.



Source: "Solar Engineering of Thermal Processes", Duffie & Beckman

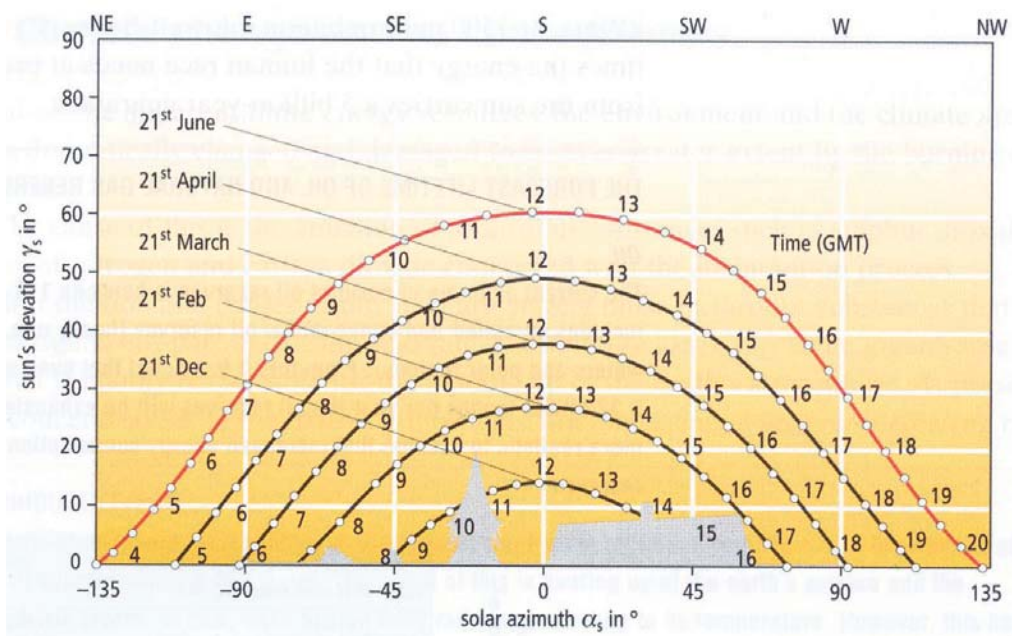
Direction of Beam Radiation: Nomogram



Nomogram to determine time of sunset and day length. Adapted from Whillier

Source: "Solar Engineering of Thermal Processes", Duffie & Beckman

Sun Chart

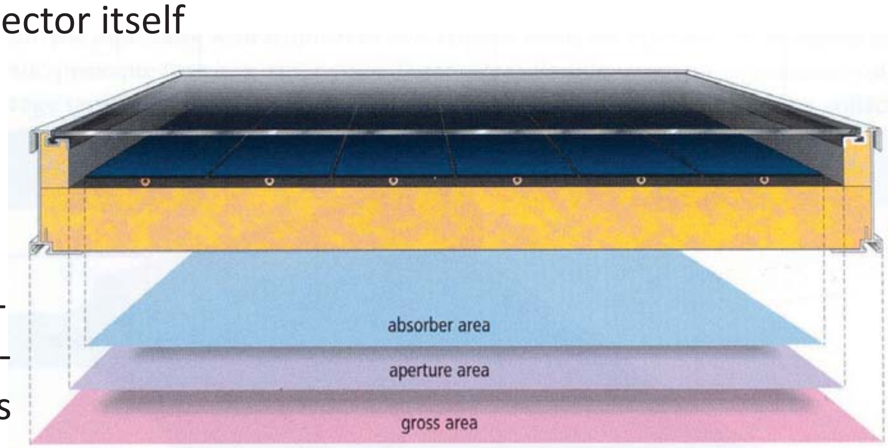


Solar altitude diagram with example silhouettes of objects (for a latitude of about 50°)

Source: "Planning and Installing Solar Thermal Systems", James & James/Earthscan, London, UK

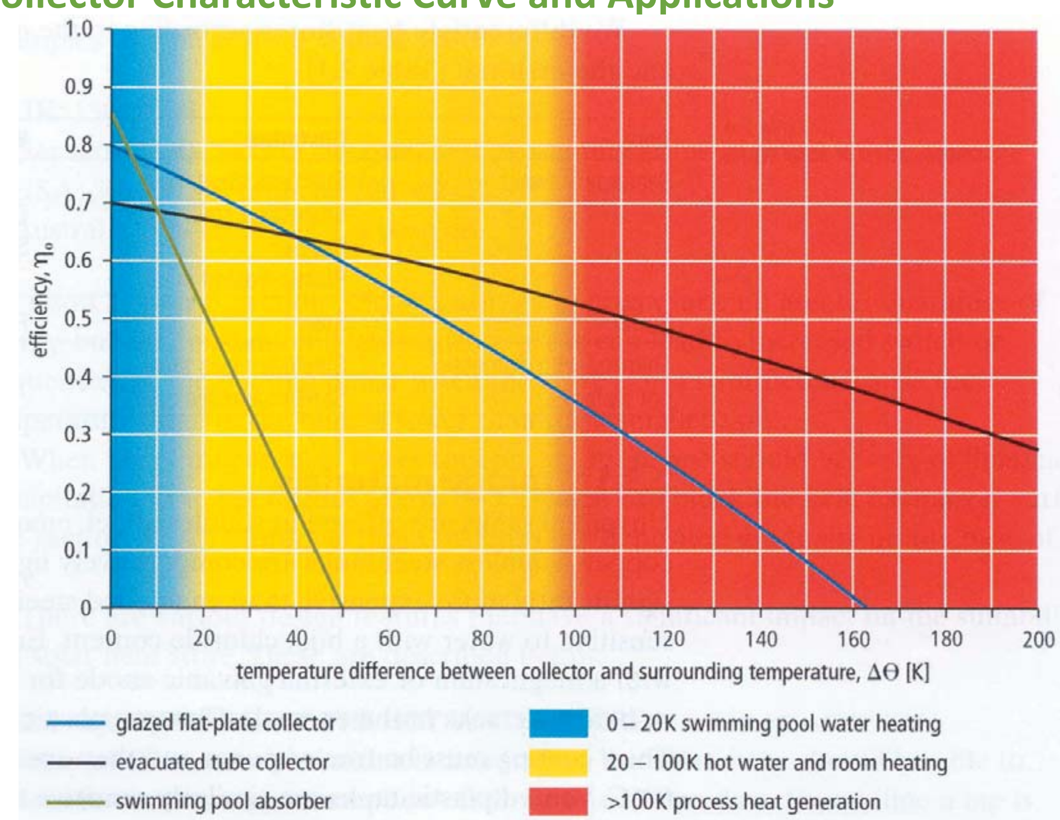
Geometry of Solar Collectors

- the gross surface area (collector area) is the product of the outside dimensions, and defines the minimum amount of roof area that is required for mounting
- the aperture area corresponds to the light entry area of the collector – that is the area through which the solar radiation passes to the collector itself
- the absorber area (also called the *effe-ctive* collector area) corresponds to the area of the actual ab-sorber panel; the ma-ximum energy yield is defined by this area



cross-section of a flat-plate collector

Collector Characteristic Curve and Applications



Heat Removal Factor (F_R)

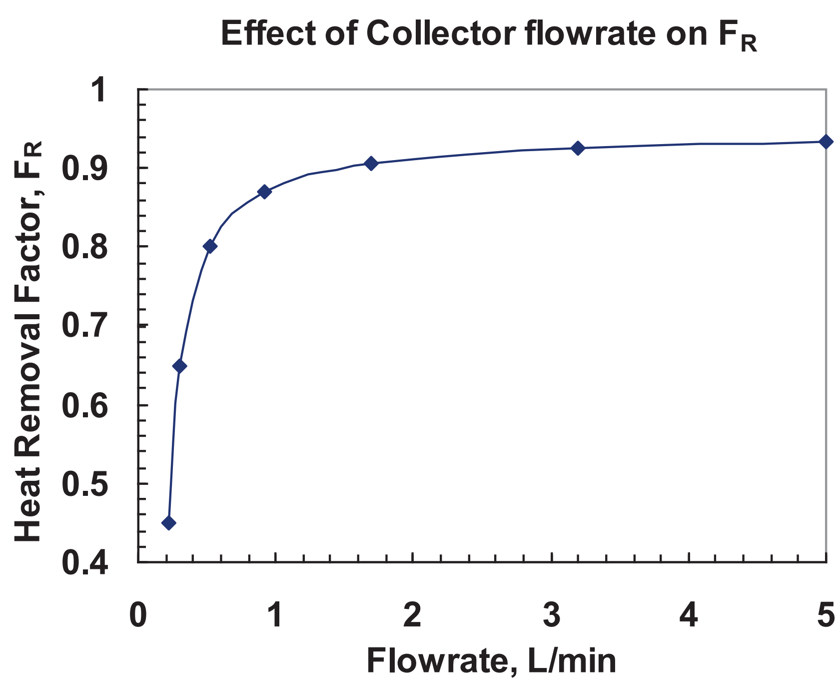
- during testing and modelling it is hard to calculate the exit fluid temperature, T_{fe}, (an iterative solution is required)
- in North America, the inlet fluid temperature is used to characterize solar collector performance, i.e., the efficiency equation is written as,

$$\eta = \frac{Q_u}{G_T \cdot A_c} \approx \left[F_R \cdot (\tau\alpha) - F_R \cdot U_L \frac{(T_{fi} - T_{amb})}{(G_T)} \right] \qquad (1)$$

- where F_R is called the "heat removal factor" and accounts for the fact that the fluid entering the collector is heated in the direction of flow and consequently is not all the same temperature as the inlet fluid
- it is also unit-less and has a value between 0 and 1; we want F_R to have as high of a value as possible; it is a strong function of flow rate through the solar collector
- the terms $F_R(\tau\alpha)$ and $F_R U_L$ are usually experimentally determined

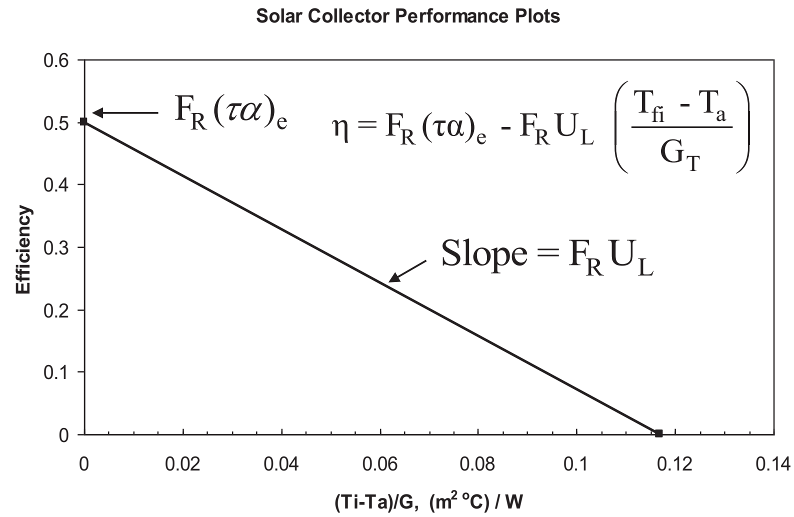
Collector Flow Rate Effect

- therefore F_R varies with flow rate as shown in the example plot below



Solar Collector Efficiency Plot

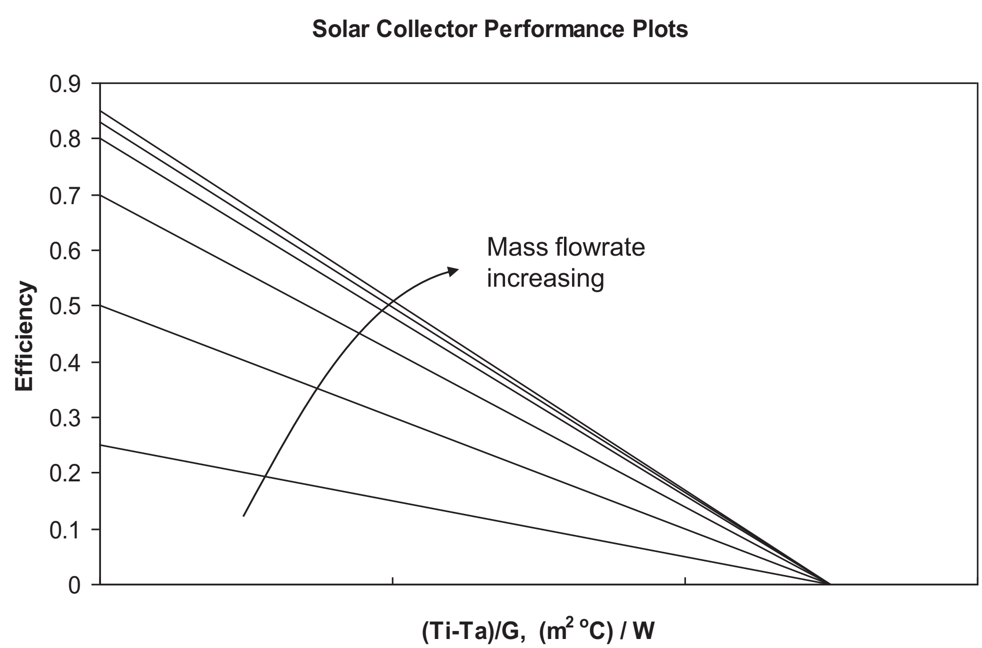
- the HWB representation of solar collector performance is based on T_{fi} ; an example plot for a fixed mass flow rate is shown below



- values of η from Eq. (1) can be represented by a linear equation of the form y = mx + b; in such a case, y = η, m = F_RU_L and b =F_R $(\tau\alpha)_e$
- results are usually plotted as η vs. $(T_{fi}-T_{amb})/G_T$, i.e., x = $(T_{fi}-T_{amb})/G_T$

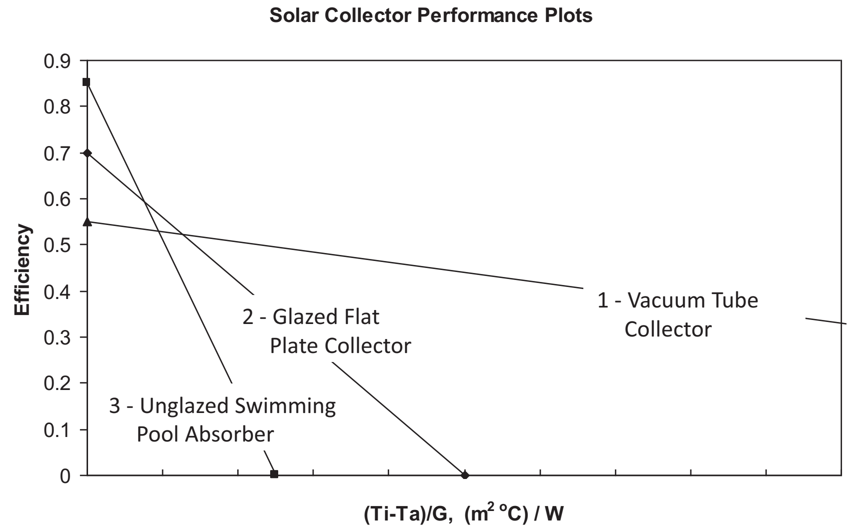
Collector Flow Rate Effect

- an example plot showing the effect of increasing flowrate is given below



Typical Efficiencies of Collectors

- solar collector efficiencies generally fall within specific ranges



#	$F_R \cdot (\tau\alpha)_e$	$F_R \cdot U_L$ (W/m²°C)	
1	0.5 - 0.75	1 - 2	Depends on tube spacing
2	0.65 - 0.8	3 - 8	Depends on # of covers and absorber coating
3	0.8 - 0.95	10 - 20	Depends on wind speed

SOLAR THERMAL SYSTEMS

Incidence Angle Effects

- the full HWB equation accounts for the above characteristics and the level of diffuse radiation from the sky and reflected from the ground and the direct beam radiation

- the rate of energy collection, Q_U (in W):

$$Q_U = A_C\left[F_R(\tau\alpha)\cdot G_T\cdot K_{(\tau\alpha)} - F_R\cdot U_L\cdot(T_{fi}\text{-}T_{amb})\right]$$

$$Q_U = A_C\left[F_R(\tau\alpha)\cdot(k_b(\theta)\cdot G_b + k_d\cdot G_d) - F_R\cdot U_L\cdot(T_{fi}\text{-}T_{amb})\right]$$

- the collector efficiency, η:

$$\eta = \frac{Q_U}{G_T\cdot A_C} = \left[F_R(\tau\alpha)\cdot\frac{K_{(\tau\alpha)}}{G_T} - F_R\cdot U_L\frac{(T_{fi}-T_{amb})}{G_T}\right]$$

$$\eta = \frac{Q_U}{G_T\cdot A_C} = \left[F_R(\tau\alpha)\cdot\left(\frac{k_b(\theta)\cdot G_b + k_d\cdot G_d}{G_T}\right) - F_R\cdot U_L\frac{(T_{fi}\text{-}T_{amb})}{G_T}\right]$$

ε

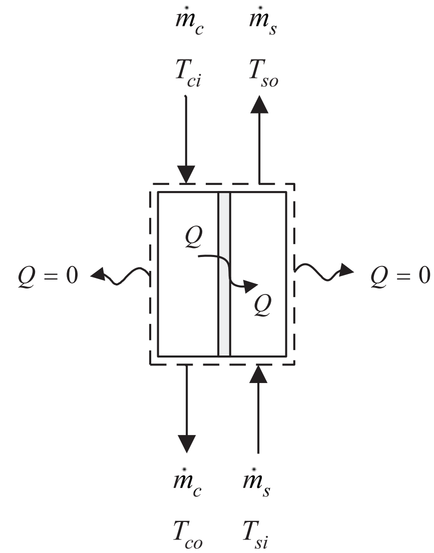
Heat Exchanger Performance and Energy Balance

- the performance of heat exchangers operating under forced flow conditions is defined by the amount of heat transferred between the two fluid streams, Q, and is characterized by the heat transfer coefficient-area product and the effectiveness, ε

- the rate of heat transfer between the two fluid streams in the heat exchanger, Q, is,

$$Q = (\dot{m}c_p)_s (T_{so} - T_{si}) = (\dot{m}c_p)_c (T_{ci} - T_{co})$$

where $\dot{m}c_p$ is the heat capacity rate of one of the fluid streams



Heat Exchanger Effectiveness

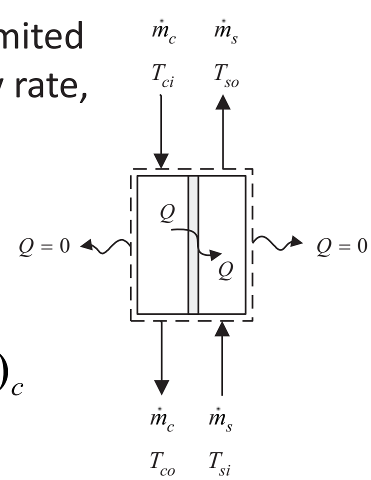
- the heat exchanger effectiveness, ε, is defined as the ratio of the rate of heat transfer in the exchanger, Q, to the maximum theoretical rate of heat transfer, $Q_{\max}$, i.e.,

$$\varepsilon = \frac{Q}{Q_{\max}}$$

- the maximum theoretical rate of heat transfer is limited by the fluid stream with the smallest heat capacity rate, i.e.,

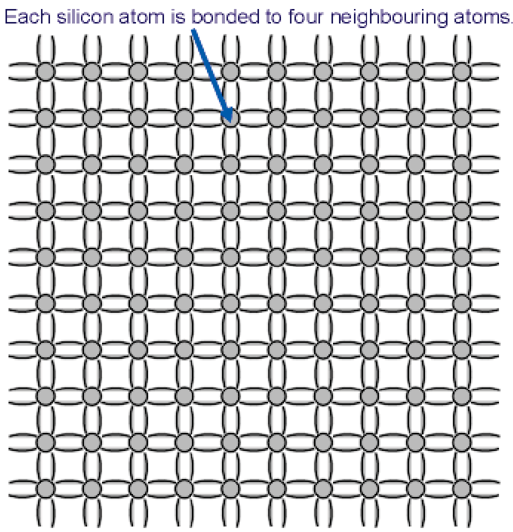
$$\varepsilon = \frac{(\dot{m}c_p)_s (T_{so} - T_{si})}{(\dot{m}c_p)_{\min} (T_{ci} - T_{si})}$$

where $(\dot{m}c_p)_{\min}$ is the smaller of $(\dot{m}c_p)_s$ or $(\dot{m}c_p)_c$

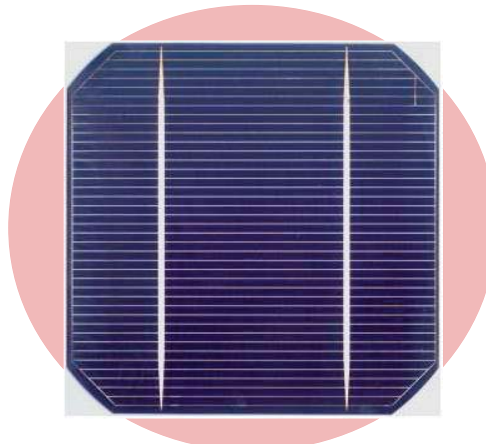


Mono-crystaline

- consists of silicon in which the crystal lattice of the entire solid is continuous, unbroken (with no grain boundaries)
- efficient and expensive

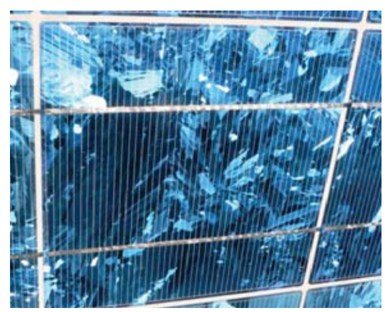


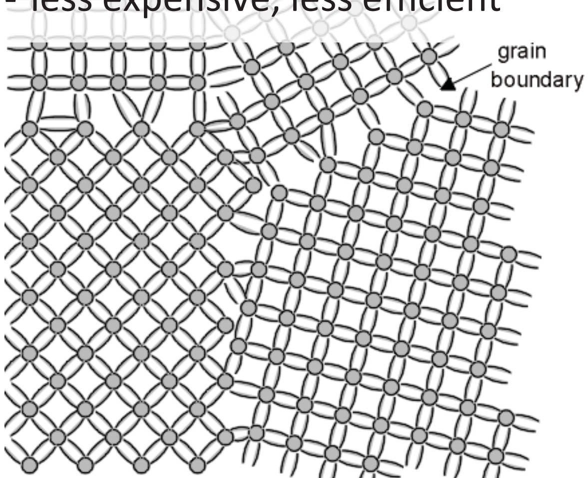
- cut from cylindrical ingots and do not completely cover a square solar cell module without a substantial waste of refined silicon

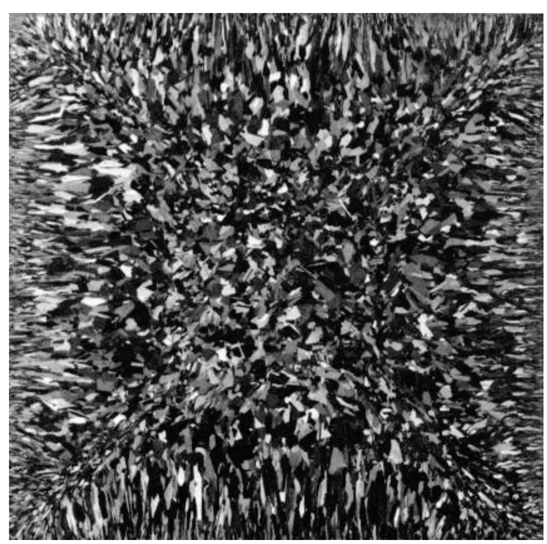


Poly-crystaline

- consists of multiple small crystals and recognized by visible grain (metal flake effect)
- made from cast square ingots (large blocks of silicon carefully cooled and solidified)
- less expensive, less efficient

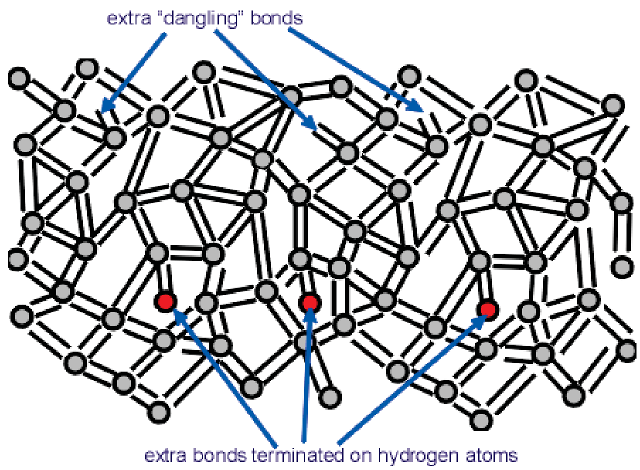






Amorphous

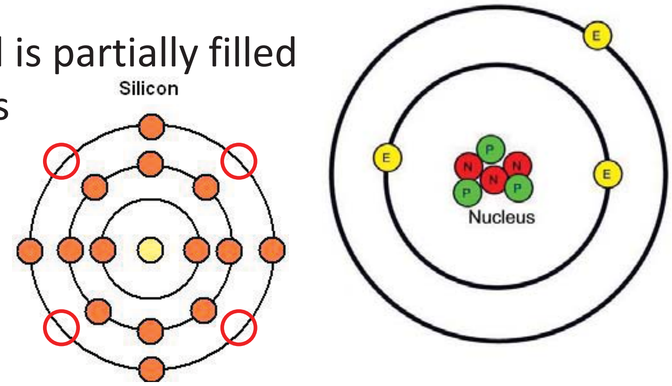
- non-crystalline allotropic form of silicon (continuous random network)
- vacuum silicon (thin film), can be deposited at low temperatures, less expensive, less efficient (compared to other two), more flexible in applications (can deposit on class, plastic, etc.)



Atomic Theory

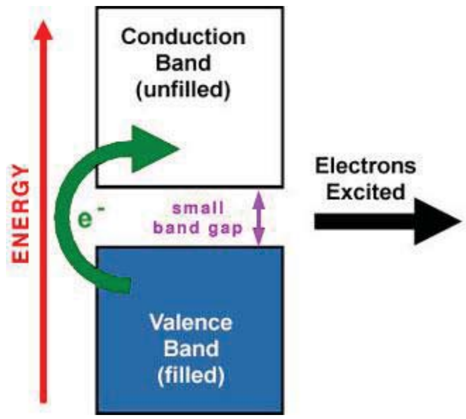
- the electrons are positioned in orbitals (bands) about the nucleus
- all inner bands holding electrons must be filled before any band farther out can be partially filled
- the energy of an electron is determined by its position
- the outermost band an electron can occupy (and maintain its association with the atom) is called the valance band
- in silicon, the valance band is partially filled and contains four electrons

  (must contain eight to be filled)



Band Gap Energy

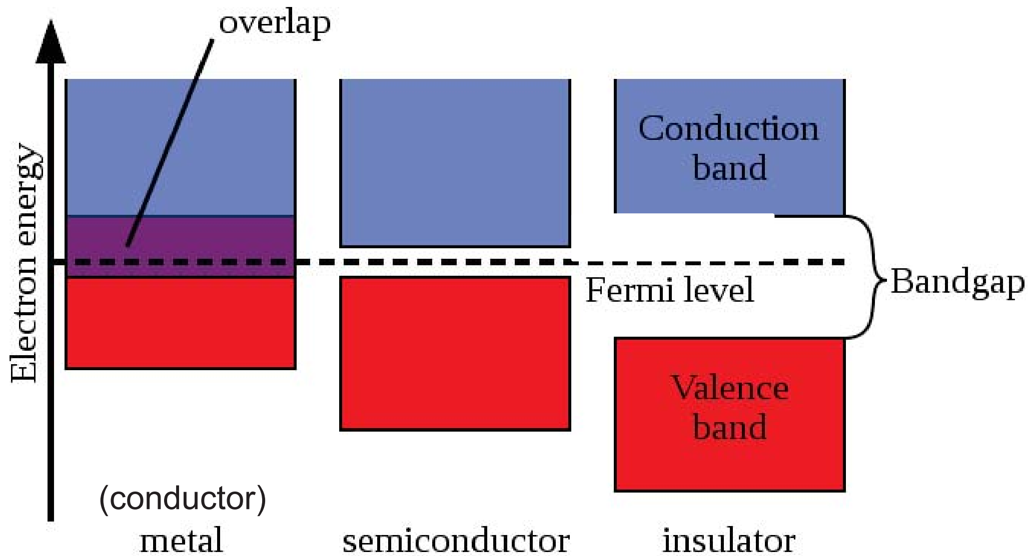
- electrons in the valence band can become so energetic that they jump into a band that is far removed from the nucleus
- this remote band is called the conduction band
- the difference in energy between an electron in the valance band and one in the conduction band is called the band gap energy



- electrons in the conduction band require only a small amount of energy to move away from the atom (this is responsible for heat and electrical conduction)

Band Gap Energy

- energy bands of electrons in a metal, semiconductor & insulator

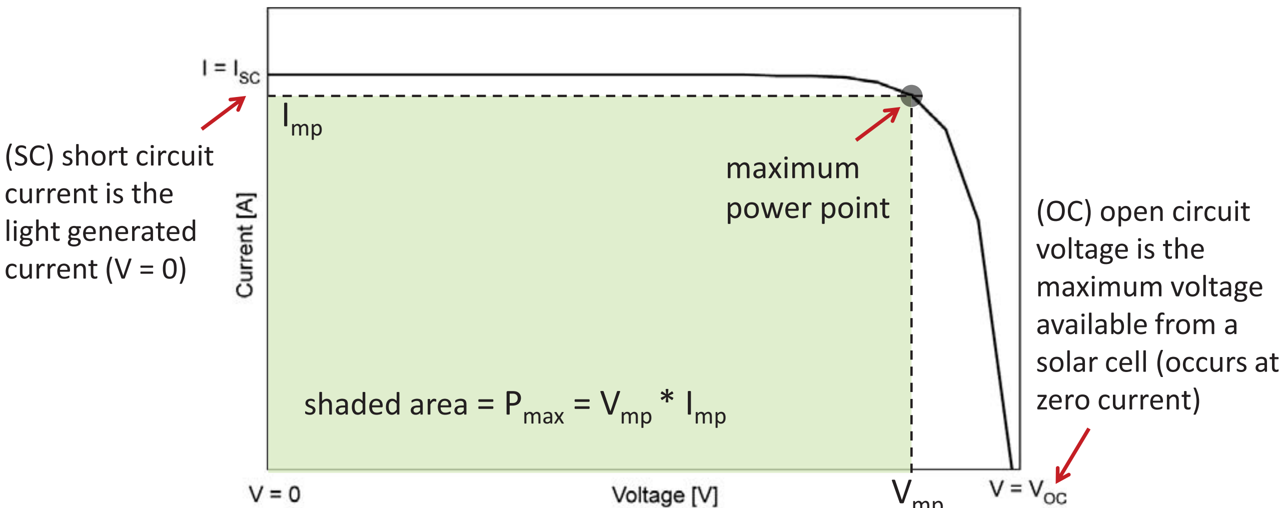


Band Gap Energy

- the previous example has important implications for the operation and efficiency of photovoltaic devices
- photons with a wavelength greater than 1.12 μm contain insufficient energy to dislodge a valence electron in silicon (no photovoltaic effect will be induced in silicon)
- photons with a wavelength less than 1.12 μm posses more energy than is required to dislodge a valence electron; however, a single photon can dislodge only a single valence electron and the difference between the band gap energy and the photon energy is absorbed by heat by the PV device

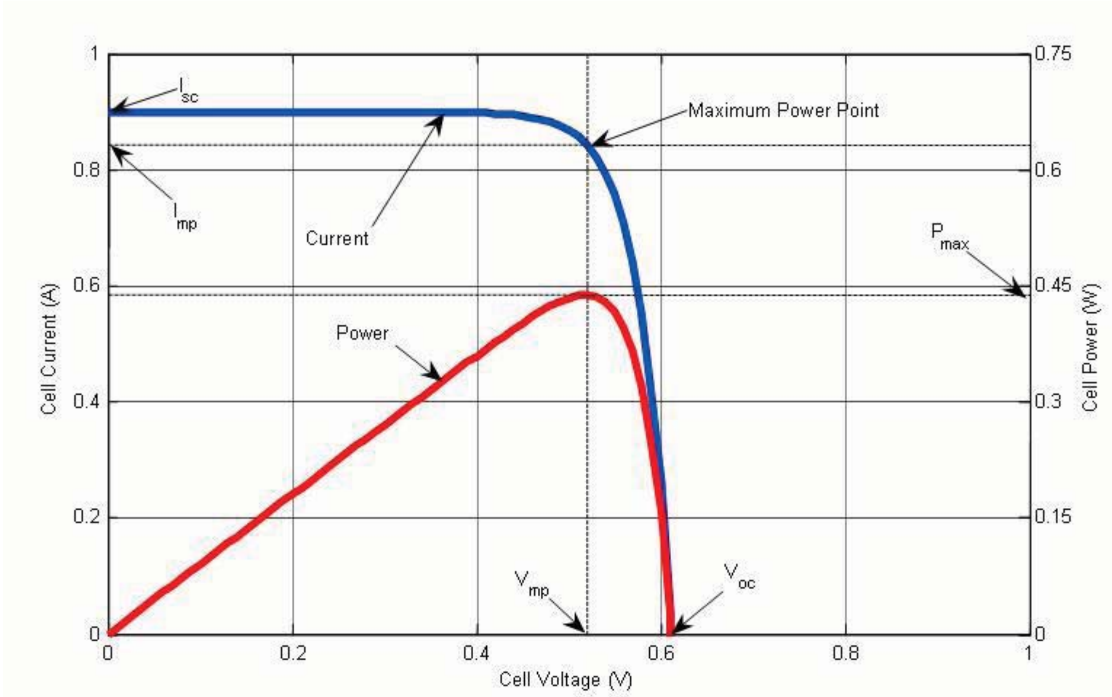
I-V Characteristics of a (Ideal) Solar Cell

- the maximum power P_{max} produced by a solar cell is reached when the product *I-V* (current * voltage) is maximum
- this can be shown graphically where the position of the maximum power point represents the largest area of the rectangle shown



Current Density Ratio and Power Ratio vs. Voltage

- I-V Characteristics Curve



Standard Test Conditions and Temperature and Irradiance Effects

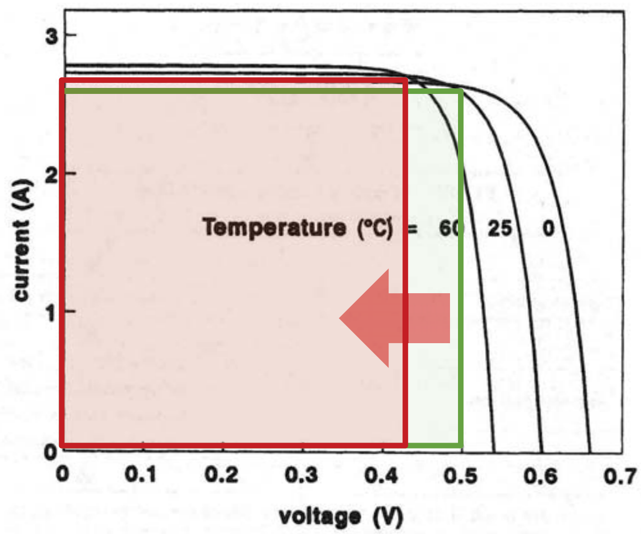
- the efficiency η of a solar cell is defined as:

$$\frac{\text{the power } P_{max} \text{ produced by the cell at the maximum power point under standard test conditions}}{\text{the power of the radiation incident upon it}}$$

- most frequent conditions are: irradiance 100 mW/cm^2, standard reference AM1.5 spectrum, and temperature 25^oC
- in practical applications, however, solar cells do not operate under standard conditions
- the two most important effects that must be allowed for are due to the variable temperature and irradiance

Temperature Effect

- temperature has an important effect on the power output
- the most significant is the temperature dependence of the voltage which decreases with increasing temperature
- P_{max} decreases as T of cell ↑
- the voltage decrease of a silicon cell is typically 2.3 mV per °C
- the temperature variation of the current is less pronounced (usually neglected in PV design)

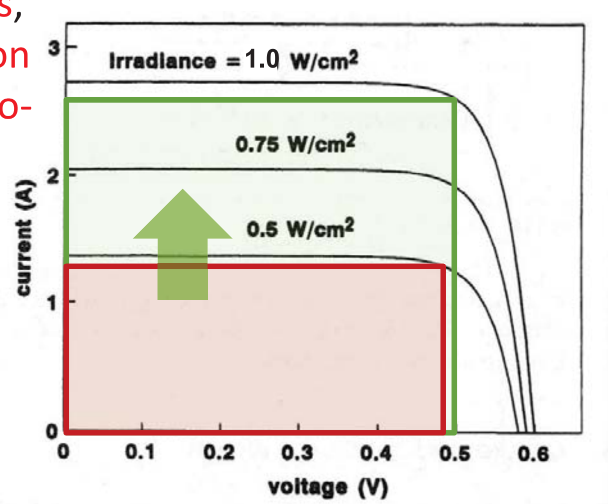


Source: Solar Electricity, Tomas Markvart

Temperature dependence of the *I*–*V* characteristic of a solar cell

Irradiance Effect

- solar cell characteristics vary under different levels of illumination
- the light generated current (flowing of electrons) is proportional to the flux of photons with above-bandgap energy
- increasing the irradiance increases, in the same proportion, the photon flux, which in turn, generates a proportionally higher current
- therefore the short circuit current is directly proportional to the irradiance
- the voltage variation is much smaller and is usually neglected

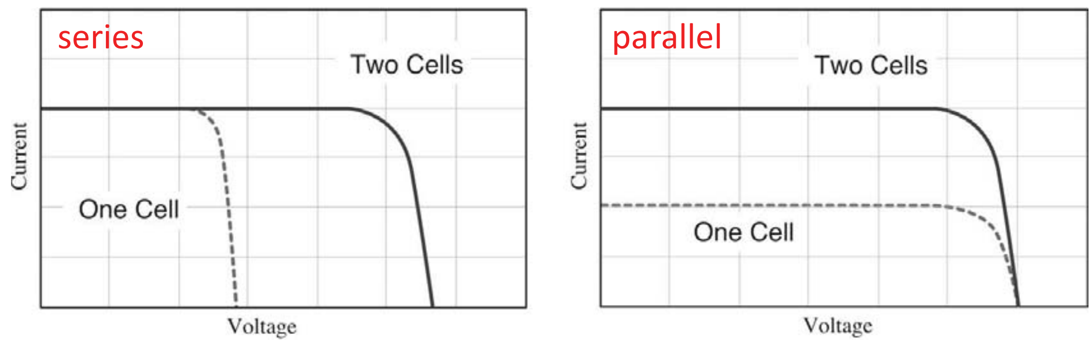


Source: Solar Electricity, Tomas Markvart

Irradiance dependence of the *I*–*V* characteristic of a solar cell

Photovoltaic Components

- series and parallel configurations of solar cells follow the same rules as series and parallel DC circuits
- for identical components placed in series, the voltages add at constant current (multiple cells in series to increase operating voltage)
- for identical components placed in parallel, the currents add at constant voltage (multiple strings in parallel increase current – used to power up to several MW)



Source: Alternative Energy Systems and Applications, B.K. Hodge

EXAMPLE

Photovoltaic cells are to be arranged to provide an output of 12 V and a power of 120 W. If the voltage and current at maximum power are 0.493 V and 5.13 A (V*I = 2.53 W), recommend an arrangement that meets the specifications.

SOLUTION

- the number of cells required for 120 W is

number of cells = 120 W / (2.53 W/cell) = 47.2 cells

- to provide the correct voltage 12 V, the number of cells in series are

cells in series = 12 V/ (0.493 V/cell) = 24.3 cells

- the number of cells in series can be rounding up to 25; two rows of 25 cells in parallel will required 50 cells with a total power of 126.5 W

Biomass Heating Systems

- a biomass heating system consists of 3 parts:
  - a heating plant
  - a heat distribution system
  - a biomass fuel supply operation

- a heating plant typically comprise of a number of different heating units:
  - ensures sufficient heating capacity
  - reduces the risk that a fuel supply interruption will endanger the supply of heat
  - maximize the use of the lowest cost heat sources

BIOMASS

Types of Heating Sources

- the four types of heat sources that may be found in a biomass heating plant are:

1 - waste heat recovery

2 - biomass combustion system

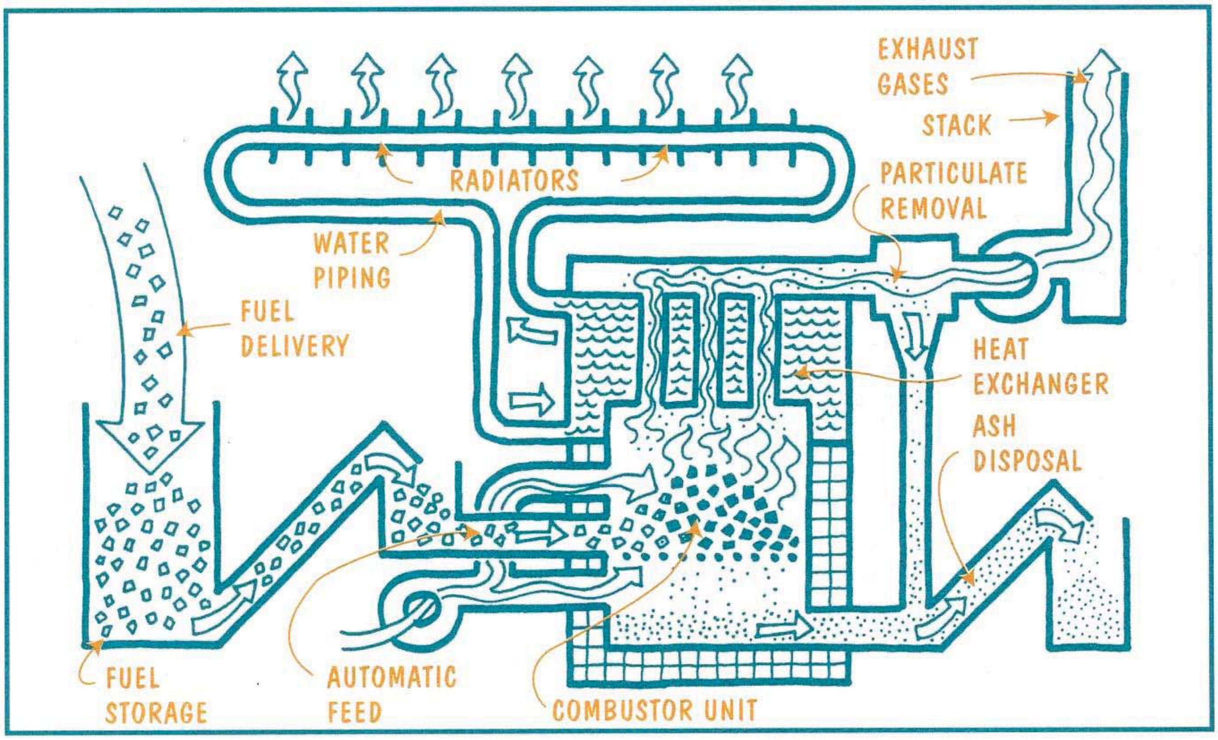
3 - peak load heating system

4 - backup heating system

increasing order of typical cost per unit of heat produced

Prof. C. A. Cruickshank, Carleton University

Biomass Combustion Systems – Typical Layout



Source "Biomass Heating", BioEnergy Series, Natural Resources Canada

EXAMPLE

How much wood (in m^3) would be required to bring one litre of water to a boil?

Data

- heat value of wood = 15 MJ/kg
- density of wood = 600 kg/m^3
- specific heat capacity of water = 4200 J/kg K
- for this case, let's assume the mass of 1 litre of water ≈ 1 kg and the initial temperature of the water is 20°C

Procedure

heat energy needed to heat 1 L of water from 20°C to 100°C

heat energy released in burning 1 m^3 of wood

volume of wood required

SOLUTION

Heat energy needed to heat 1 L of water from 20°C to 100°C

$Q_{needed} = \rho * V * C_p * (T_f - T_i)$

$Q_{needed} = 1\ kg/L * 1\ L * 4200\ J/kgK * (100 - 20)°C$

$Q_{needed} = 336\ kJ$

Heat energy released in burning 1 m^3 of wood

$Q_{released} = Q/m * \rho * V$

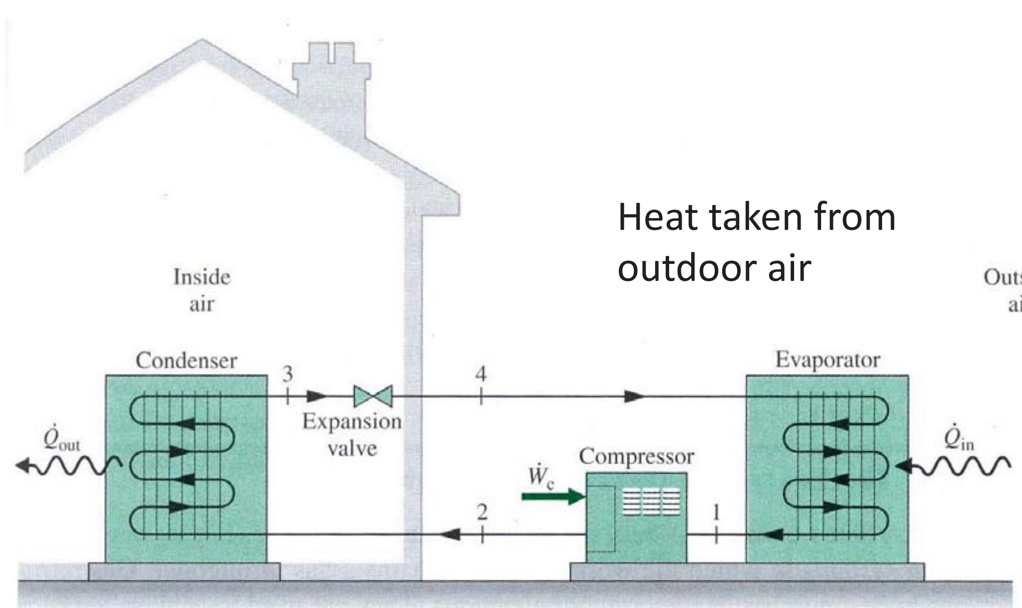
$Q_{released} = 15000\ kJ/kg * 600\ kg/m^3 * 1\ m^3$

$Q_{released} = 9 \times 10^6\ kJ$

Volume of wood required

$Q_{needed}/Q_{released} = 0.000037\ m^3 = 37\ cm^3$

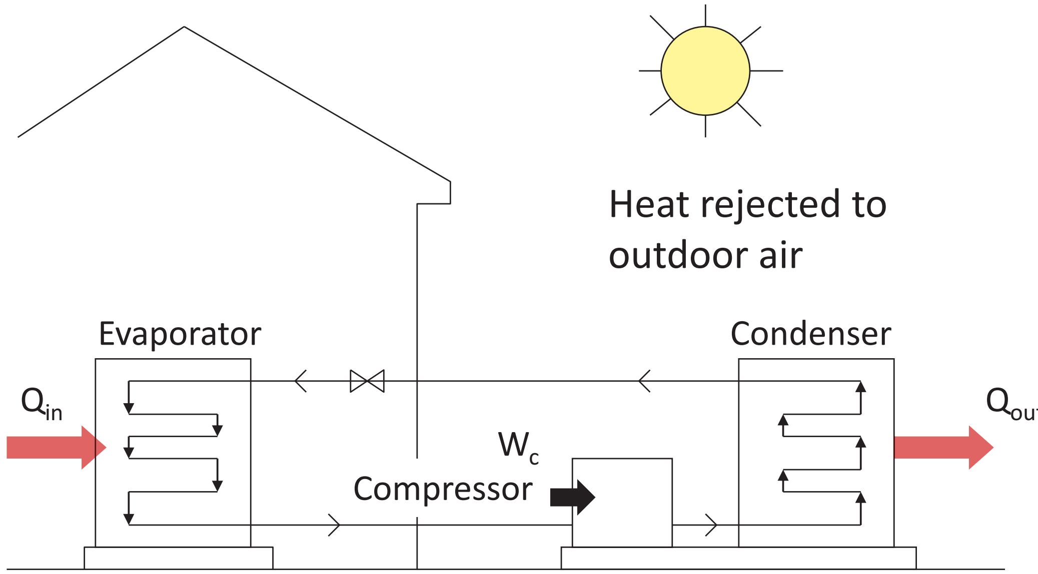
Heat Pump System (Heating Mode)



▲ **Figure 10.11** Air-source vapor-compression heat pump system.

From “Fundamentals of Engineering Thermodynamics”, Moran and Shapiro, 5th Edition

Heat Pump System (Cooling Mode)



Maximum COP

Recall the COP for a refrigeration and heat pump cycle:

<u>Refrigeration Cycle</u>

$COP_R = Q_{in} / W_{cycle}$

$COP_R = Q_{in} / (Q_{out} - Q_{in})$

You can approximate the maximum COP (Carnot cycle) to be:

$(COP_R)_{max} \approx T_{cold}/(T_{hot} - T_{cold})$

For an air source refrigeration cycle,

$(COP_R)_{max} \approx T_{evap}/(T_{cond} - T_{evap})$ (with $T_{cond} \approx T_{amb}$)

Therefore as T_{amb} ↑ COP_R ↓

<u>Heat Pump</u>

$COP_{HP} = Q_{out} / W_{cycle}$

$COP_{HP} = Q_{out} / (Q_{out} - Q_{in})$

You can approximate the maximum COP (Carnot cycle) to be:

$(COP_{HP})_{max} \approx T_{hot}/(T_{hot} - T_{cold})$

For an air source HP

$(COP_{HP})_{max} \approx T_{cond}/(T_{cond} - T_{evap})$ (with $T_{evap} \approx T_{amb}$)

Therefore as T_{amb} ↓ COP_{HP} ↓

Heat Pump Example

A heat pump cycle whose coefficient of performance is 2.5 delivers energy by heat transfer to a dwelling at a rate of 20 kW.

a) Determine the net power required to operate the heat pump, in kW.

b) Evaluating electricity at $0.08 per kW·h, determine the cost of electricity in a month when the heat pump operates for 200 hours.

$COP_{HP} = Q_{out} / W_{cycle}$
$COP_{HP} = Q_{out} / (Q_{out} - Q_{in})$

a) $W_{cycle} = Q_{out} / COP_{HP}$ = 20 kW / 2.5 = 8 kW

b) Cost = 8 kW * 200 h * $0.08 / kW·h = $128.00

Heat Pump Example 2

A heat pump cycle delivers energy by heat transfer to a dwelling at a rate of 60,000 Btu/h. The power input to the cycle is 7.8 hp (1 hp = 2546.7 Btu/h).

a) Determine the coefficient of performance of the cycle.

b) Evaluating electricity at $0.08 per kW·h, determine the cost of electricity in a month when the heat pump operates for 200 hours (1 kW·h = 3412 Btu).

$COP_{HP} = Q_{out} / W_{cycle}$
$COP_{HP} = Q_{out} / (Q_{out} - Q_{in})$

a) COP_{HP} = 60000 Btu/h / (7.8 * 2546.7 Btu/h)
$COP_{HP} = 3.02$

Heat Pump Example 2

A heat pump cycle delivers energy by heat transfer to a dwelling at a rate of 60,000 Btu/h. The power input to the cycle is 7.8 hp (1 hp = 2546.7 Btu/h).

a) Determine the coefficient of performance of the cycle.

b) Evaluating electricity at $0.08 per kW·h, determine the cost of electricity in a month when the heat pump operates for 200 hours (1 kW·h = 3412 Btu).

$COP_{HP} = Q_{out} / W_{cycle}$
$COP_{HP} = Q_{out} / (Q_{out} - Q_{in})$

b) W_{cycle} = 7.8*2546.7Btu/h * 200 h = 3972852 Btu
Cost = 3972852 Btu * 1 kWh/3412 Btu * $0.08 / kWh
Cost = $93.15

Refrigeration Example

A refrigeration cycle operates continuously and removes energy from the refrigerated space at a rate of 12,000 Btu/h. For a coefficient of performance of 2.6, determine the net power required, in Btu/h. Convert your answer to horsepower. (1 hp = 2546 Btu/h)

$COP_R = Q_{in} / W_{cycle}$

$COP_R = Q_{in} / (Q_{out} - Q_{in})$

$W_{cycle} = Q_{in} / COP_R$

W_{cycle} = 12000 Btu/h / 2.6

W_{cycle} = 4615 Btu/h

W_{cycle} = 1.81 hp

Coefficient of Performance

Can anyone think of why the COP must be at least 3?

For example:

For a natural gas power plant operating at 30% efficiency, less than 1/3 of the energy is converted to electricity. If this electricity is used to power a heat pump with a COP of 3 then we would break even with a gas furnace operating at 95% efficiency. If your COP is not 3+, then you would obtain more heat using natural gas than with a heat pump.

Thank you and GOOD LUCK!!