## Functions of several variables

- A function of two variables $f(x,y)$ assigns a value to each point $(x,y)$

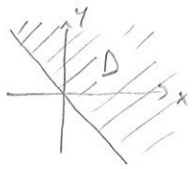
  eg. $f(x,y) = \sqrt{x+y}$ $\quad f(0,0) = 0$, $f(2,3) = \sqrt{2+3} = \sqrt{5}$, $f(0,1) = \sqrt{0+1} = 1$, etc.

- The <u>domain</u> $D$ is the set of points $\{(x,y)\}$ that can be put into the function - sometimes this is given explicitly, but usually it is understood to be all points where the expression $f(x,y)$ gives a well-defined real number.

  The <u>range</u> $R$ is the set of values that $f$ can take

  eg. for $f(x,y) = \sqrt{x+y}$, $D = \{(x,y) : x+y \geq 0\}$

  $R = \{z : z \geq 0\}$



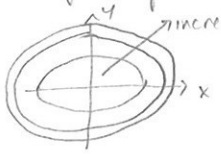

- The function might represent, for example, - temperature at point $(x,y)$ on a sheet of metal $T = f(x,y)$
  - height of land at longitude / latitude $(x,y)$ $z = f(x,y)$
  - air pressure at height $z$ at time $t$ $p = f(z,t)$

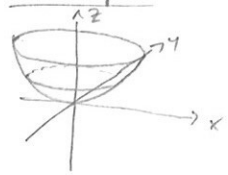
- A function of three variables $f(x,y,z)$ assigns a value to each point $(x,y,z)$ in its domain. Functions of more variables are possible, eg. $V = f(x,y,z,t)$ might represent wind speed at position $(x,y,z)$ at time $t$.

- Functions of two variables are often represented with a <u>contour plot</u> or a <u>graph</u> (ie the surface $z = f(x,y)$)

→ eg. $f(x,y) = \frac{x^2}{2} + y^2$

$D = \mathbb{R}^2$

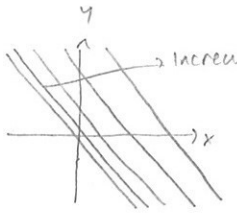


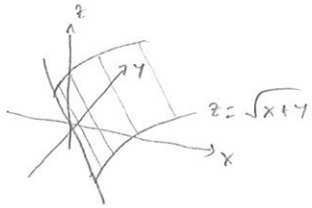




[ Contour lines are also called <u>level curves</u> ]

→ eg. $f(x,y) = \sqrt{x+y}$

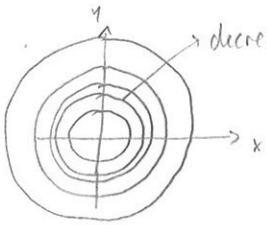
$D = \{(x,y) : x+y \geq 0\}$

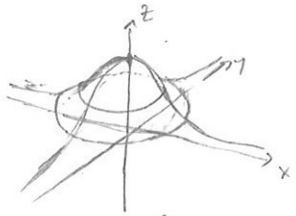


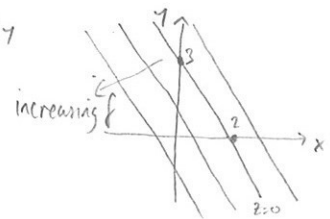

eg. $f(x,y) = e^{-x^2-y^2}$

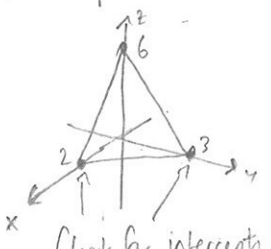
$D = \mathbb{R}^2$




eg. $f(x,y) = 6 - 3x - 2y$




The graph of a <u>linear function</u>

$f(x,y) = ax + by + c$

is a <u>plane</u>

[ Look for intercepts to help draw the graph ]