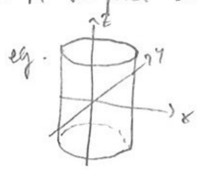
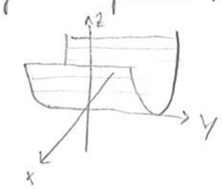
Cylinders & Quadric Surfaces

- A surface constructed of lines parallel to a given line is called a cylinder, ie the extrusion of a curve

eg.  $x^2+y^2=1$  $z=x^2$ If the cylinder is aligned with one of the coordinate axes, its equation is independent of that variable

- A surface described by a quadratic equation in x, y and z is a quadric surface.

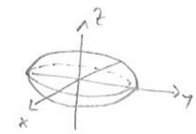
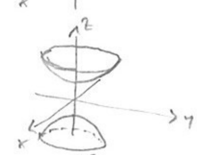
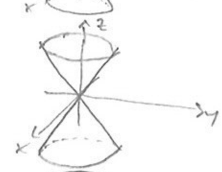
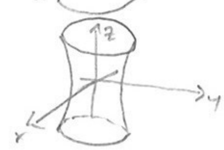
There are 2 broad types: $Ax^2+By^2+Cz^2=D$ & $z=Ax^2+By^2$

[any other quadratic equation can be converted to one of these types by a translation and rotation, which correspond to a relabelling of the variables.

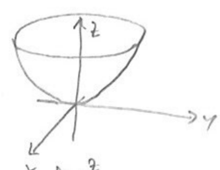
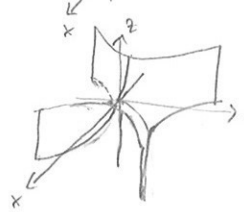
eg. $x^2-4x+y+z^2=0$

$\Rightarrow (x-2)^2-4+y+z^2=0$ Redefine $\hat{x}=x-2$, $\hat{y}=z$, $\hat{z}=-y+4$, then $\hat{z}=\hat{x}^2+\hat{y}^2$]

Of the first type there is

- $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{c^2}{z^2}=1$  Ellipsoid (a sphere in the special case $a=b=c$)
- $\frac{z^2}{c^2}-\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$  Hyperboloid of two sheets
- $\frac{z^2}{c^2}-\frac{x^2}{a^2}-\frac{y^2}{b^2}=0$  Cone
- $\frac{z^2}{c^2}-\frac{x^2}{a^2}-\frac{x^2}{b^2}=-1$  Hyperboloid of one sheet

Of the second type

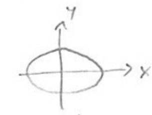
- $\frac{z}{c}=\frac{x^2}{a^2}+\frac{y^2}{b^2}$  Elliptic paraboloid
- $\frac{z}{c}=\frac{x^2}{a^2}-\frac{y^2}{b^2}$  Hyperbolic paraboloid (Saddle).

Sketching surfaces

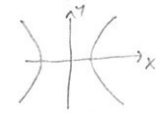
eg. sketch $x^2 + \frac{y^2}{4} + z^2 = 1$

Think about trace curves or level curves, when one of the variables is set equal to a constant – these are the 'slices' in the xy, xz and yz planes. The z traces can be thought of like contour lines.

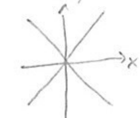
[Remember $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \text{constant}$ is an ellipse



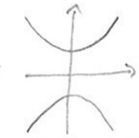
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \text{constant}$ is a hyperbola



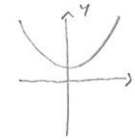
or



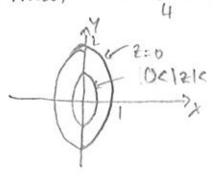
or



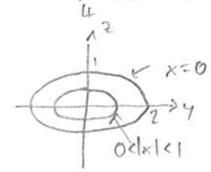
$\frac{y}{b} - \frac{x^2}{a^2} = \text{constant}$ is a parabola


]

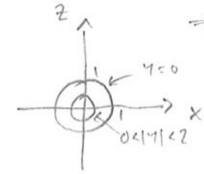
z traces $x^2 + \frac{y^2}{4} = 1 - z^2 \Rightarrow$ ellipses



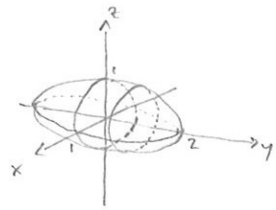
x traces $\frac{y^2}{4} + z^2 = 1 - x^2 \Rightarrow$ ellipses



y traces $x^2 + z^2 = 1 - \frac{y^2}{4}$ $\Rightarrow$ circles



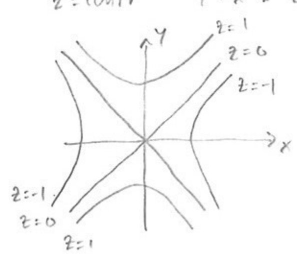
$\Rightarrow$



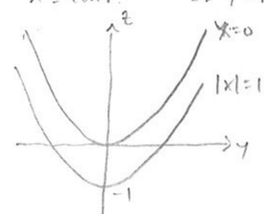
Ellipsoid

eg. $z = y^2 - x^2$

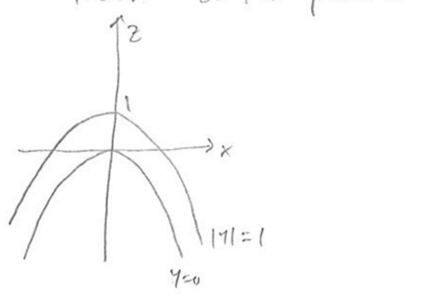
$z = \text{const}$ $y^2 - x^2 = z$ hyperbolas



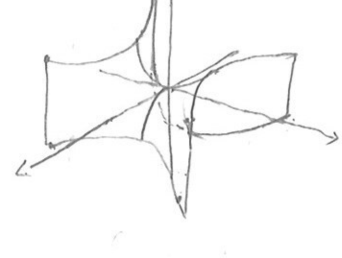
$x = \text{const}$ $z = y^2 - x^2$ parabolas



$y = \text{const}$ $z = y^2 - x^2$ parabolas



$\Rightarrow$



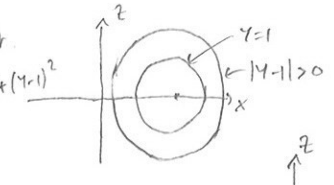
Hyperbolic paraboloid.

eg. $(x-2)^2 + z^2 - (y-1)^2 = 1$

$y = \text{const}$

$(x-2)^2 + z^2 = 1 + (y-1)^2$

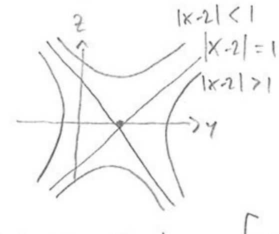
circles



$x = \text{const}$

$z^2 - (y-1)^2 = 1 - (x-2)^2$

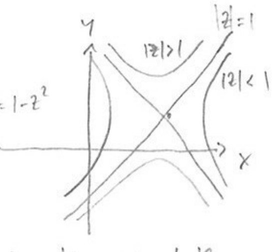
hyperbolas



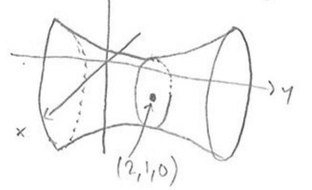
$z = \text{const}$

$(x-2)^2 - (y-1)^2 = 1 - z^2$

hyperbolas



$\Rightarrow$



Hyperboloid of one sheet

[note the y axis takes the role of the z axis in the standard example since $(y-1)^2$ has opposite sign to $(x-2)^2$ and z^2]