

CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Section(s)	
Mathematics	206/2	All	
Examination	Date	Time	Pages
Final	December 2011	3 Hours	2
Instructors	Course Examiner		
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Special Instructions

- ▷ Only approved calculators are allowed.

MARKS

- [4] 1. Simplify the expressions below. Do not use a calculator.

$$(a) -4\sqrt{27} + \sqrt{12} + 3\sqrt{75} \quad (b) \frac{2}{3} \log_2 8 - \log_2 (3^2 - 1)$$

- [4] 2. Rationalize the denominator:

$$(a) \frac{\sqrt{2}}{\sqrt{7} + 2} \quad (b) \frac{2 + \sqrt{3}}{2 - \sqrt{3}}$$

- [6] 3. Simplify the expressions:

$$(a) x(8x^3 - 2x^2 + 6x - 2) + 5x(3x^4 - 2x^3 + x^2 + x) \quad (b) \frac{x^2 + 4x - 5}{x^2 - 2x + 1}$$

- [8] 4. Factor the polynomials completely:

$$(a) 6x^2 + 8x + 2 \quad (b) 1 - x^4$$

- [4] 5. Perform the arithmetic operations and simplify:

$$\frac{3}{x^2 + x} - \frac{x + 4}{x^2 + 2x + 1}$$

- [9] 6. Solve the equations:

$$(a) \frac{5}{x + 4} = 4 + \frac{3}{x - 2} \quad (b) \log_2 (x + 6) - \log_2 (x - 4) = \log_2 x$$

$$(c) 3^{x^2 - 7} = (27)^{2x}$$

- [8] 7. Solve the inequalities, express your answer using set notation or interval notation:

$$(a) 0 < \frac{3x + 2}{2} < 4 \quad (b) 3 - |x + 1| < \frac{1}{2}$$

- [4] 8. Solve the system of equations:

$$\begin{aligned}x^2 + y^2 &= 13 \\x^2 - y &= 7\end{aligned}$$

- [8] 9. (a) Which of the points $A(3, 2)$, $B(5, 3)$ is closer to the point $C(2, 7)$?
(b) Show that the equation $x^2 + y^2 + 4x - 6y + 12 = 0$ represents a circle. Find coordinates of the center and radius of the circle.

- [6] 10. Find the domain and range of the functions (do not graph):

$$(a) f(x) = \frac{x}{x^2 + 2x - 3} \quad (b) g(x) = \sqrt{2 - x} \quad (c) h(x) = |x^2 - 4|$$

- [5] 11. Sketch the graph of the function $f(x) = -\log(x - 2)$, starting from the graph of the function $g(x) = \log x$ and using appropriate transformations.

- [8] 12. Let $f(x) = \frac{x - 5}{x + 1}$ and $g(x) = \frac{x + 2}{x - 3}$. Find:

$$(a) fg \quad (b) \frac{f}{g} \quad (c) f \circ g \quad (d) g \circ f$$

- [8] 13. (a) Find the inverse of the function $f(x) = \frac{3x + 4}{2x - 3}$.

(b) Find the vertical and horizontal asymptotes of both f and f^{-1} above.

- [5] 14. A bank loaned out \$48,000, part of it at the rate of 8% per year and the rest at the rate of 18% per year. If the interest received totaled \$4000, how much was loaned at 8%?

- [5] 15. The length of fence required to enclose a rectangular field is 1500 meters. What are the dimensions of the field if it is known that the difference between its length and width is 25 meters?

- [8] 16. Fruit flies are placed in a half-pint milk bottle with a banana (for food) and yeast plants (for food and to provide a stimulus to lay eggs). Suppose that the fruit fly population after t days is given by

$$P(t) = \frac{230}{1 + 56.5e^{-0.37t}}$$

- (a) Determine the initial population.
(b) What is the population after 5 days?
(c) How long does it take for the population to reach 180?