Contents

CHAPTER 2

Stress and Strain-Axial Loading

End

Stress & Strain: Axial Loading

- Suitability of a structure or machine may depend on the deformations in the structure as well as the stresses induced under loading. Statics analyses alone are not sufficient.

- Considering structures as deformable allows determination of member forces and reactions which are statically indeterminate.

- Determination of the stress distribution within a member also requires consideration of deformations in the member.

- Chapter 2 is concerned with deformation of a structural member under axial loading. Later chapters will deal with torsional and pure bending loads.

Normal Strain

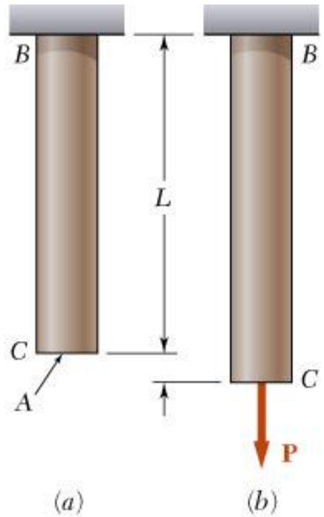


Fig. 2.1

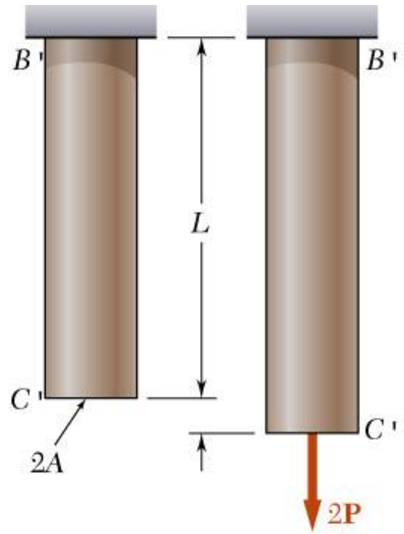


Fig. 2.3

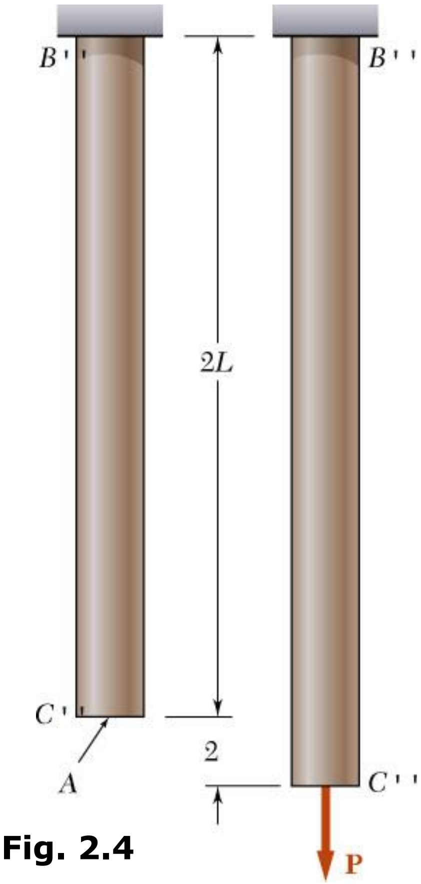


Fig. 2.4

$$\sigma = \frac{P}{A} = \text{stress}$$

$$\varepsilon = \frac{\delta}{L} = \text{normal strain}$$

$$\sigma = \frac{2P}{2A} = \frac{P}{A}$$

$$\varepsilon = \frac{\delta}{L}$$

$$\sigma = \frac{P}{A}$$

$$\varepsilon = \frac{2\delta}{2L} = \frac{\delta}{L}$$

Stress-Strain Test

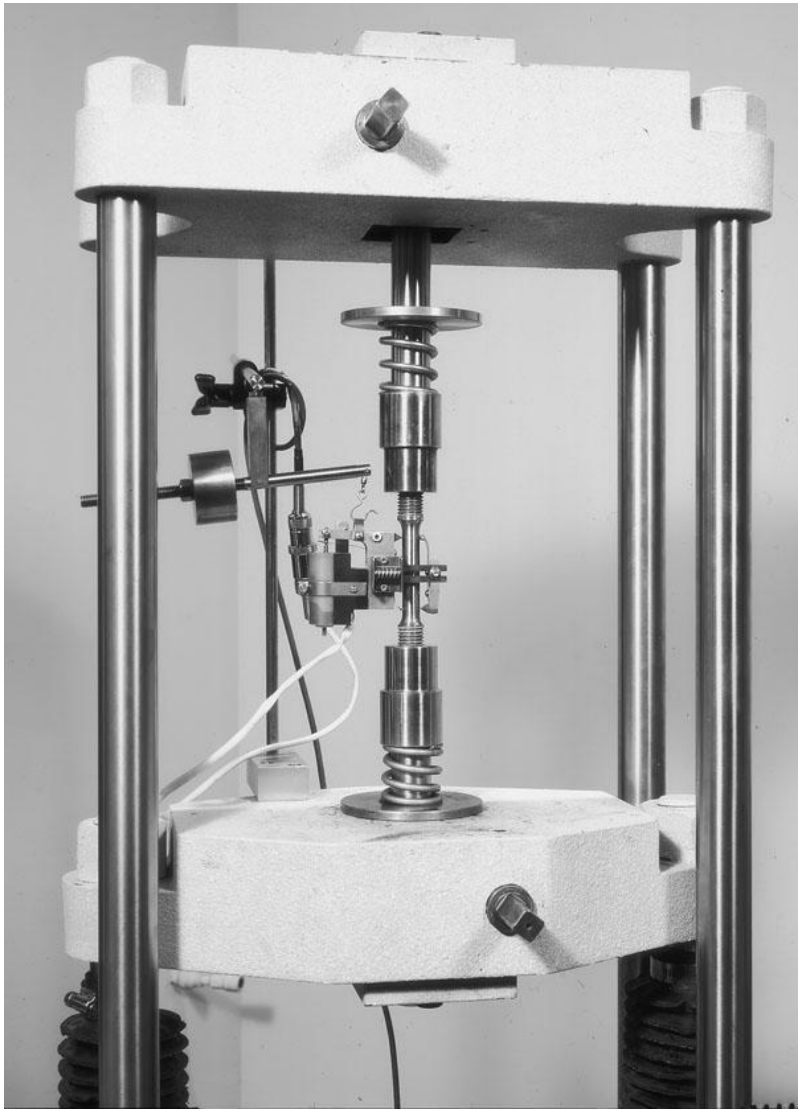


Fig 2.7 This machine is used to test tensile test specimens, such as those shown in this chapter.

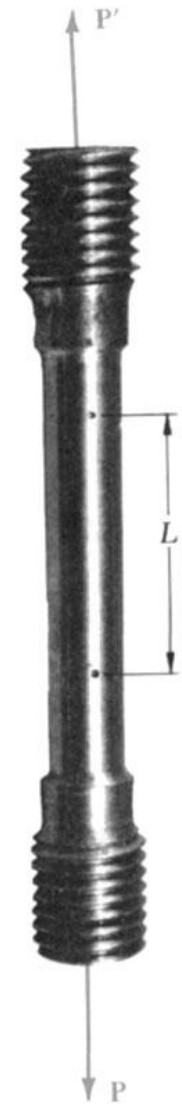
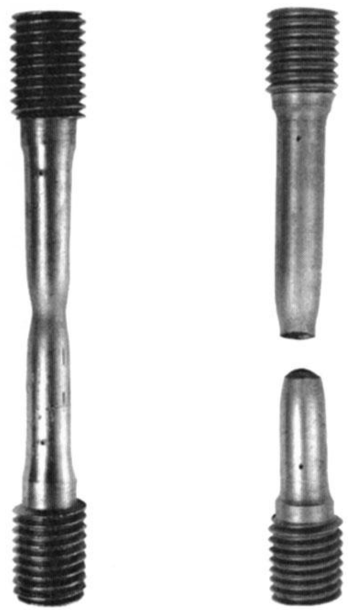
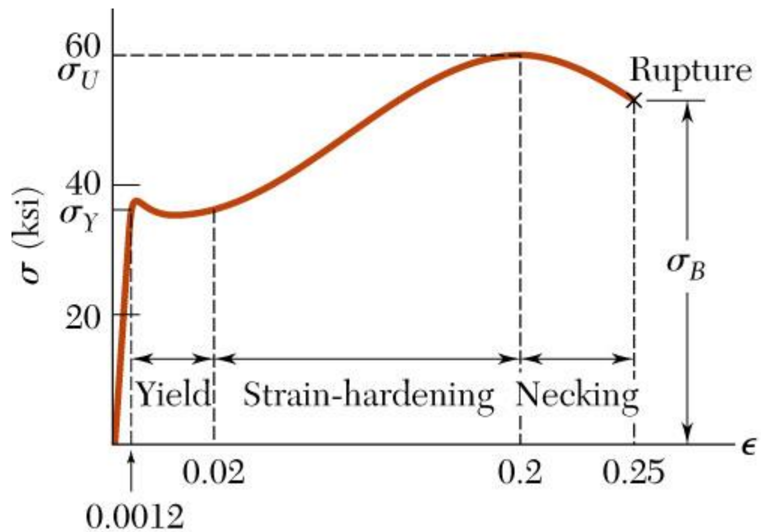


Fig 2.8 Test specimen with tensile load.

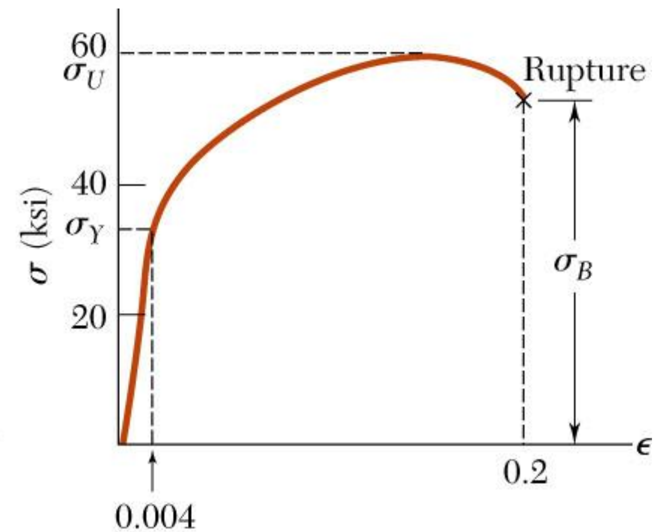
Stress-Strain Diagram: Ductile Materials



(*a*) (*b*)

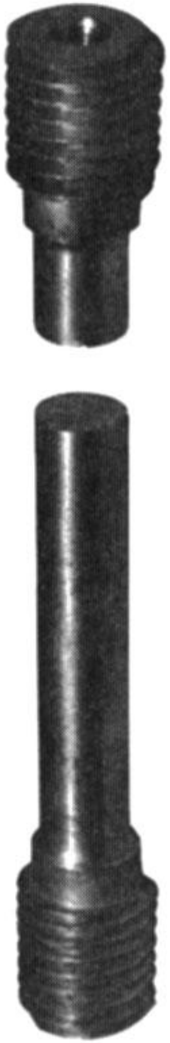


(*a*) Low-carbon steel



(*b*) Aluminum alloy

Stress-Strain Diagram: Brittle Materials



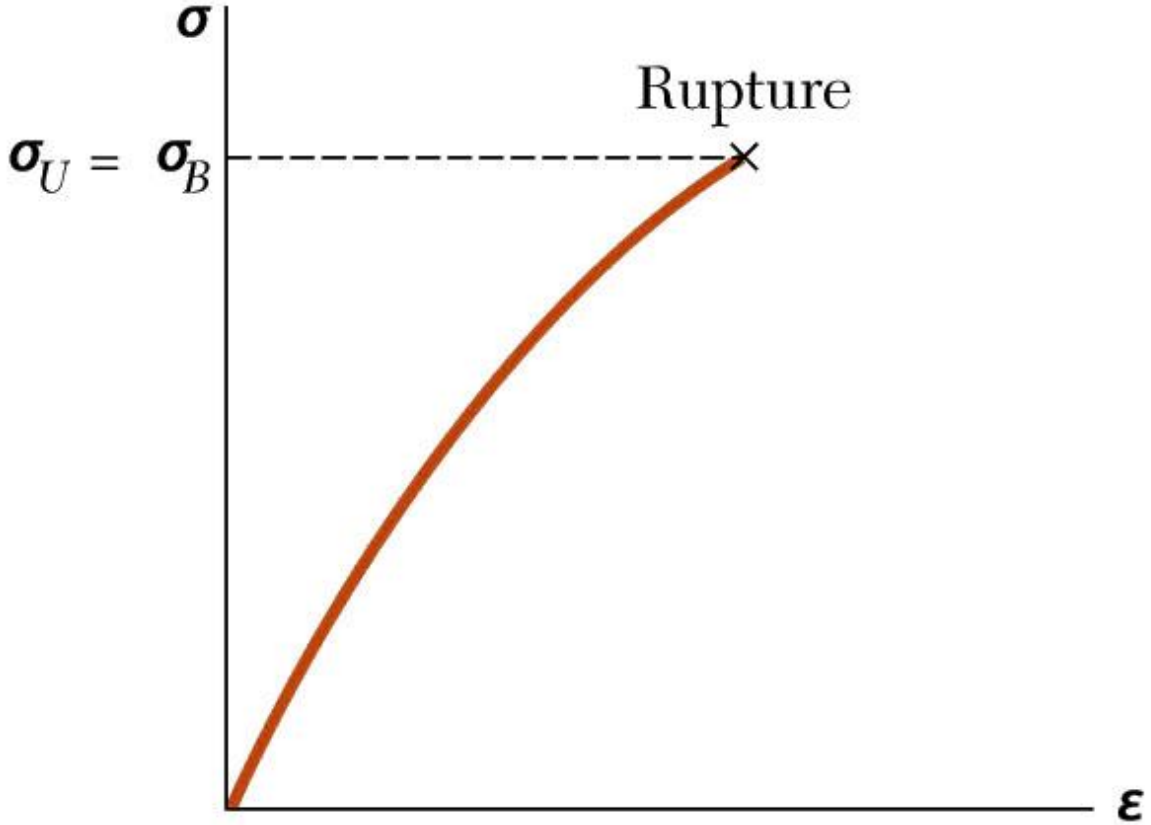


Fig 2.1 Stress-strain diagram for a typical brittle material.

Hooke's Law: Modulus of Elasticity

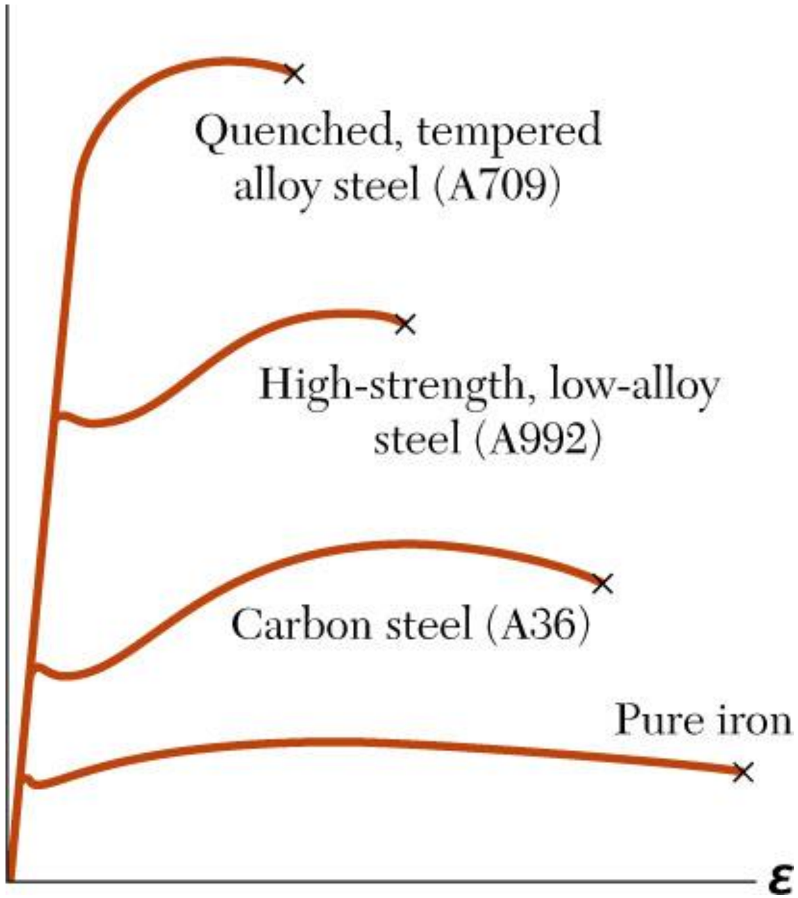


Fig 2.16 Stress-strain diagrams for iron and different grades of steel.

- Below the yield stress

$$\sigma = E\varepsilon$$

E = Youngs Modulus or Modulus of Elasticity

- Strength is affected by alloying, heat treating, and manufacturing process but stiffness (Modulus of Elasticity) is not.

Elastic vs. Plastic Behavior

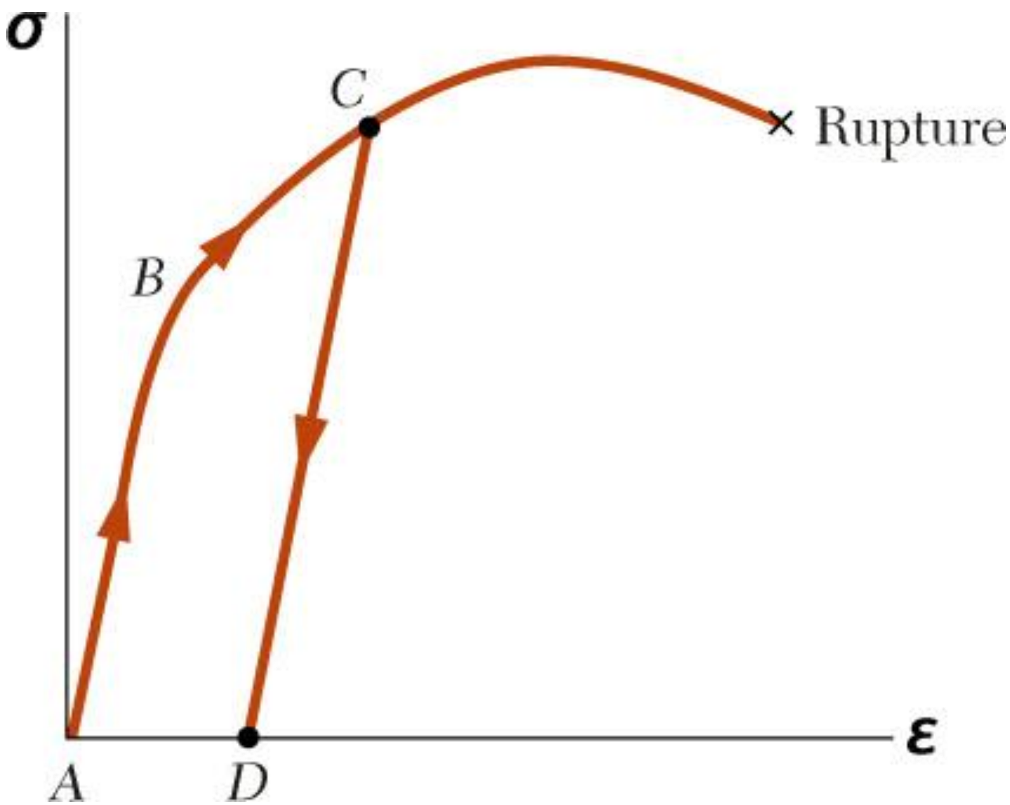


Fig. 2.18

- If the strain disappears when the stress is removed, the material is said to behave *elastically*.
- The largest stress for which this occurs is called the *elastic limit*.
- When the strain does not return to zero after the stress is removed, the material is said to behave *plastically*.

Deformations Under Axial Loading

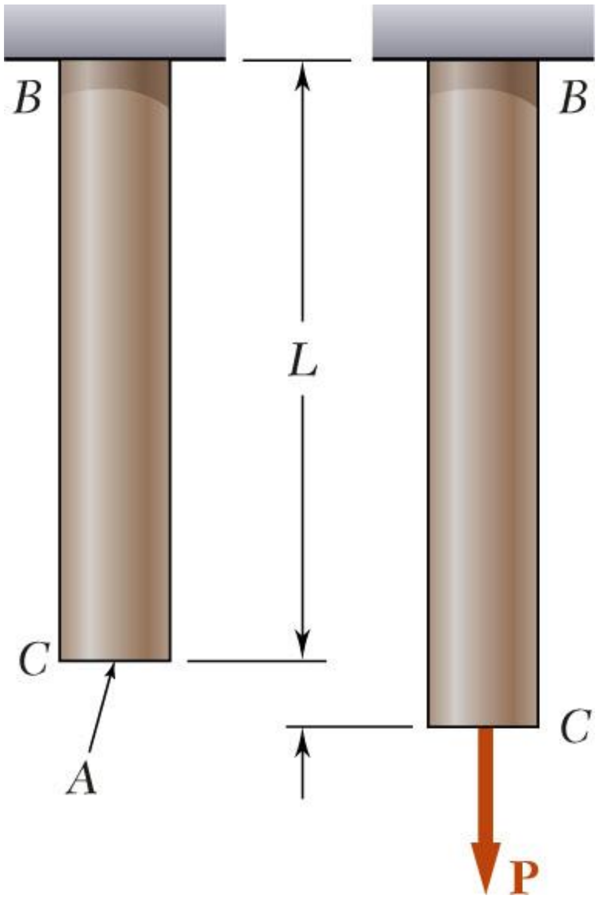


Fig. 2.22

- From Hooke's Law:

$$\sigma = E\varepsilon \qquad \varepsilon = \frac{\sigma}{E} = \frac{P}{AE}$$

- From the definition of strain:

$$\varepsilon = \frac{\delta}{L}$$

- Equating and solving for the deformation,

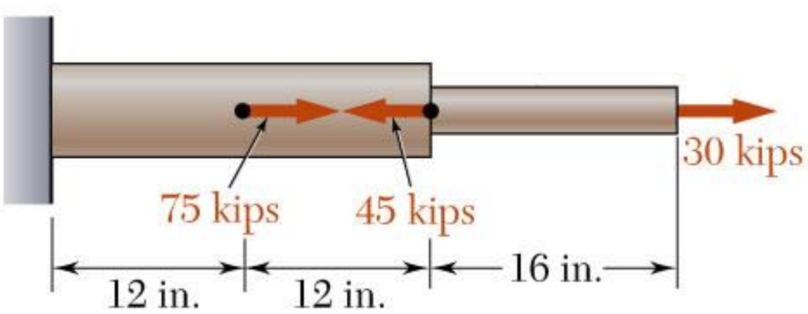
$$\delta = \frac{PL}{AE}$$

- With variations in loading, cross-section or material properties,

$$\delta = \sum_i \frac{P_i L_i}{A_i E_i}$$

End

Example 2.01



$$E = 29 \times 10^{-6}\,\text{psi}$$

$$D = 1.07\,\text{in.} \quad d = 0.618\,\text{in.}$$

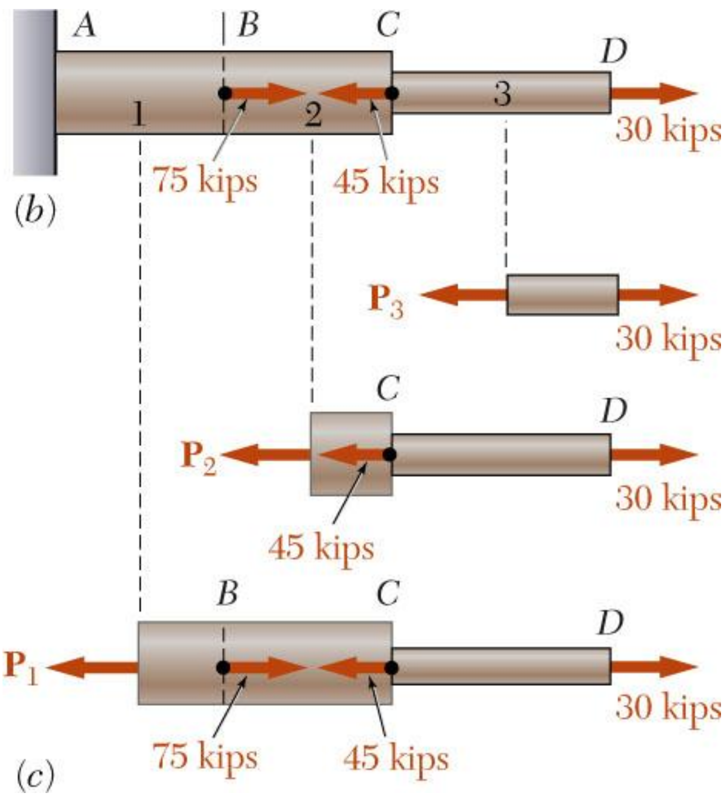
Determine the deformation of the steel rod shown under the given loads.

SOLUTION:

- Divide the rod into components at the load application points.
- Apply a free-body analysis on each component to determine the internal force
- Evaluate the total of the component deflections.

SOLUTION:

- Divide the rod into three components:



$L_1 = L_2 = 12\,\text{in.}$ $\quad L_3 = 16\,\text{in.}$

$A_1 = A_2 = 0.9\,\text{in}^2$ $\quad A_3 = 0.3\,\text{in}^2$

- Apply free-body analysis to each component to determine internal forces,

$$P_1 = 60\times10^3\,\text{lb}$$

$$P_2 = -15\times10^3\,\text{lb}$$

$$P_3 = 30\times10^3\,\text{lb}$$

- Evaluate total deflection,

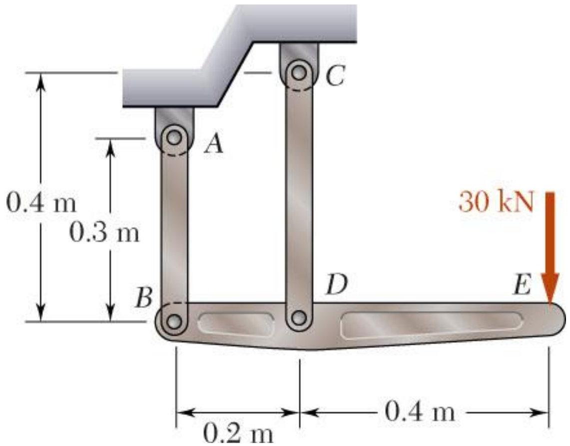
$$\delta = \sum_i \frac{P_i L_i}{A_i E_i} = \frac{1}{E}\left(\frac{P_1 L_1}{A_1} + \frac{P_2 L_2}{A_2} + \frac{P_3 L_3}{A_3}\right)$$

$$= \frac{1}{29\times10^6}\left[\frac{(60\times10^3)12}{0.9} + \frac{(-15\times10^3)12}{0.9} + \frac{(30\times10^3)16}{0.3}\right]$$

$$= 75.9\times10^{-3}\,\text{in.}$$

$$\boxed{\delta = 75.9\times10^{-3}\,\text{in.}}$$

Sample Problem 2.1



The rigid bar *BDE* is supported by two links *AB* and *CD*.

Link *AB* is made of aluminum (*E* = 70 GPa) and has a cross-sectional area of 500 mm². Link *CD* is made of steel (*E* = 200 GPa) and has a cross-sectional area of (600 mm²).

For the 30-kN force shown, determine the deflection a) of *B*, b) of *D*, and c) of *E*.

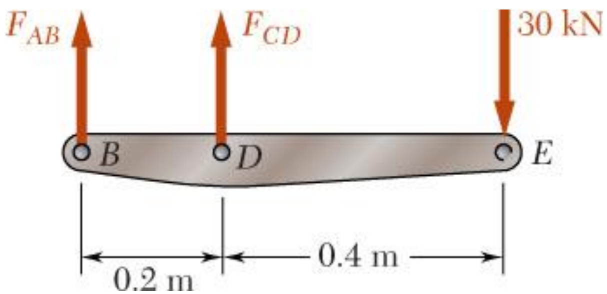
SOLUTION:

- Apply a free-body analysis to the bar *BDE* to find the forces exerted by links *AB* and *DC*.
- Evaluate the deformation of links *AB* and *DC* or the displacements of *B* and *D*.
- Work out the geometry to find the deflection at E given the deflections at B and D.

Sample Problem 2.1

SOLUTION:

Free body: Bar *BDE*



$$+\circlearrowleft \sum M_B = 0$$

$$0 = -(30\,\text{kN} \times 0.6\,\text{m}) + F_{CD} \times 0.2\,\text{m}$$

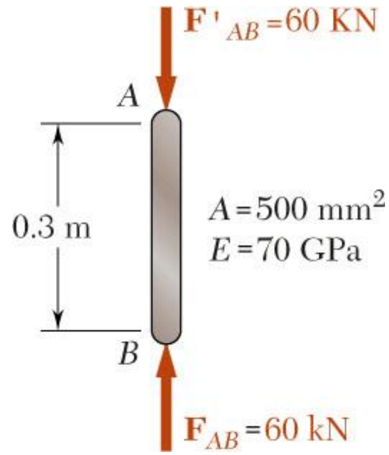
$$F_{CD} = +90\,\text{kN} \quad \textit{tension}$$

$$+\circlearrowleft \sum \text{M}_\text{D} = 0$$

$$0 = -(30\,\text{kN} \times 0.4\,\text{m}) - F_{AB} \times 0.2\,\text{m}$$

$$F_{AB} = -60\,\text{kN} \quad \textit{compression}$$

Displacement of *B*:



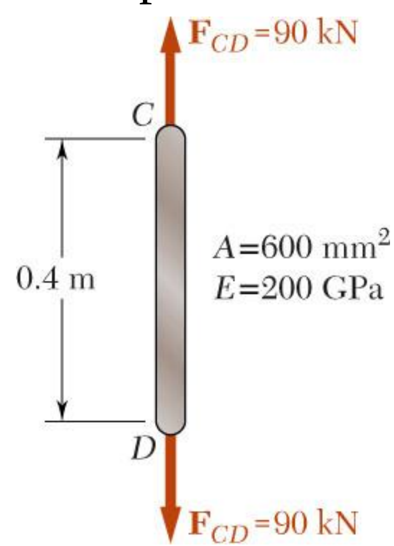
$$\delta_B = \frac{PL}{AE}$$

$$= \frac{\left(-60\times10^3\,\text{N}\right)(0.3\,\text{m})}{\left(500\times10^{-6}\,\text{m}^2\right)\left(70\times10^9\,\text{Pa}\right)}$$

$$= -514\times10^{-6}\,\text{m}$$

$$\boxed{\delta_B = 0.514\,\text{mm}\uparrow}$$

Displacement of *D*:



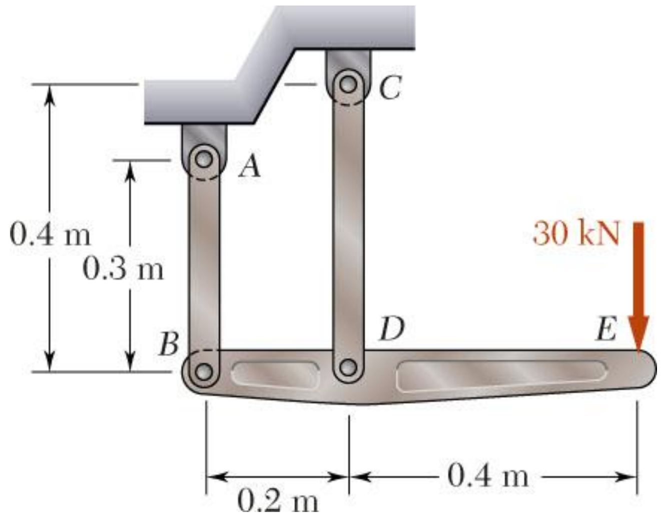
$$\delta_D = \frac{PL}{AE}$$

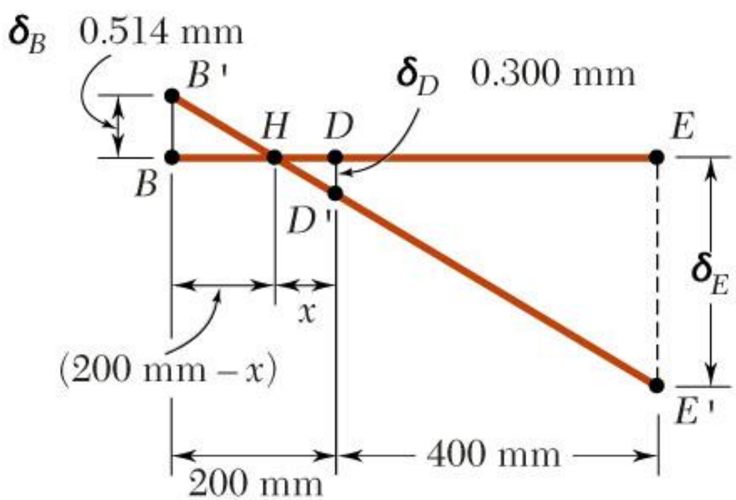
$$= \frac{\left(90\times10^3\,\text{N}\right)(0.4\,\text{m})}{\left(600\times10^{-6}\,\text{m}^2\right)\left(200\times10^9\,\text{Pa}\right)}$$

$$= 300\times10^{-6}\,\text{m}$$

$$\boxed{\delta_D = 0.300\,\text{mm}\downarrow}$$

Sample Problem 2.1





Displacement of D:

$$\frac{BB'}{DD'} = \frac{BH}{HD}$$

$$\frac{0.514\,\text{mm}}{0.300\,\text{mm}} = \frac{(200\,\text{mm}) - x}{x}$$

$$x = 73.7\text{ mm}$$

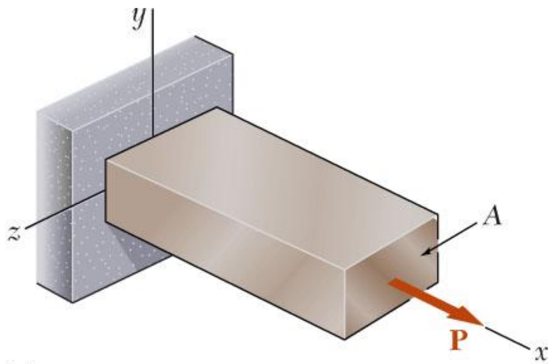
$$\frac{EE'}{DD'} = \frac{HE}{HD}$$

$$\frac{\delta_E}{0.300\,\text{mm}} = \frac{(400 + 73.7)\text{mm}}{73.7\text{ mm}}$$

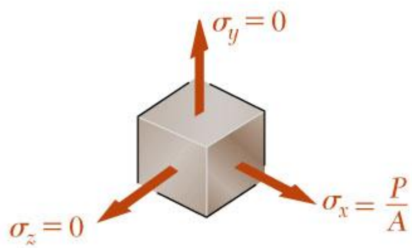
$$\delta_E = 1.928\,\text{mm}$$

$$\boxed{\delta_E = 1.928\,\text{mm}\downarrow}$$

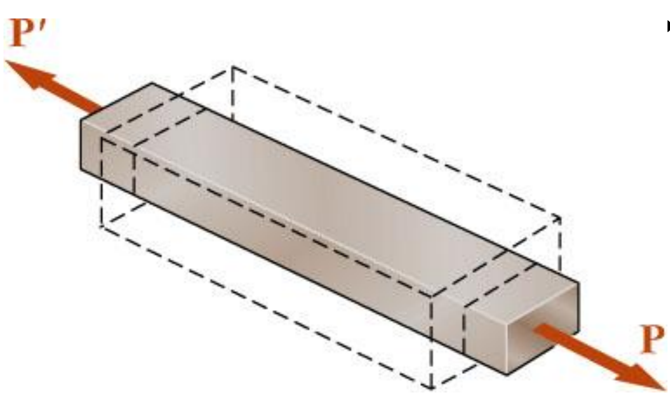
Poisson's Ratio



(a)



(b)



- For a slender bar subjected to axial loading:

$$\varepsilon_x = \frac{\sigma_x}{E} \qquad \sigma_y = \sigma_z = 0$$

- The elongation in the x-direction is accompanied by a contraction in the other directions. Assuming that the material is isotropic (no directional dependence),

$$\varepsilon_y = \varepsilon_z \neq 0$$

- Poisson's ratio is defined as

$$\nu = \left| \frac{\text{lateral strain}}{\text{axial strain}} \right| = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}$$

Shearing Strain

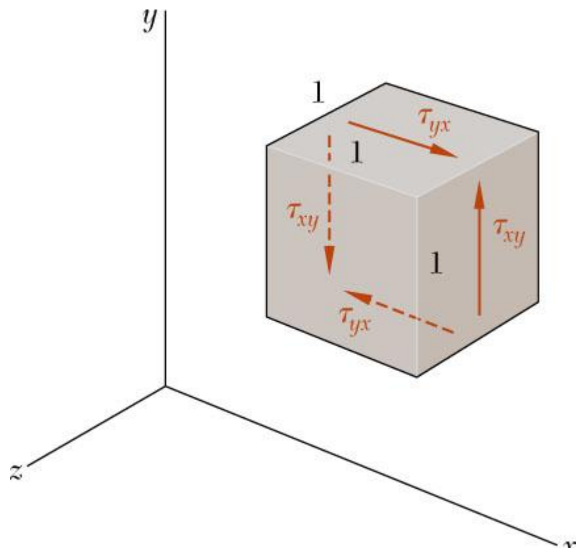


Fig. 2-46

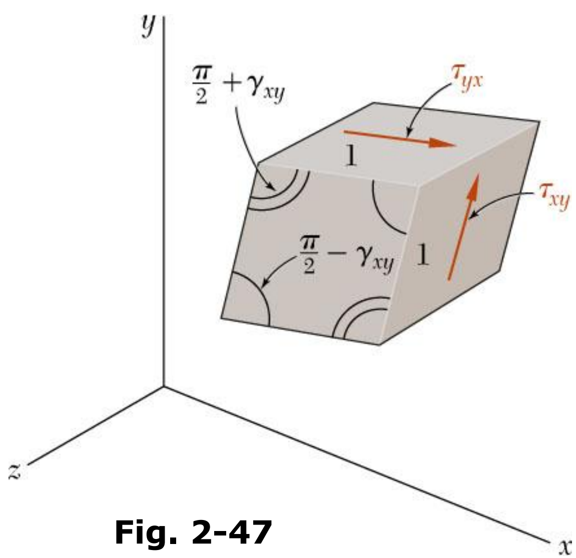


Fig. 2-47

- A cubic element subjected to a shear stress will deform into a rhomboid. The corresponding *shear* strain is quantified in terms of the change in angle between the sides,

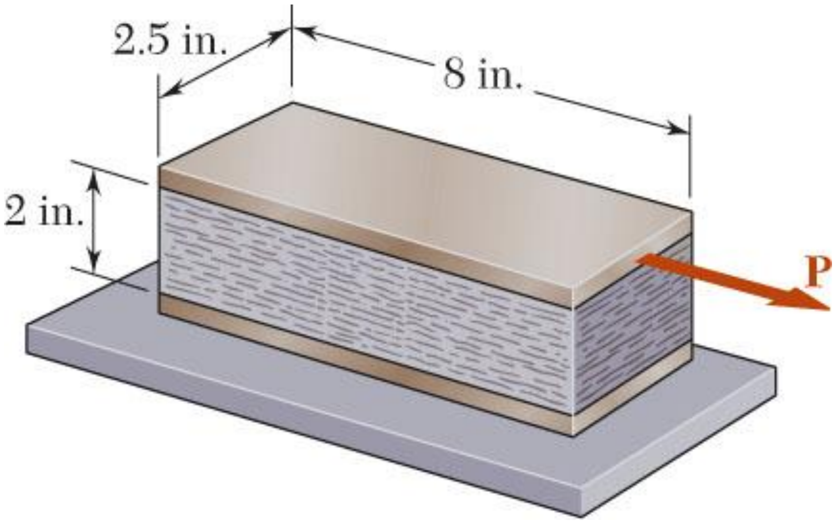
$$\tau_{xy} = f(\gamma_{xy})$$

- A plot of shear stress vs. shear strain is similar to the previous plots of normal stress vs. normal strain except that the strength values are approximately half. For small strains,

$$\tau_{xy} = G\gamma_{xy} \quad \tau_{yz} = G\gamma_{yz} \quad \tau_{zx} = G\gamma_{zx}$$

where G is the modulus of rigidity or shear modulus.

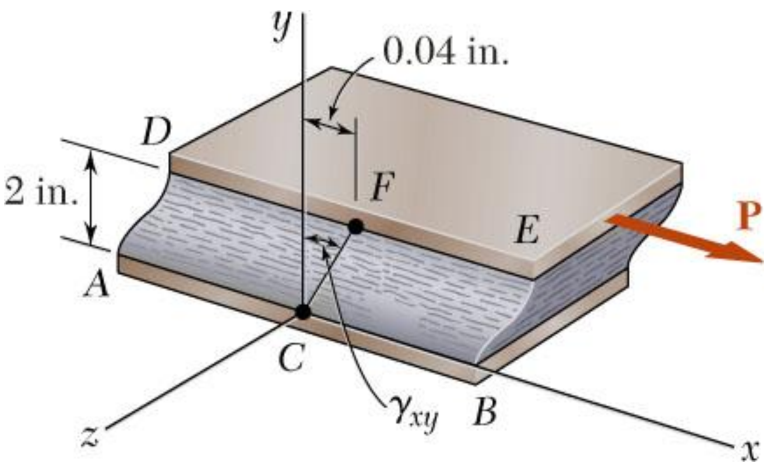
Example 2.10



A rectangular block of material with modulus of rigidity G = 90 ksi is bonded to two rigid horizontal plates. The lower plate is fixed, while the upper plate is subjected to a horizontal force P. Knowing that the upper plate moves through 0.04 in. under the action of the force, determine a) the average shearing strain in the material, and b) the force P exerted on the plate.

SOLUTION:

- Determine the average angular deformation or shearing strain of the block.
- Apply Hooke's law for shearing stress and strain to find the corresponding shearing stress.
- Use the definition of shearing stress to find the force P.



- Determine the average angular deformation or shearing strain of the block.

$$\gamma_{xy} \approx \tan\gamma_{xy} = \frac{0.04\,\text{in.}}{2\,\text{in.}} \qquad \gamma_{xy} = 0.020\,\text{rad}$$

- Apply Hooke's law for shearing stress and strain to find the corresponding shearing stress.

$$\tau_{xy} = G\gamma_{xy} = \left(90\times10^{3}\,\text{psi}\right)\left(0.020\,\text{rad}\right) = 1800\,\text{psi}$$

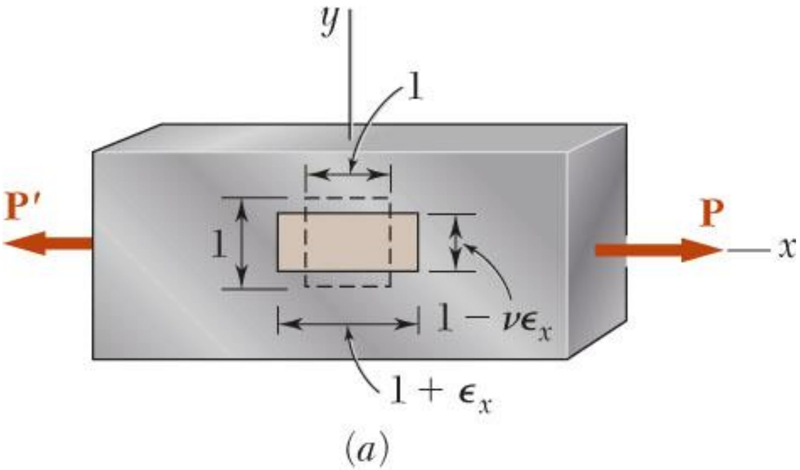
- Use the definition of shearing stress to find the force P.

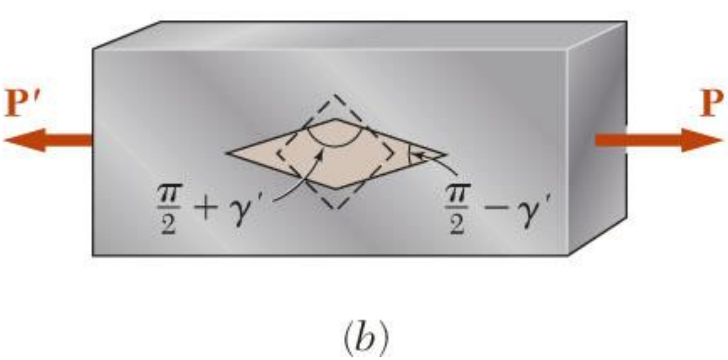
$$P = \tau_{xy}A = \left(1800\,\text{psi}\right)\left(8\,\text{in.}\right)\left(2.5\,\text{in.}\right) = 36\times10^{3}\,\text{lb}$$

$$\boxed{P = 36.0\,\text{kips}}$$

End

Relation Among E, ν, and G

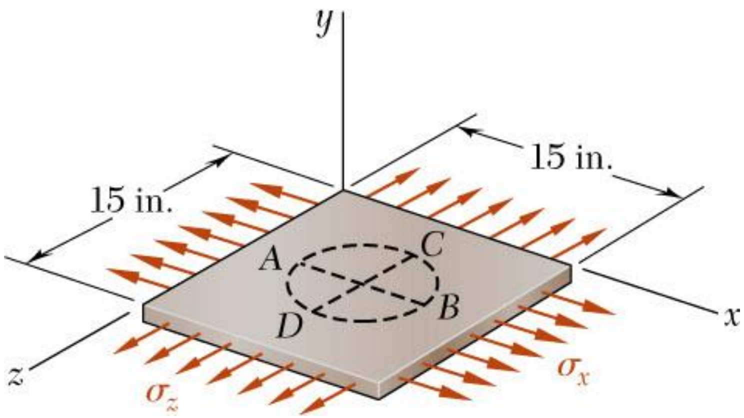




- An axially loaded slender bar will elongate in the axial direction and contract in the transverse directions.
- An initially cubic element oriented as in top figure will deform into a rectangular parallelepiped. The axial load produces a normal strain.
- If the cubic element is oriented as in the bottom figure, it will deform into a rhombus. Axial load also results in a shear strain.
- Components of normal and shear strain are related,

$$\frac{E}{2G} = (1 + \nu)$$

Sample Problem 2.5



A circle of diameter d = 9 in. is scribed on an unstressed aluminum plate of thickness t = 3/4 in. Forces acting in the plane of the plate later cause normal stresses σ_x = 12 ksi and σ_z = 20 ksi.

For $E = 10\text{x}10^6$ psi and ν = 1/3, determine the change in:

a) the length of diameter *AB*,

b) the length of diameter *CD*,

c) the thickness of the plate, and

d) the volume of the plate.

SOLUTION:

- Apply the generalized Hooke's Law to find the three components of normal strain.

$$\varepsilon_x = +\frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$= \frac{1}{10\times10^6\,\text{psi}}\left[(12\,\text{ksi}) - 0 - \frac{1}{3}(20\,\text{ksi})\right]$$

$$= +0.533\times10^{-3}\,\text{in./in.}$$

$$\varepsilon_y = -\frac{\nu\sigma_x}{E} + \frac{\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$= -1.067\times10^{-3}\,\text{in./in.}$$

$$\varepsilon_z = -\frac{\nu\sigma_x}{E} - \frac{\nu\sigma_y}{E} + \frac{\sigma_z}{E}$$

$$= +1.600\times10^{-3}\,\text{in./in.}$$

- Evaluate the deformation components.

$$\delta_{B/A} = \varepsilon_x d = \left(+0.533\times10^{-3}\,\text{in./in.}\right)(9\,\text{in.})$$

$$\boxed{\delta_{B/A} = +4.8\times10^{-3}\,\text{in.}}$$

$$\delta_{C/D} = \varepsilon_z d = \left(+1.600\times10^{-3}\,\text{in./in.}\right)(9\,\text{in.})$$

$$\boxed{\delta_{C/D} = +14.4\times10^{-3}\,\text{in.}}$$

$$\delta_t = \varepsilon_y t = \left(-1.067\times10^{-3}\,\text{in./in.}\right)(0.75\,\text{in.})$$

$$\boxed{\delta_t = -0.800\times10^{-3}\,\text{in.}}$$

- Find the change in volume

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z = 1.067\times10^{-3}\,\text{in}^3/\text{in}^3$$

$$\Delta V = eV = 1.067\times10^{-3}(15\times15\times0.75)\,\text{in}^3$$

$$\boxed{\Delta V = +0.187\,\text{in}^3}$$