1. In this question, $z = 2 + i$, $w = -1 - 2i$, and $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & i & -1 \\ 0 & 1 & 1 \end{bmatrix}$.

   Give the answer to each part. Simplify your answer as much as possible.

   (a) $w + \overline{z} =$____________________.

   (b) $\dfrac{z}{w} =$____________________.

   (c) $|z| =$____________________.

   (d) $(1 - i)\begin{bmatrix} z \\ w \end{bmatrix} - \begin{bmatrix} 1 \\ -2i \end{bmatrix} =$____________________.

   (e) $\det(A^8) =$____________________.

   (f) The nullity of A is____________________.

   (g) The rank of $A - I_3$, where I_3 is the 3×3 identity matrix, is____________________.

   (h) The number of distinct eigenvalues of A is ____________________.

2. Let $A = \begin{bmatrix} -1 & 1 & 0 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 2 & 0 \\ 2 & 0 & 0 & 2 \end{bmatrix}$.

   (a) Give a basis for the column space of A. Show your work.

   (b) Give a basis for the nullspace of A.

3. Let $A = \begin{bmatrix} 1 & 3 & 3 \\ -1 & -2 & -2 \\ 2 & 3 & 4 \end{bmatrix}$.

   (a) Find A^{-1}.

   (b) Find $\det(A)$.

4. Let $T\colon \mathbb{R}^{2\times 2} \to \mathbb{R}^3$ be a linear transformation with $T\left(\begin{bmatrix} 1 & 0 \\ -2 & -1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ and $T\left(\begin{bmatrix} -2 & 1 \\ 4 & 1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 2 \\ -4 \end{bmatrix}$.

   (a) Find $T\left(\begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}\right)$.

(b) Show that T has a nontrivial kernel.

5. Let $S\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 2x_2 + 3x_3 + x_4 \\ x_1 - 2x_2 + 2x_3 - 2x_4 \end{bmatrix}$.

(a) Give a matrix A such that $S(x) = Ax$.

(b) Give a basis for the kernel of S, $\ker(S)$.

(c) Is S one-to-one (injective)? Is S onto (surjective)?

(d) Is $\begin{bmatrix} \pi \\ \log_{10}(2013) \end{bmatrix}$ in the range of S? Explain.

6. Let $A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 4 & 1 \\ -2 & 2 & 4 \end{bmatrix}$.

(a) Give the characteristic polynomial of A in the form $a_3x^3 + a_2x^2 + a_1x + a_0$.

(b) Diagonalize A, if possible, i.e. write it explicitly as a product $A = PDP^{-1}$ or equivalently, write D as the product $D = S^{-1}AS$, where D is a diagonal matrix. You do not need to evaluate P^{-1} or S^{-1}.

7. Let $S = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} -5 \\ -10 \\ 9 \end{bmatrix} \right\}$.

(a) How many vectors are in S? How many vectors are in $\text{Span}(S)$?

(b) Is S a linearly independent set? If not, give a dependence relation between the vectors in S.

(c) is $\begin{bmatrix} -2 \\ -4 \\ 13 \end{bmatrix}$ in $\text{Span}(S)$?

(d) Give two sets of linearly independent vectors in $\mathbb{R}^3$.

(e) Let $B = \begin{bmatrix} 1 & -1 & -5 & -2 \\ 2 & -2 & -10 & -4 \\ 3 & 3 & 9 & 13 \end{bmatrix}$.

i. Find a basis of $\text{Col}(B)$.

ii. Find two bases of Row(B), the vectors of one must contain many zeroes.

iii. Find a basis of Null(B).

iv. Suppose T is a linear transformation where $T(x) = Bx$. Is T one-to-one (injective)? Is T onto (surjective)? Is T invertible?

8. Let $W = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} : x_1 = x_2 = x_3 \right\}$. Verify that W is a subspace of $\mathbb{R}^3$.