MAT 2379, Introduction to Biostatistics

Chapter 3. Axioms of Probability

In this chapter, we study some simple techniques which allow us to calculate the probabilities of events.

3.1. Venn Diagrams

The **Venn diagram** is a graphical method used for representing subsets of a set S. When using this method, we represent the sample space S (i.e. the set of all possible outcomes of a random experiment) as a rectangle, and an event A as a closed curve inside the rectangle.

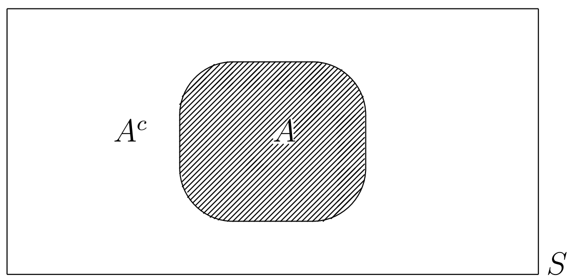

Figure 1: The shaded region represents the event A

We denote by A' the event that "A fails". A' is called the *complement* of A. We have

$$P(S) = 1 \text{ and } P(A) + P(A') = 1.$$

Example 1. A family has 3 children. Let A be the event "they have only boys". Then A' is the event "they have at least one girl". We know that $P(A) = 1/8$. Hence

$$P(A') = 1 - 1/8 = 7/8.$$

Let us consider two events A and B which can not occur in the same time. These are called *mutually exclusive* (or disjoint) events. When we draw the Venn diagrams, the regions inside the two curves representing A and B are not overlapping.

$$P(A \cup B) = P(A \text{ or } B) = P(A) + P(B) \quad \text{if } A, B \text{ are mutually exclusive}$$
$$P(A \cap B) = P(A \text{ and } B) = P(\emptyset) = 0 \quad \text{if } A, B \text{ are mutually exclusive}$$

When A and B are mutually exclusive, the event $A \cap B$ ("A and B occur") is the *impossible event* which is denoted by $\emptyset$ (the empty set symbol). The probability of the impossible event is 0.

Example 2. Among Canadians, 42% have blood type A, 9% have blood type B, 3% have blood type AB and 46% have blood type O. A new patient is admitted into a hospital and needs a blood transfusion. We are interested in the event that this patient has blood type A or B. We have

$$P(A \cup B) = P(A \text{ or } B) = P(A) + P(B) = 0.42 + 0.09 = 0.51$$

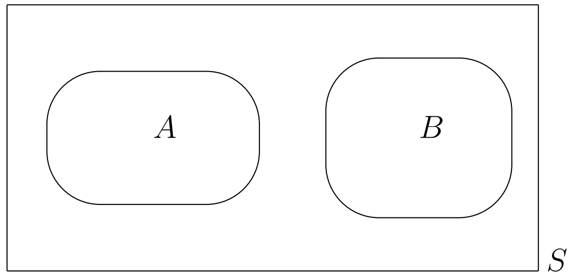

Figure 2: The Venn diagrams of mutually exclusive events A and B

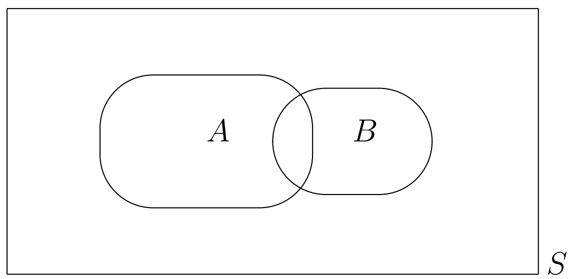

Figure 3: The Venn diagrams of events A and B which may occur in the same time

Let us consider two events A and B which may occur in the same time (e.g. A="patient has leukemia", B="his white blood cell count is high"). In this case, we draw the Venn diagram such that the regions inside the two curves representing A and B are overlapping.

We will explain in class which are the regions which correspond to the following events: $A \cap B$ ("A **and** B occur"), $A \cup B$ ("A **or** B occur"), $A \cap B'$ ("A occurs but B does not occur"), $A' \cap B$ ("B occurs but A does not occur").

3.2. Addition Rule

For calculating the probability of $A \cup B$ ("A or B occur"), we use the following **addition rule**:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

Example 3. In a group of 100 German men, 70 have blond hair, 77 have blue eyes and 55 have blond hair and blue eyes. We would like to calculate the probability that a randomly selected man in this group has blond hair or blue eyes. Let A be the event "the man has blond hair" and B be the event "the man has blue eyes". Since $P(A) = 0.70$, $P(B) = 0.77$ and $P(A \cap B) = 0.55$,

$$P(A \cup B) = 0.70 + 0.77 - 0.55 = 0.92.$$

The probability that the randomly selected man does not have blond hair, but has blue eyes is

$$P(A' \cap B) = P(B) - P(A \cap B) = 0.77 - 0.55 = 0.22$$

The probability that a randomly selected man does not have blond hair and does not have blue eyes is:

$$P(A' \cap B') = 1 - P(A \cup B) = 1 - 0.92 = 0.08$$