TEST 1 SOL'NS

1 for format

(1) $LS = \dfrac{1+\cot x}{1+\tan x}$

$= \dfrac{1+\frac{\cos x}{\sin x}}{1+\frac{\sin x}{\cos x}}$ 1 1

$= \dfrac{\frac{\sin x}{\sin x}+\frac{\cos x}{\sin x}}{\frac{\cos x}{\cos x}+\frac{\sin x}{\cos x}}$

$= \dfrac{\frac{\sin x+\cos x}{\sin x}}{\frac{\cos x+\sin x}{\cos x}}$

$= \dfrac{\cancel{\sin x+\cos x}}{\sin x}\cdot\dfrac{\cos x}{\cancel{\sin x+\cos x}}$

2 for work

$= \dfrac{\cos x}{\sin x}$ 1

$= \cot x$

$= RS$ 1

(2) (a) $\lim\limits_{x\to4}\dfrac{x-4}{x^2-16} = \lim\limits_{x\to4}\dfrac{x-4}{(x-4)(x+4)}$ 2 $= \lim\limits_{x\to4}\dfrac{1}{x+4}$ 1 $= \dfrac{1}{8}$ 1

(b) $\lim\limits_{x\to2}\dfrac{x+2}{x^2-4} = \lim\limits_{x\to2}\dfrac{x+2}{(x+2)(x-2)}$ 2 $= \lim\limits_{x\to2}\dfrac{1}{x-2}$ 1 $=$ DNE 1

This is because $\lim\limits_{x\to2^+}\dfrac{1}{x-2} \neq \lim\limits_{x\to2^-}\dfrac{1}{x-2}$

2

(c) $\lim_{x\to 0} \frac{x\sin x}{\sin 2x} = \lim_{x\to 0} \underbrace{\frac{\sin x}{1}\cdot\frac{1}{\sin 2x}\cdot\frac{x}{1}\cdot\frac{2x}{2x}}_{1}$

$= \lim_{x\to 0} \underbrace{\frac{\sin x}{x}\cdot\frac{2x}{\sin x}\cdot\frac{x}{2}}_{1} = \underbrace{\lim_{x\to 0}\frac{\sin x}{x}\cdot\lim_{x\to 0}\frac{2x}{\sin x}\cdot\lim_{x\to 0}\frac{x}{2}}_{1}$

$= (1)(1)(0) = \underline{\underline{0}}$ 1

(d) $\lim_{x\to -\infty} \frac{(x^2+1)^{\frac12}-x}{1}\cdot\frac{(x^2+1)^{\frac12}+x}{(x^2+1)^{\frac12}+x}$ 2

$= \lim_{x\to -\infty} \frac{x^2+1-x^2}{(x^2+1)^{\frac12}+x}$ 1 $= \lim_{x\to -\infty}\frac{1}{(x^2+1)^{\frac12}+x} = \underline{\underline{0}}$ 1

(by dominance)

(3) If $\left|\frac{x}{x-2}\right| < \frac12$, then $-\frac12 < \frac{x}{x-2} < \frac12$ 1

You can make this easier $\Rightarrow -1 < \frac{2x}{x-2} < 1$.

CASE I If $x-2>0$ 1 $\Rightarrow \boxed{x>2}$ 1

then $-x+2 < 2x < x-2$

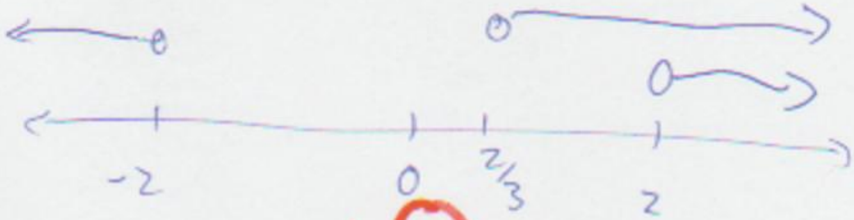
$-x+2 < 2x$

$2 < 3x$

$\boxed{x > \frac{2}{3}}$ 1

$2x < x-2$

$\boxed{x < -2}$ 1



$\therefore$ NO SOLUTION! (1)

CASE II If $x-2<0 \Rightarrow \boxed{x<2}$ (1)

$-x+2 > 2x > x-2$

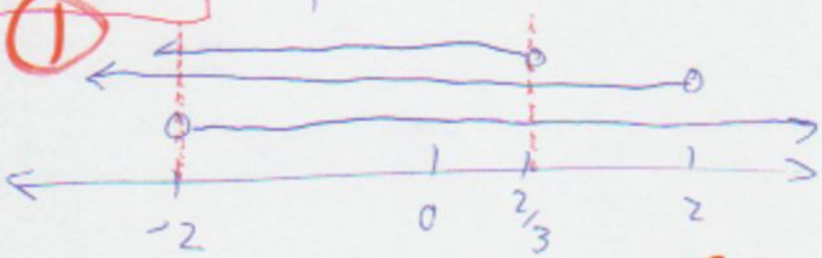
$-x+2 > 2x$

$2 > 3x$

$\boxed{x < \frac{2}{3}}$ (1)

$2x > x-2$

$\boxed{x > -2}$ (1)



$\therefore -2 < x < 2/3$ (1)

So for x to satisfy the inequality, $\underline{-2<x<\frac{2}{3}}$.

(4) (a) I/ where is $f(x)$ not define?

$f(x) = \csc x = \frac{1}{\sin x}$ (2)

$\sin x = 0$ if $x = 0, \pm\pi, \pm 2\pi, \ldots$ so $f(x)$ is undefined if $x = 0, \pm\pi, \pm 2\pi, \ldots$ (2)

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(b) I/ Where is $g(x)$ undefined?

$g(x)$ is always defined, but we suspect $x=-4$.

1

So $g(-4) = -5$ 1

II/ $\lim_{x\to -4^+} g(x) \stackrel{?}{=} \lim_{x\to 1^-} g(x) \stackrel{?}{=} L$

$$\lim_{x\to -4^+} \frac{x^2+3x-4}{x+4} = \lim_{x\to -4^+} \frac{(x-1)\cancel{(x+4)}}{\cancel{x+4}} = -5 \longrightarrow = L$$

1

$$\lim_{x\to -4^-} \frac{x^2+3x-4}{x+4} = \lim_{x\to -4^-} \frac{(x-1)\cancel{(x+4)}}{\cancel{x+4}} = -5$$

1 2

III/ $g(-4) = -5 \xleftrightarrow{\text{equal}} -5 = L$

1

$\therefore$ $g(x)$ is continuous.

1