

MATH1005C — Test 3 — 4:35–5:25, Mar. 20 2013

Name:

Student Number:

Total: 20 marks

Closed book, no GRAPHING calculator!

Question 1. Two linearly independent solutions of the system [CANCEL] $\begin{cases} x' = 3x + y \\ y' = -4x + 2y \end{cases}$ are

- a) $e^{-t} \begin{pmatrix} 1 \\ -4 \end{pmatrix}, e^{2t} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ b) $e^{-2t} \begin{pmatrix} 1 \\ -4 \end{pmatrix}, e^t \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ c) $e^{-2t} \begin{pmatrix} -4 \\ -1 \end{pmatrix}, e^{-t} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ d) $e^{-t} \begin{pmatrix} -1 \\ -4 \end{pmatrix}, e^{2t} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

Question 2. [3] The sum of the series $\sum_{n=0}^{\infty} \frac{3^{n+1}}{4^n}$ is

- a) 9 b) 12 (**) c) 4 d) 1

Question 3. [3] The general solution of $y''' - 5y'' + 17y' - 13y = 0$ is given by

- a) $c_1 e^{2x} + e^{2x}[c_2 \cos(3x) + c_3 \sin(3x)]$ b) $c_1 e^x + e^{2x}[c_2 \cos(2x) + c_3 \sin(2x)]$ (**)
c) $c_1 x e^x + e^{2x}[c_2 \cos(3x) + c_3 \sin(3x)]$ d) $c_1 x e^x + e^{2x}[c_2 \cos(2x) + c_3 \sin(2x)]$

Question 4. Determine whether the following series is convergent or divergent. Justify your answer.

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n^2 + 1} [2], \quad \sum_{n=1}^{\infty} n^2 \left(\frac{-3}{8} \right)^n [2], \quad \sum_{n=1}^{\infty} \frac{n+1}{3n+5} [2]$$

Sol:

$\sum_{n=0}^{\infty} \frac{(-1)^n}{n^2+1}$ is alternating series with $b_n = \frac{1}{n^2+1}$ which is decreasing with $\lim_{n \rightarrow \infty} \frac{1}{n^2+1} = 0$, so series converges.

$\sum_{n=1}^{\infty} n^2 \left(\frac{-3}{8} \right)^n =$ converges according to ratio test where

$$L = \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left((n+1)^2 \left(\frac{3}{8} \right)^{n+1} \right) \left(\frac{1}{n^2 \left(\frac{3}{8} \right)^n} \right) = \lim_{n \rightarrow \infty} \frac{(n+1)^2 \left(\frac{3}{8} \right)}{n^2} = \frac{3}{8} < 1$$

$\sum_{n=1}^{\infty} \frac{n+1}{3n+5}$ diverges since $\lim_{n \rightarrow \infty} \frac{n+1}{3n+5} \neq 0$ (n^{th} term test)

Question 5. For each one of the following series, determine whether it converges absolutely, converges conditionally, or diverges. Justify your answer.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{5^n} [2], \quad \sum_{n=1}^{\infty} \frac{(-2)^n}{3^n - 1} [2], \quad \sum_{n=0}^{\infty} \frac{(-1)^n}{n^2 + 1} [2], \quad \sum_{n=1}^{\infty} \frac{(-2)^n}{n!} [2]$$

Sol:

$\sum_{n=1}^{\infty} \frac{(-1)^n}{5^n}$ is alternating series, with $b_n = \frac{1}{5^n}$ decreasing and $\sum_{n=1}^{\infty} \frac{1}{5^n} = 0$, so series converges. $\sum_{n=1}^{\infty} \left| \frac{(-1)^n}{5^n} \right| = \sum_{n=1}^{\infty} \frac{1}{5^n}$ which converges based on ratio test where

$$L = \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \frac{1}{5^{n+1}} \frac{5^n}{1} = \frac{1}{5} < 1$$

so series converges absolutely. (or show by using geometric series)

$\sum_{n=1}^{\infty} \frac{(-2)^n}{3^n - 1}$ converges absolutely since $\sum_{n=1}^{\infty} \left| \frac{(-2)^n}{3^n - 1} \right| = \sum_{n=1}^{\infty} \frac{2^n}{3^n - 1}$ which converges.

$\sum_{n=0}^{\infty} \frac{(-1)^n}{n^2 + 1}$ is converges absolutely since $\sum_{n=0}^{\infty} \left| \frac{(-1)^n}{n^2 + 1} \right| = \sum_{n=0}^{\infty} \frac{1}{n^2 + 1}$ which converges according to comparison test that $\frac{1}{n^2 + 1} < \frac{1}{n^2}$ which converges based on p-series with $p = 2 > 1$.

$\sum_{n=1}^{\infty} \left| \frac{(-2)^n}{n!} \right| = \sum_{n=1}^{\infty} \frac{2^n}{n!}$ converges according to ratio test where

$$L = \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \frac{2^{n+1}}{(n+1)!} \frac{n!}{2^n} = \lim_{n \rightarrow \infty} \frac{2}{n+1} = 0 < 1$$

so the series converges absolutely.