

MATH 3705C - Mathematical Methods I - Winter 2011
Tutorial 1 Answers

1. Find $\mathcal{L}\{f(t)\}$, where $f(t) = \sqrt{t} + 2t^3 - e^{-3t} \cos(4t)$.

Answer: $F(s) = \frac{\sqrt{\pi}}{2s^{3/2}} + \frac{12}{s^4} - \frac{s+3}{(s+3)^2 + 16}$.

2. Find $\mathcal{L}\{f(t)\}$, where $f(t) = \left\{ \begin{array}{l} t, 0 \leq t \leq 2 \\ e^{2t} \sin[3(t-2)], t \geq 2 \end{array} \right\}$.

Answer: $F(s) = \frac{1}{s^2} + e^{-2s} \left[\frac{3e^4}{(s-2)^2 + 9} - \frac{1}{s^2} - \frac{2}{s} \right]$.

3. Solve the initial-value problem $y'' + 2y' + y = e^{-t}$, $y(0) = 1$, $y'(0) = -1$.

Answer: $y(t) = e^{-t} + \frac{1}{2}t^2e^{-t}$.

4. Find $\mathcal{L}^{-1} \left\{ \frac{e^{-3s}}{s(s^2 + 2s + 2)} \right\}$.

Answer: $f(t) = u(t-3) \left[\frac{1}{2} - \frac{1}{2}e^{-(t-3)} \cos(t-3) - \frac{1}{2}e^{-(t-3)} \sin(t-3) \right]$.

5. Find $\mathcal{L}\{f(t)\}$, where $f(t) = \left\{ \begin{array}{l} 0, t < 0 \\ t-1, 0 \leq t < 2 \\ t^2, t \geq 2 \end{array} \right\}$.

Answer: $F(s) = \frac{1}{s^2} - \frac{1}{s} + e^{-2s} \left(\frac{2}{s^3} + \frac{3}{s^2} + \frac{3}{s} \right)$.