UNIVERSITY OF OTTAWA
Principles of Physics II
PHY1322 Winter2013
Dr. A. Czajkowski

STUDENT #: ___________________

NAME: _________________________

Assignment 3: Birth of Modern Physics

Assigned: Jan 28 14:30 *Due: Feb4 17:30*

1. The radius of our Sun is 6.96 × 10⁸ m, and its total power output is 3.77 × 10²⁶ W. (a) Assuming that the Sun's surface emits as a black body, calculate its surface temperature. (b) Using the result of part (a), find λ_{max} for the Sun.

(a)

$$P = eA\sigma T^4 \quad T = \left(\frac{P}{eA\sigma}\right)^{1/4} = \left[\frac{3.77\times10^{26}\ \text{W}}{1\left[4\pi\left(6.96\times10^{8}\ \text{m}\right)^2\right]\left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)}\right]^{1/4} = \boxed{5.75\times10^{3}\ \text{K}}$$

$$\lambda_{max} = \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{T} = \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{5.75\times10^{3}\ \text{K}} = 5.04\times10^{-7}\ \text{m} = \boxed{504\ \text{nm}}$$

2. Consider a black body of surface area 20.0 cm² and temperature 5 000 K. (a) How much power does it radiate? (b) At what wavelength does it radiate most intensely? Find the spectral power per wavelength at (c) this wavelength and at wavelengths of (d) 1.00 nm (an x- or γ ray), (e) 5.00 nm (ultraviolet light or an x-ray), (f) 400 nm (at the boundary between UV and visible light), (g) 700 nm (at the boundary between visible and infrared light), (h) 1.00 mm (infrared light or a microwave) and (i) 10.0 cm (a microwave or radio wave). (j) About how much power does the object radiate as visible light?

(a) $$P = eA\sigma T^4 = 1\left(20.0\times10^{-4}\ \text{m}^2\right)\left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)\left(5\,000\ \text{K}\right)^4 = \boxed{7.09\times10^{4}\ \text{W}}$$

(b) $$\lambda_{max}T = \lambda_{max}\left(5\,000\ \text{K}\right) = 2.898\times10^{-3}\ \text{m}\cdot\text{K} \Rightarrow \lambda_{max} = \boxed{580\ \text{nm}}$$

(c) We compute: $$\frac{hc}{k_B T} = \frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{\left(1.38\times10^{-23}\ \text{J/K}\right)\left(5\,000\ \text{K}\right)} = 2.88\times10^{-6}\ \text{m}$$

The power per wavelength interval is $P(\lambda) = AI(\lambda) = \dfrac{2\pi hc^2 A}{\lambda^5\left[\exp\left(hc/\lambda k_B T\right)-1\right]}$, and

$$2\pi hc^2 A = 2\pi\left(6.626\times10^{-34}\right)\left(3.00\times10^{8}\right)^2\left(20.0\times10^{-4}\right) = 7.50\times10^{-19}\ \text{J}\cdot\text{m}^4/\text{s}$$

$$P(580\ \text{nm}) = \frac{7.50\times10^{-19}\ \text{J}\cdot\text{m}^4/\text{s}}{\left(580\times10^{-9}\ \text{m}\right)^5\left[\exp\left(2.88\ \mu\text{m}/0.580\ \mu\text{m}\right)-1\right]} = \frac{1.15\times10^{13}\ \text{J/m}\cdot\text{s}}{e^{4.973}-1}$$
$$= \boxed{7.99\times10^{10}\ \text{W/m}}$$

(d)–(i) The other values are computed similarly: see the other page

(j)

3 A sodium-vapor lamp has a power output of 10.0 W. Using 589.3 nm as the average wavelength of this source, calculate the number of photons emitted per second

$$E = hf = \frac{hc}{\lambda} = \frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{589.3\times10^{-9}\ \text{m}} = 3.37\times10^{-19}\ \text{J/photon}$$

$$n = \frac{P}{E} = \frac{10.0\ \text{J/s}}{3.37\times10^{-19}\ \text{J/photon}} = \boxed{2.96\times10^{19}\ \text{photons/s}}$$

Reverse page Problem 2 cont.
d-i

λ	$\frac{hc}{\lambda k_B T}$	$e^{hc/\lambda k_B T}-1$	$\frac{2\pi hc^2 A}{\lambda^5}$	$P(\lambda)$, W/m
1.00 nm	2882.6	7.96×10^{1251}	7.50×10^{26}	9.42×10^{-1226}
5.00 nm	576.5	2.40×10^{250}	2.40×10^{23}	1.00×10^{-227}
400 nm	7.21	1347	7.32×10^{13}	5.44×10^{10}
580 nm	4.97	143.5	1.15×10^{13}	7.99×10^{10}
700 nm	4.12	60.4	4.46×10^{12}	7.38×10^{10}
1.00 mm	0.00288	0.00289	7.50×10^{-4}	0.260
10.0 cm	2.88×10^{-5}	2.88×10^{-5}	7.50×10^{-14}	2.60×10^{-9}

(j) We approximate the area under the $P(\lambda)$ versus λ curve, between 400 nm and 700 nm, as two trapezoids:

$$P = \frac{\left[(5.44+7.99)\times10^{10}\ \text{W/m}\right]\left[(580-400)\times10^{-9}\ \text{m}\right]}{2} + \frac{\left[(7.99+7.38)\times10^{10}\ \text{W/m}\right]\left[(700-580)\times10^{-9}\ \text{m}\right]}{2}$$

$P = 2.13\times10^{4}$ W so the power radiated as visible light is **approximately 20 kW**.

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Assignment 3 CONT.

4Calculate the energy, in electron volts, of a photon whose frequency is
a) 620 THz,
b) 3.10 GHz,
c) 46.0 MHz.
d) Determine the corresponding wavelengths for these photons and state the classification of each on the electromagnetic spectrum.

(a) $$E = hf = \left(6.626\times 10^{-34}\ \text{J}\cdot\text{s}\right)\left(620\times 10^{12}\ \text{s}^{-1}\right)\left(\frac{1.00\ \text{eV}}{1.60\times 10^{-19}\ \text{J}}\right) = \boxed{2.57\ \text{eV}}$$

(b) $$E = hf = \left(6.626\times 10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.10\times 10^{9}\ \text{s}^{-1}\right)\left(\frac{1.00\ \text{eV}}{1.60\times 10^{-19}\ \text{J}}\right) = \boxed{1.28\times 10^{-5}\ \text{eV}}$$

(c) $$E = hf = \left(6.626\times 10^{-34}\ \text{J}\cdot\text{s}\right)\left(46.0\times 10^{6}\ \text{s}^{-1}\right)\left(\frac{1.00\ \text{eV}}{1.60\times 10^{-19}\ \text{J}}\right) = \boxed{1.91\times 10^{-7}\ \text{eV}}$$

(d) $$\lambda = \frac{c}{f} = \frac{3.00\times 10^{8}\ \text{m/s}}{620\times 10^{12}\ \text{Hz}} = 4.84\times 10^{-7}\ \text{m} = \boxed{484\ \text{nm, visible light (blue)}}$$

$$\lambda = \frac{c}{f} = \frac{3.00\times 10^{8}\ \text{m/s}}{3.10\times 10^{9}\ \text{Hz}} = 9.68\times 10^{-2}\ \text{m} = \boxed{9.68\ \text{cm, radio wave}}$$

$$\lambda = \frac{c}{f} = \frac{3.00\times 10^{8}\ \text{m/s}}{46.0\times 10^{6}\ \text{Hz}} = \boxed{6.52\ \text{m, radio wave}}$$

5 Molybdenum has a work function of 4.20 eV.
a) Find the cutoff wavelength and cutoff frequency for the photoelectric effect.
b) What is the stopping potential if the incident light has a wavelength of 180 nm?

(a) $$\lambda_c = \frac{hc}{\phi} = \frac{\left(6.626\times 10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times 10^{8}\ \text{m/s}\right)}{\left(4.20\ \text{eV}\right)\left(1.60\times 10^{-19}\ \text{J/eV}\right)} = \boxed{296\ \text{nm}}\ ;\ f_c = \frac{c}{\lambda_c} = \frac{3.00\times 10^{8}\ \text{m/s}}{296\times 10^{-9}\ \text{m}} = \boxed{1.01\times 10^{15}\ \text{Hz}}$$

(b) $$\frac{hc}{\lambda} = \phi + e\Delta V_S:\ \frac{\left(6.626\times 10^{-34}\right)\left(3.00\times 10^{8}\right)}{180\times 10^{-9}} = \left(4.20\ \text{eV}\right)\left(1.60\times 10^{-19}\ \text{J/eV}\right) + \left(1.60\times 10^{-19}\right)\Delta V_S\ \boxed{\Delta V_S = 2.71\ \text{V}}$$

6 Electrons are ejected from a metallic surface with speeds ranging up to 4.60 × 10^5 m/s when light with a wavelength of 625 nm is used.
a)What is the work function of the surface?
b) What is the cutoff frequency for this surface?

$$K_{\text{max}} = \frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}\left(9.11\times 10^{-31}\right)\left(4.60\times 10^{5}\right)^2 = 9.64\times 10^{-20}\ \text{J} = 0.602\ \text{eV}$$

(a) $$\phi = E - K_{\text{max}} = \frac{1\,240\ \text{eV}\cdot\text{nm}}{625\ \text{nm}} - 0.602\ \text{nm} = \boxed{1.38\ \text{eV}}$$

(b) $$f_c = \frac{\phi}{h} = \frac{1.38\ \text{eV}}{6.626\times 10^{-34}\ \text{J}\cdot\text{s}}\left(\frac{1.60\times 10^{-19}\ \text{J}}{1\ \text{eV}}\right) = \boxed{3.34\times 10^{14}\ \text{Hz}}$$