UNIVERSITY OF OTTAWA
Principles of Physics II
PHY1322 Winter2012
Dr. A. Czajkowski

STUDENT #: ________________

NAME: ________________

Assignment 2: Nuclear Physics

Assigned: Jan 18 14:30 *Due: Jan 25 11:30*

1. Calculate the binding energy per nucleon for (a) ^{2}H, , (b) ^{56}Fe, and (c) ^{238}U.
Using atomic masses as given in Table A.3,

(a) For $^{2}_{1}\mathrm{H}$: $\frac{-2.014\,102+1(1.008\,665)+1(1.007\,825)}{2}$

$$\frac{E_b}{A}=(0.001\,194\text{ u})\left(\frac{931.5\text{ MeV}}{\text{u}}\right)=\boxed{1.11\text{ MeV/nucleon}}.$$

(c) For $^{56}_{26}\mathrm{Fe}$: $30(1.008\,665)+26(1.007\,825)-55.934\,942=0.528\text{ u}$

$$\frac{E_b}{A}=\frac{0.528}{56}=0.009\,44\text{ u}c^2=\boxed{8.79\text{ MeV/nucleon}}.$$

(c) For $^{238}_{92}\mathrm{U}$: $146(1.008\,665)+92(1.007\,825)-238.050\,783=1.934\,2\text{ u}$

$$\frac{E_b}{A}=\frac{1.934\,2}{238}=0.008\,13\text{ u}c^2=\boxed{7.57\text{ MeV/nucleon}}.$$

2. The iron isotope ^{56}Fe is near the peak of the stability curve. This is why iron is generally prominent in the spectrum of the Sun and stars. Show that ^{56}Fe has a higher binding energy per nucleon than its neighbors ^{55}Mn and ^{59}Co. (calculate BE/A in MeV) for all three isotopes. Compare your results with the results from diagram.

$$\Delta M=Zm_{\mathrm{H}}+Nm_{\mathrm{n}}-M \qquad \frac{\mathrm{BE}}{A}=\frac{\Delta M(931.5)}{A}$$

Nuclei	Z	N	M in u	ΔM in u	$\frac{\mathrm{BE}}{A}$ in MeV
$^{55}\mathrm{Mn}$	25	30	54.938 050	0.517 5	8.765
$^{56}\mathrm{Fe}$	26	30	55.934 942	0.528 46	8.790
$^{59}\mathrm{Co}$	27	32	58.933 200	0.555 35	8.768

$\therefore$ $^{56}\mathrm{Fe}$ has a greater $\frac{\mathrm{BE}}{A}$ than its neighbors. This tells us finer detail than is shown diagram of (BE per nucleon) vs. A

3. A certain African artifact is found to have a carbon-14 activity of (0.12 ± 0.01) Bq per gram of carbon. Assume the uncertainty is negligible in the half-life of ^{14}C (5 730 yr) and in the activity of atmospheric carbon (0.25 Bq per gram). The age of the object lies within what range?

$$N=N_0e^{-\lambda t} \qquad \left|\frac{dN}{dt}\right|=R=\left|-\lambda N_0e^{-\lambda t}\right|=R_0e^{-\lambda t}$$

$$e^{-\lambda t}=\frac{R}{R_0} \qquad e^{\lambda t}=\frac{R_0}{R} \qquad \lambda t=\ln\left(\frac{R_0}{R}\right)=\frac{\ln 2}{T_{1/2}}t \qquad t=T_{1/2}\frac{\ln(R_0/R)}{\ln 2}$$

If $R=0.13\text{ Bq}$, $t=5\,730\text{ yr}\frac{\ln(0.25/0.13)}{0.693}=5\,406\text{ yr}$.

If $R=0.11\text{ Bq}$, $t=5\,730\text{ yr}\frac{\ln(0.25/0.11)}{0.693}=6\,787\text{ yr}$.

The range is most clearly written as $\boxed{\text{between 5 400 yr and 6 800 yr}}$, without understatement.

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4. The half-life of ^{131}I is 8.04 days. On a certain day, the activity of an iodine-131 sample is 6.40 mCi. What is its activity 40.2 days later?

$$R = R_0 e^{-\lambda t} = (6.40\text{ mCi})e^{-(\ln 2/8.04\text{ d})(40.2\text{ d})} = (6.40\text{ mCi})\left(e^{-\ln 2}\right)^5 = (6.40\text{ mCi})\left(\frac{1}{2^5}\right) = \boxed{0.200\text{ mCi}}$$

5. A living specimen in equilibrium with the atmosphere contains one atom of ^{14}C (half-life = 5 730 yr) for every 7.7 × 10^{11} stable carbon atoms. An archeological sample of wood (cellulose, $C_{12}H_{22}O_{11}$) contains 21.0 mg of carbon. When the sample is placed inside a shielded beta counter with 88.0% counting efficiency, 837 counts are accumulated in one week. Assuming that the cosmic-ray flux and the Earth's atmosphere have not changed appreciably since the sample was formed, find the age of the sample.

$$N_C = \left(\frac{0.021\,0\text{ g}}{12.0\text{ g/mol}}\right)\left(6.02\times10^{23}\text{ molecules/mol}\right)$$

$$\left(N_C = 1.05\times10^{21}\text{ carbon atoms}\right)\text{ of which 1 in } 7.70\times10^{11}\text{ is a } ^{14}\text{C atom}$$

$$(N_0)_{\text{C-14}} = 1.37\times10^9, \qquad \lambda_{\text{C-14}} = \frac{\ln 2}{5\,730\text{ yr}} = 1.21\times10^{-4}\text{ yr}^{-1} = 3.83\times10^{-12}\text{ s}^{-1}$$

$$R = \lambda N = \lambda N_0 e^{-\lambda t}$$

At $t = 0$, $R_0 = \lambda N_0 = \left(3.83\times10^{-12}\text{ s}^{-1}\right)\left(1.37\times10^9\right)\left[\frac{7(86\,400\text{ s})}{1\text{ week}}\right] = 3.17\times10^3\text{ decays/week}$.

At time t, $R = \frac{837}{0.88} = 951\text{ decays/week}$. Taking logarithms, $\ln\frac{R}{R_0} = -\lambda t$ so $t = \frac{-1}{\lambda}\ln\left(\frac{R}{R_0}\right)$

$$t = \frac{-1}{1.21\times10^{-4}\text{ yr}^{-1}}\ln\left(\frac{951}{3.17\times10^3}\right) = \boxed{9.96\times10^3\text{ yr}}.$$

6. A theory of nuclear astrophysics proposes that all the elements heavier than iron are formed in supernova explosions ending the lives of massive stars. Assume that at the time of the explosion the amounts of ^{235}U and ^{238}U were equal. How long ago did the star(s) explode that released the elements that formed our Earth? The present ^{235}U/^{238}U ratio is 0.007 25. The half-lives of ^{235}U and ^{238}U are 0.704 × 10^9 yr and 4.47 × 10^9 yr.

We have $N_{235} = N_{0,\,235}e^{-\lambda_{235}t}$ and $N_{238} = N_{0,\,238}e^{-\lambda_{238}t}$

$\frac{N_{235}}{N_{238}} = 0.007\,25 = e^{\left(-(\ln 2)t/T_{h,\,235} + (\ln 2)t/T_{h,\,238}\right)}$. Taking logarithms,

$$-4.93 = \left(-\frac{\ln 2}{0.704\times10^9\text{ yr}} + \frac{\ln 2}{4.47\times10^9\text{ yr}}\right)t \quad\text{or}\quad -4.93 = \left(-\frac{1}{0.704\times10^9\text{ yr}} + \frac{1}{4.47\times10^9\text{ yr}}\right)(\ln 2)t$$

$$t = \frac{-4.93}{\left(-1.20\times10^{-9}\text{ yr}^{-1}\right)\ln 2} = \boxed{5.94\times10^9\text{ yr}}$$