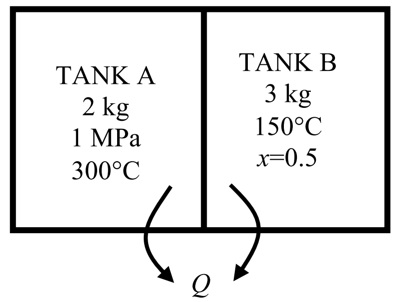
ENGR 251: Thermodynamics

Solution to Tutorial 4: Chapter 4

4-42 Two tanks initially separated by a partition contain steam at different states. Now the partition is removed and they are allowed to mix until equilibrium is established. The temperature and quality of the steam at the final state and the amount of heat lost from the tanks are to be determined.

Assumptions **1** The tank is stationary and thus the kinetic and potential energy changes are zero. **2** There are no work interactions.



Analysis (*a*) We take the contents of both tanks as the system. This is a closed system since no mass enters or leaves. Noting that the volume of the system is constant and thus there is no boundary work, the energy balance for this stationary closed system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{system}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$-Q_{out} = \Delta U_A + \Delta U_B = [m(u_2 - u_1)]_A + [m(u_2 - u_1)]_B \quad (\text{since } W = \text{KE} = \text{PE} = 0)$$

The properties of steam in both tanks at the initial state are (Tables A-4 through A-6)

$$\left.\begin{aligned} P_{1,A} &= 1000 \text{ kPa} \\ T_{1,A} &= 300°\text{C} \end{aligned}\right\} \begin{aligned} v_{1,A} &= 0.25799 \text{ m}^3/\text{kg} \\ u_{1,A} &= 2793.7 \text{ kJ/kg} \end{aligned}$$

$$\left.\begin{aligned} T_{1,B} &= 150°\text{C} \\ x_1 &= 0.50 \end{aligned}\right\} \begin{aligned} v_f &= 0.001091, \quad v_g = 0.39248 \text{ m}^3/\text{kg} \\ u_f &= 631.66, \quad u_{fg} = 1927.4 \text{ kJ/kg} \end{aligned}$$

$$v_{1,B} = v_f + x_1 v_{fg} = 0.001091 + [0.50 \times (0.39248 - 0.001091)] = 0.19679 \text{ m}^3/\text{kg}$$

$$u_{1,B} = u_f + x_1 u_{fg} = 631.66 + (0.50 \times 1927.4) = 1595.4 \text{ kJ/kg}$$

The total volume and total mass of the system are

$$V = V_A + V_B = m_A v_{1,A} + m_B v_{1,B} = (2 \text{ kg})(0.25799 \text{ m}^3/\text{kg}) + (3 \text{ kg})(0.19679 \text{ m}^3/\text{kg}) = 1.106 \text{ m}^3$$

$$m = m_A + m_B = 3 + 2 = 5 \text{ kg}$$

Now, the specific volume at the final state may be determined

$$v_2 = \frac{V}{m} = \frac{1.106 \text{ m}^3}{5 \text{ kg}} = 0.22127 \text{ m}^3/\text{kg}$$

which fixes the final state and we can determine other properties

$$\left.\begin{aligned} P_2 &= 300 \text{ kPa} \\ v_2 &= 0.22127 \text{ m}^3/\text{kg} \end{aligned}\right\} \begin{aligned} T_2 &= T_{sat @ 300 \text{ kPa}} = \mathbf{133.5°C} \\ x_2 &= \frac{v_2 - v_f}{v_g - v_f} = \frac{0.22127 - 0.001073}{0.60582 - 0.001073} = \mathbf{0.3641} \\ u_2 &= u_f + x_2 u_{fg} = 561.11 + (0.3641 \times 1982.1) = 1282.8 \text{ kJ/kg} \end{aligned}$$

(*b*) Substituting,

$$\begin{aligned} -Q_{out} &= \Delta U_A + \Delta U_B = [m(u_2 - u_1)]_A + [m(u_2 - u_1)]_B \\ &= (2 \text{ kg})(1282.8 - 2793.7)\text{kJ/kg} + (3 \text{ kg})(1282.8 - 1595.4)\text{kJ/kg} = -3959 \text{ kJ} \end{aligned}$$

or $\quad Q_{out} = \mathbf{3959 \ kJ}$

4-63 The internal energy change of hydrogen gas during a heating process is to be determined using an empirical specific heat relation, constant specific heat at average temperature, and constant specific heat at room temperature.

Analysis (*a*) Using the empirical relation for $\bar{c}_p(T)$ from Table A-2c and relating it to $\bar{c}_v(T)$,

$$\bar{c}_v(T) = \bar{c}_p - R_u = (a - R_u) + bT + cT^2 + dT^3$$

where a = 29.11, b = -0.1916×10^{-2}, c = 0.4003×10^{-5}, and d = -0.8704×10^{-9}. Then,

$$\begin{aligned}
\Delta\bar{u} &= \int_1^2 \bar{c}_v(T)dT = \int_1^2 \left[(a - R_u) + bT + cT^2 + dT^3\right]dT \\
&= (a - R_u)(T_2 - T_1) + \tfrac{1}{2}b(T_2^2 + T_1^2) + \tfrac{1}{3}c(T_2^3 - T_1^3) + \tfrac{1}{4}d(T_2^4 - T_1^4) \\
&= (29.11 - 8.314)(800 - 200) - \tfrac{1}{2}(0.1961\times10^{-2})(800^2 - 200^2) \\
&\quad + \tfrac{1}{3}(0.4003\times10^{-5})(800^3 - 200^3) - \tfrac{1}{4}(0.8704\times10^{-9})(800^4 - 200^4) \\
&= 12{,}487 \text{ kJ/kmol}
\end{aligned}$$

$$\Delta u = \frac{\Delta\bar{u}}{M} = \frac{12{,}487 \text{ kJ/kmol}}{2.016 \text{ kg/kmol}} = \mathbf{6194\ kJ/kg}$$

(*b*) Using a constant c_p value from Table A-2b at the average temperature of 500 K,

$$\begin{aligned}
c_{v,\text{avg}} &= c_{v@500\text{ K}} = 10.389 \text{ kJ/kg}\cdot\text{K} \\
\Delta u &= c_{v,\text{avg}}(T_2 - T_1) = (10.389 \text{ kJ/kg}\cdot\text{K})(800 - 200)\text{K} = \mathbf{6233\ kJ/kg}
\end{aligned}$$

(*c*) Using a constant c_p value from Table A-2a at room temperature,

$$\begin{aligned}
c_{v,\text{avg}} &= c_{v@300\text{ K}} = 10.183 \text{ kJ/kg}\cdot\text{K} \\
\Delta u &= c_{v,\text{avg}}(T_2 - T_1) = (10.183 \text{ kJ/kg}\cdot\text{K})(800 - 200)\text{K} = \mathbf{6110\ kJ/kg}
\end{aligned}$$

4-66 A resistance heater is to raise the air temperature in the room from 7 to 23°C within 15 min. The required power rating of the resistance heater is to be determined.

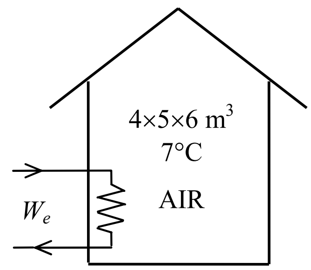
Assumptions **1** Air is an ideal gas since it is at a high temperature and low pressure relative to its critical point values of -141°C and 3.77 MPa. **2** The kinetic and potential energy changes are negligible, $\Delta ke \cong \Delta pe \cong 0$. **3** Constant specific heats at room temperature can be used for air. This assumption results in negligible error in heating and air-conditioning applications. **4** Heat losses from the room are negligible. **5** The room is air-tight so that no air leaks in and out during the process.

Properties The gas constant of air is R = 0.287 kPa.m^3/kg.K (Table A-1). Also, c_v = 0.718 kJ/kg.K for air at room temperature (Table A-2).

Analysis We take the air in the room to be the system. This is a closed system since no mass crosses the system boundary. The energy balance for this stationary constant-volume closed system can be expressed as

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$W_{\text{e,in}} = \Delta U \cong mc_{v,\text{avg}}(T_2 - T_1) \quad (\text{since } Q = \text{KE} = \text{PE} = 0)$$



or,

$$\dot{W}_{e,\text{in}}\Delta t = mc_{v,\text{avg}}(T_2 - T_1)$$

The mass of air is

$$V = 4 \times 5 \times 6 = 120 \text{ m}^3$$

$$m = \frac{P_1 V}{RT_1} = \frac{(100 \text{ kPa})(120 \text{ m}^3)}{(0.287 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(280 \text{ K})} = 149.3 \text{ kg}$$

Substituting, the power rating of the heater becomes

$$\dot{W}_{e,\text{in}} = \frac{(149.3 \text{ kg})(0.718 \text{ kJ/kg}\cdot^\circ\text{C})(23-7)^\circ\text{C}}{15 \times 60 \text{ s}} = \mathbf{1.91 \text{ kW}}$$

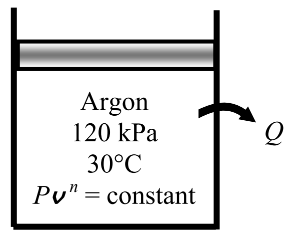
Discussion In practice, the pressure in the room will remain constant during this process rather than the volume, and some air will leak out as the air expands. As a result, the air in the room will undergo a constant pressure expansion process. Therefore, it is more proper to be conservative and to use ΔH instead of using ΔU in heating and air-conditioning applications.

4-74 Argon is compressed in a polytropic process. The work done and the heat transfer are to be determined.

Assumptions **1** Argon is an ideal gas since it is at a high temperature and low pressure relative to its critical point values of 151 K and 4.86 MPa. **2** The kinetic and potential energy changes are negligible, $\Delta ke \cong \Delta pe \cong 0$.

Properties The properties of argon are $R = 0.2081$kJ/kg·K and $c_v = 0.3122$ kJ/kg·K (Table A-2*a*).

Analysis We take argon as the system. This is a *closed system* since no mass crosses the boundaries of the system. The energy balance for this system can be expressed as



$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{system}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc. energies}}}$$

$$Q_{in} - W_{b,out} = \Delta U = mc_v(T_2 - T_1)$$

Using the boundary work relation for the polytropic process of an ideal gas gives

$$w_{b,out} = \frac{RT_1}{1-n}\left[\left(\frac{P_2}{P_1}\right)^{(n-1)/n} - 1\right] = \frac{(0.2081\,\text{kJ/kg}\cdot\text{K})(303\,\text{K})}{1\text{-}1.2}\left[\left(\frac{1200}{120}\right)^{0.2/1.2} - 1\right] = -147.5\,\text{kJ/kg}$$

Thus,

$$w_{b,in} = \mathbf{147.5\ kJ/kg}$$

The temperature at the final state is

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(n\text{-}1)/n} = (303\,\text{K})\left(\frac{1200\,\text{kPa}}{120\,\text{kPa}}\right)^{0.2/1.2} = 444.7\,\text{K}$$

From the energy balance equation,

$$q_{in} = w_{b,out} + c_v(T_2 - T_1) = -147.5\,\text{kJ/kg} + (0.3122\,\text{kJ/kg}\cdot\text{K})(444.7 - 303)\text{K} = -103.3\,\text{kJ/kg}$$

Thus,

$$q_{out} = \mathbf{103.3\ kJ/kg}$$