FINAL EXAM

Instructions:

- Time = 3 hours
- **CLOSED BOOK TEST.**
- **Non programmable calculators without built-in functions for integration differentiation, simultaneous equation solvers, solver are allowed.**
- **Programmable calculators are not**
- Equation sheets are attached in the back
- ***In the numerical problems you need to clearly show all your calculation, tables of the results are not satisfactory.***
- **Unless other wise stated, all calculations are to be carried using at least four decimal places, otherwise loose at least 20% of the marks.**
- **If at all possible answer the numerical questions within the question sheets, you can write on both sides.**
- **The exam question sheets are to be handed in along with any booklets.**
- ***If a set of simultaneous equations need to be solved use the two iterations of the Gauss Seidel method.***

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*The calculator brand and model that I am using is*_____

I _____*have read the above instructions*

Print name and student number

Signature

PART A (28 points)

Read carefully and answer 7 out of the 8 questions

1. (4 points) What is an empirical model? Can a model be both empirical and mechanistic? EXPLAIN.

2. (4 points) In regressing linear models list the four best ways of assessing if a particular linear model fits the data better than another linear model.

3. (4 points) Why efficient computer codes use factorization methods to solve matrices rather than Gauss-Elimination? What kind of method works just for tridiagonal matrices, such as those produced by the finite difference methods for boundary value ODE problems?

4. (4 points) The textbook covers linear splines, quadratic splines and cubic splines. Why are fourth order splines not used? EXPLAIN.

5. (4 points) Describe in detail the two key advantages of the Runge-Kutta Fehlberg Method over the standard fourth order Runge-Kutta ?

6.(4 points) What are the characteristics of the Brent method?

7. (4 points) What is the most efficient (i.e. highest accuracy per number of function evaluations) method for the integration of functions? EXPLAIN

8. (4 points) What is the generic name for errors arising from the way computers conduct arithmetic operations? Describe one situation involving a specific numerical method when this type of problem may arise.

PART B

B-1. (9 points) A nonlinear equation solving technique yields the following results

Iteration #	Xnew
Initial guess	0.1
1	0.06136452
2	0.05482266
3	0.05471793

a) (8 points) Given that the true answer is 0.05471792, determine the rate of convergence of this technique based on the results of the three iterations shown.
b) (1 point) Which technique have we covered that has similar rates of convergence?

B-2. (9 points) Solve for x and y at t=0.2 using the midpoint method with $\Delta t=0.2$

$$\frac{dx}{dt} = -y + t^2 \qquad x(0) = 1$$

$$\frac{dy}{dt} = x + 1 \qquad y(0) = 0$$

B3. (6 points) Indicate **in detail** (but do not calculate) the method or combination of methods you will give the **greatest accuracy integration** of the data set below (i.e., for the integration from x=0 to x=21).

x	F(x)
0	0
1	1
2	2
4	4
6	6
7	17
10	10
12	12
14	14
16	16
18	18
20	20
21	17

B.3 (17 points)

a. (7.5 points) Complete the following forward difference table:

x	y	$\in^1$y	$\in^2$y	$\in^3$y	$\in^4$y	$\in^5$y
0	_____					
		_____				
5	_____		0.0013			
		0.0888		_____		
10	_____		_____		0.0002	
		_____		_____		-0.0002
15	_____		_____		_____	
		_____		0.0017		
20	_____		_____			
		_____				
25	0.4663					

b. (8 points) Given the experimental data (x,y) provided in the above table, use the Newton-Gregory forward difference interpolating polynomials of degree three to estimate f(2.5)

c. (1.5 points) Explain the criteria for selecting the degree of polynomial to be used.

B-4. (18 points) Estimate the first derivative of f(x) at x=2 with an error of magnitude $O(h^6)$ using the Richardson Extrapolation Method (coupled with the central difference scheme). Let the first h =2.

$$f(x) = 2^x$$

SHOW ALL YOUR CALCULATIONS IN DETAIL OR LOOSE MARKS
Carry all calculations to at least six decimal places

B5 (13 POINTS) Write the VBA code to implement a general Regula Falsi Method for solving a non-linear function that contains three constants (called A, P, and n) that are also arguments of the subroutine. Write in detail not just a pseudo-code. Assume the function is the rearranged annual worth equation

$$\mathbf{f(i)} = 0 = 1 - \frac{\mathbf{A}}{\mathbf{P}} \bullet \frac{\mathbf{i} \cdot (\mathbf{i}+1)^{\mathbf{n}}}{(1+\mathbf{i})^{\mathbf{n}} - 1}$$ and we want to solve for i.